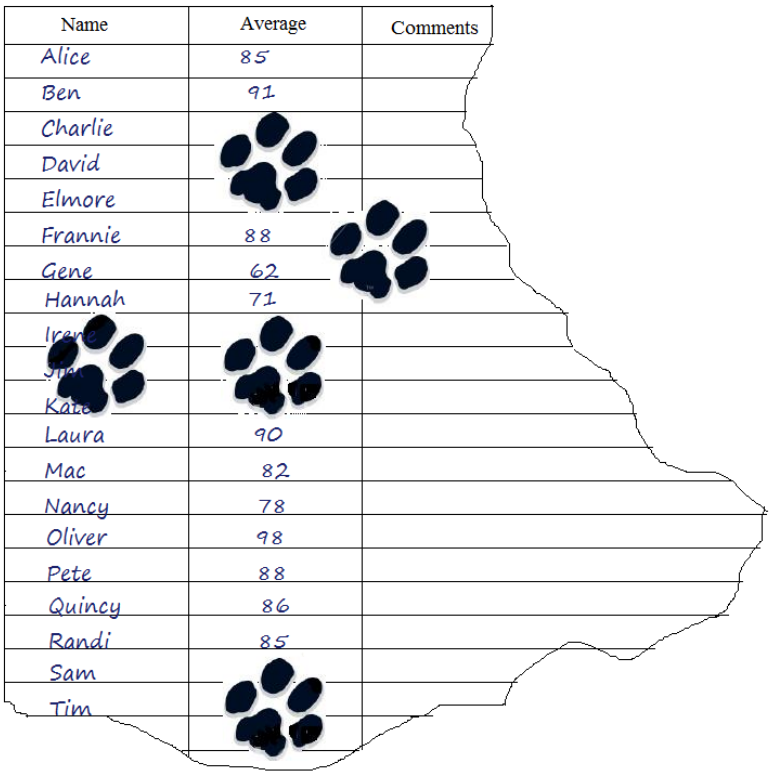
Statistics Review

Practice Questions and Exercises (with Solutions)



Name	Average	Comments
Alice	85	
Ben	91	
Charlie		
David		
Elmore		
Frannie	88	
Gene	62	
Hannah	71	
Irene		
Jack		
Kate		
Laura	90	
Mac	82	
Nancy	78	
Oliver	98	
Pete	88	
Quincy	86	
Randi	85	
Sam		
Tim		

Topics include mean, median, mode, range, standard deviation, z-score, probability, box and whiskers, and more.

Statistics Review Questions

1) Find the mean, median, mode, range, variance, and standard deviation of this sample.

3, 5, 10, 7, 5, 14, 12

2) Find the mean, median, mode, range, variance, and standard deviation of this population.

-6, 4, 0, 6, 8, 14, 14, 8

Assuming this is a normal distribution, what is the z-score of 10?

a z-score of -1 signifies what value?

3) Find the mean and median of: 8, -6, -4, 2

a) mean: -5 median: -5

b) mean: 0 median: -5

c) mean: 0 median: -1

d) mean: 0 median: 1

e) mean: -5 median: 0

4) In each pair of data samples, determine whether the statistic is >, <, or =

Interpreting mean, median, mode and range

A 5 5 5 5

B 60 60 60 60

mean: A ____ B

variance: A ____ B

range: A ____ B

C 1 2 3 4 5

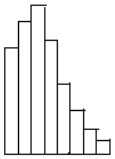
D 1 3 3 3 5

mean: C ____ D

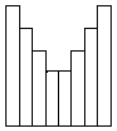
variance: C ____ D

range: C ____ D

E



F



mean: E _____ F

variance: E _____ F

range: E _____ F

G 1 1 1 10

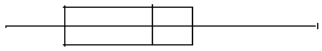
H 1 10 10 10

mean: G _____ H

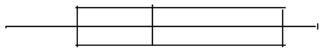
variance: G _____ H

range: G _____ H

I



J



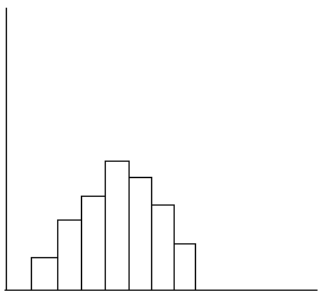
mean: I _____ J

range: I _____ J

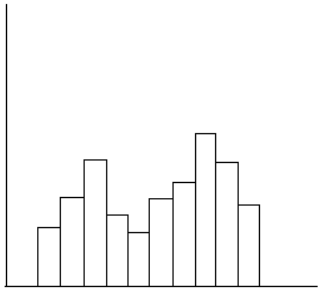
median: I _____ J

5) Match each histogram to the corresponding boxplot

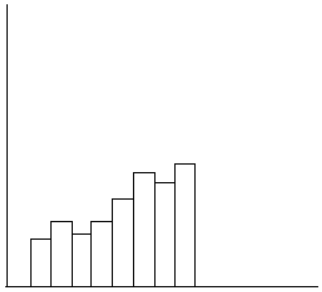
1)



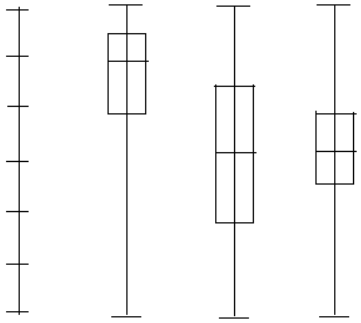
2)



3)



A B C



Mean, Median, Mode, and Range

6) Determine the changes....

a) If data in a sample is all increased by 5, then the

mean

median

mode

standard deviation

range

b) If data in a sample is all tripled, then the

mean

median

mode

standard deviation

range

c) If the largest term in the data is removed, then the

mean

median

mode

standard deviation

range

7) A sample of 100 households were asked "how many cars do you own?"
The table shows the response.

# of cars	Frequency
0	12
1	39
2	31
3	15
4	3

Determine the a) mean

b) median

c) mode

d) range

8) For the set of values, 23, 28, 34, 37,

a) what value would increase the range to 20?

b) What values would increase the median to 34?

9) Create data set for n = 11 (i.e. 11 terms)
median 15
bi-modal with modes 4 and 7

10) If the first 5 scores are 38, 51, 42, 50, and 45,

a) what score is needed to have a range of 15?

b) what score is needed to have a mean of 50?

Statistics Review

My dog ate part of the teacher's grade book.
Here is a portion of a page containing names and grades from the statistics class.

Answer the following:

1) Sample size

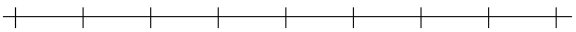
2) Mean, Median, Mode, and Range

3) Variance and Standard Deviation

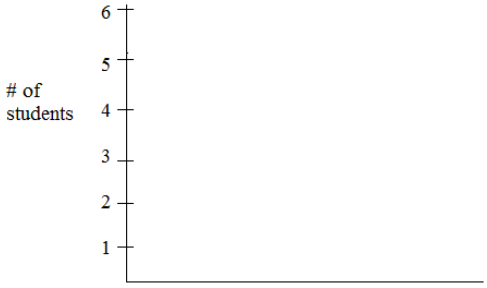
Name	Average	Comments
Alice	85	
Ben	91	
Charlie		
David		
Elmore		
Frannie	88	
Gene	62	
Hannah	71	
Irene		
[illegible]		
Kate		
Laura	90	
Mac	82	
Nancy	78	
Oliver	98	
Pete	88	
Quincy	86	
Randi	85	
Sam		
Tim		

Sketch the following:

a) boxplot ("box and whiskers") -- identify the quartiles, IQR (Interquartile Range), and any outliers



b) histogram showing grade distribution



c) stem and leaf diagram

Stem	*Leaves*

Data Analysis and measurements

A) Sketch the boxplot ('box and whiskers') and find the outliers in the set:

14, 32, 40, 41, 42, 44, 46, 54, 70

B) Find the standard deviation of 7, 11, 13, 16, 18. (NO CALCULATOR)

C) In a normal distribution, what percentage is

a) above the mean

b) is within 1 standard deviation of the mean

c) is less than the value marking 1 standard deviation below the mean

D) Given the sample set { 3, 7, 6, 4, 10, 4, 1 }

Find the following: mean, median, mode, range

box and whiskers plot

Interquartile range. outliers?

standard deviation

z-score of x = 6

E) Here is a stem and leaf diagram...

Construct a box plot (box and whiskers).
Label the median, quartiles, and any outliers

0	2, 4, 5
1	2, 4, 4, 6, 7
2	3, 5, 5
3	0, 1
4	2
5	9

Probability and Counting Questions

1) Two teams meet in a 7 game world series.
If the teams are evenly matched (i.e. the chance of either winning a game is 1/2),
then what is the probability the series goes 7 games?

2) A game: Flip 5 coins simultaneously... If 1, 2, 3, or 4 coins are heads, you win 1 dollar...
If 0 or all 5 coins are heads, then you lose 12 dollars...

Is it a 'good' bet?

3) Which odds are better: A - drawing an Ace or a Club
B - rolling a 2, 3, 7, or 11 with two ordinary dice ?

4) Box #1 has 2 red blocks and 1 white block

Box #2 has 3 red blocks and 2 white blocks

A person randomly picks a block from Box #1 and places it into Box #2...
Then, if a block is randomly picked from Box #2, what is the probability it is red?

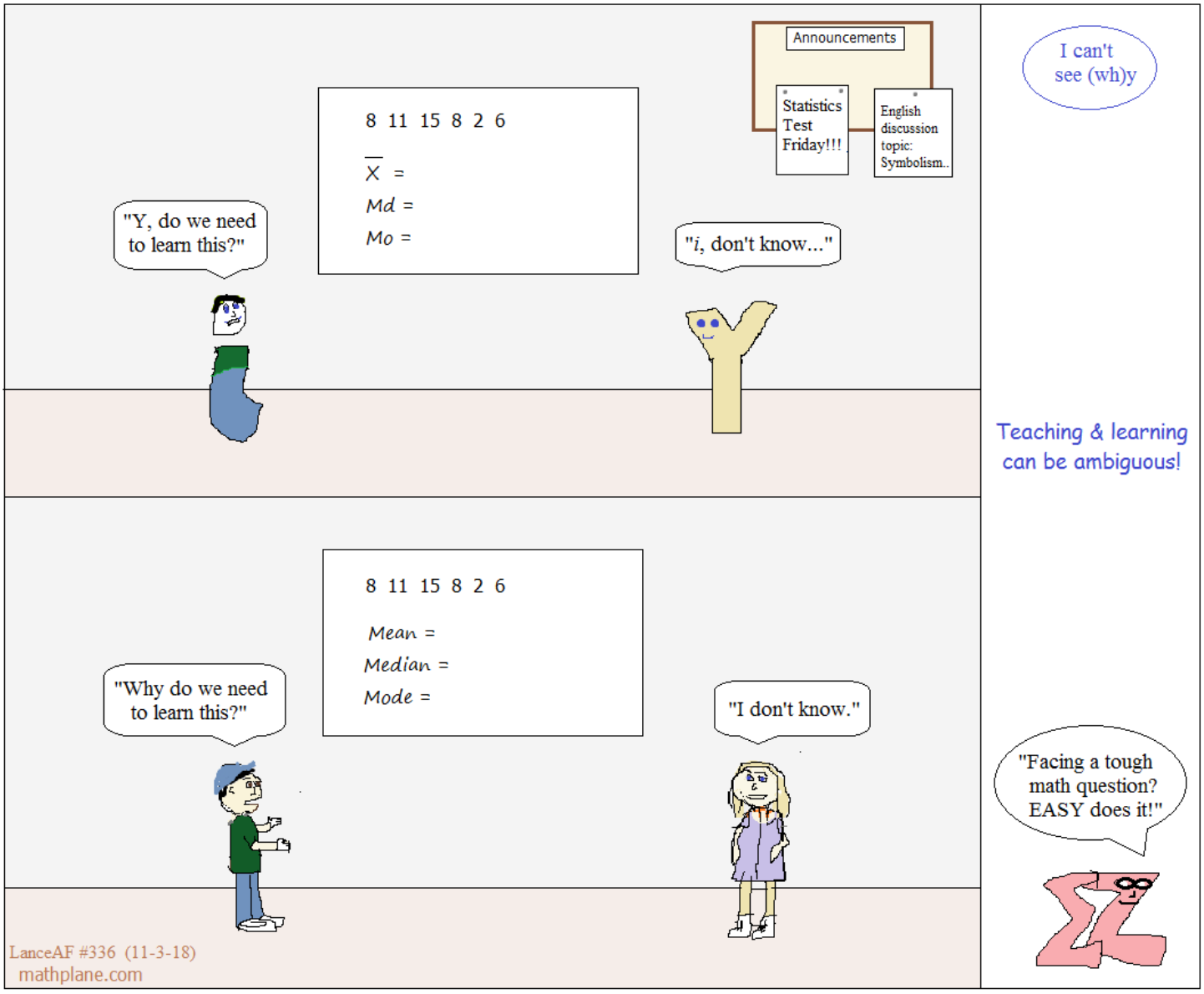
5) Joe is batting .350 and Carl is batting .250...

What is the P(Carl goes 1 for 4)?

What is the P(Joe goes 1 for 4)?

What is the P(both go 1 for 4)?

What is the P(EITHER goes 3 for 4)?


Announcements
Statistics Test Friday!!!
English discussion topic: Symbolism..
I can't see (wh)y
8 11 15 8 2 6
X̄ =
Md =
Mo =
"Y, do we need to learn this?"
"i, don't know..."
Teaching & learning can be ambiguous!
8 11 15 8 2 6
Mean =
Median =
Mode =
"Why do we need to learn this?"
"I don't know."
"Facing a tough math question? EASY does it!"
LanceAF #336 (11-3-18)
mathplane.com

SOLUTIONS-→

Statistics Review Questions

SOLUTIONS

1) Find the mean, median, mode, range, variance, and standard deviation of this sample.

3, 5, 10, 7, 5, 14, 12

mean: $\frac{3+5+10+7+5+14+12}{7} = 8$

median: the middle term is 7

mode: the most often term is 5

range: the span from min to max is 14 - 3 = 11

to find variance of sample:

$$\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

$n = 7$

$\bar{x} = 8$

x_i	3	5	10	7	5	14	12	
$x_i - \bar{x}$	-5	-3	2	-1	-3	6	4	
$(x_i - \bar{x})^2$	25	9	4	1	9	36	16	= 100

variance is $\frac{100}{7} = 14.28$ std dev is $s\sqrt{\frac{100}{6}} = 4.08$

2) Find the mean, median, mode, range, variance, and standard deviation of this population.

-6, 4, 0, 6, 8, 14, 14, 8

mean: $\frac{-6+4+0+6+8+14+14+8}{8} = 6$

median: since there are even number of terms, the average of the middle terms: 6 & 8 ---> 7

mode: there are 2 modes: 8 and 14

range: -6 to 14 ----> 20

$n = 8$ $\mu = 6$

x_i	-6	4	0	6	8	14	14	8	
$x_i - \mu$	-12	-2	-6	0	2	8	8	2	
$(x_i - \mu)^2$	144	4	36	0	4	64	64	4	= 320

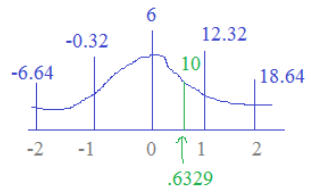
variance is $\frac{320}{8} = 40$ std dev is $\sigma\sqrt{\frac{320}{8}} = 6.32$

Assuming this is a normal distribution, what is the z-score of 10?

$z = \frac{x - \mu}{\sigma}$ $\frac{\text{term} - \text{mean}}{\text{standard deviation}} = \frac{10 - 6}{6.32} = .6329$

a z-score of -1 signifies what value?

$z = \frac{x - \mu}{\sigma}$ $-1 = \frac{x - 6}{6.32}$ ----> $x = -.32$



3) Find the mean and median of: 8, -6, -4, 2

a) mean: -5 median: -5

b) mean: 0 median: -5

c) mean: 0 median: -1

d) mean: 0 median: 1

e) mean: -5 median: 0

The mean is the "average": $\frac{8 + (-6) + (-4) + 2}{4} = \frac{0}{4} = 0$

The median is the "middle term":

in ascending order: -6, -4, 2, 8
since there is an even number of terms, take the average of the middle terms

$\frac{(-4) + 2}{2} = -1$

4) In each pair of data samples, determine whether the statistic is >, <, or =

SOLUTIONS

Interpreting mean, median, mode and range

A 5 5 5 5

B 60 60 60 60

mean: A < B

variance: A = B

range: A = B

C 1 2 3 4 5

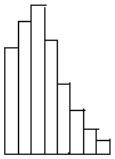
D 1 3 3 3 5

mean: C = D

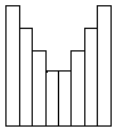
variance: C > D

range: C = D

E



F



mean: E < F

variance: E < F

range: E = F

G 1 1 1 10

H 1 10 10 10

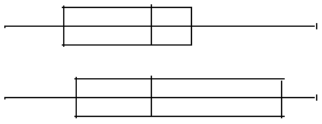
mean: G < H

variance: G = H

range: G = H

I

J



mean: I < J

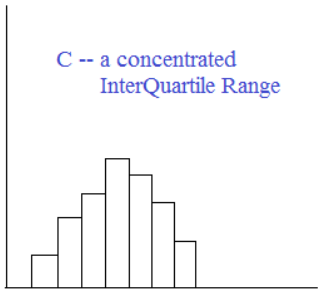
range: I = J

median: I = J

1/4 of J is at the highest part

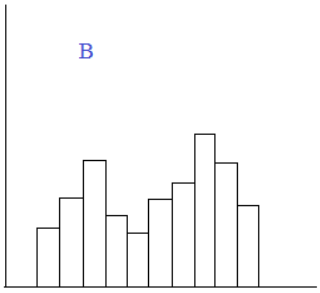
5) Match each histogram to the corresponding boxplot

1)



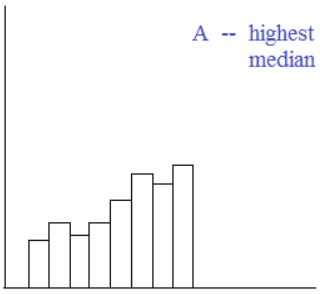
C -- a concentrated InterQuartile Range

2)

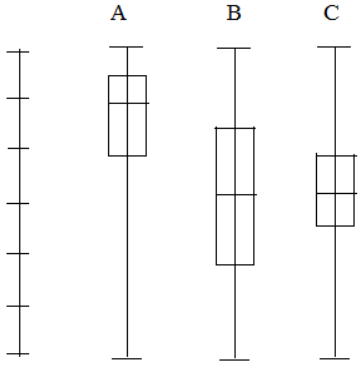


B

3)



A -- highest median



Mean, Median, Mode, and Range

6) Determine the changes....

a) If data in a sample is all increased by 5, then the

mean	up 5
median	up 5
mode	up 5
standard deviation	same
range	same

b) If data in a sample is all tripled, then the

mean	tripled
median	tripled
mode	tripled
standard deviation	tripled
range	tripled

c) If the largest term in the data is removed, then the

mean	lessens
median	same or lower
mode	same or lower
standard deviation	lower
range	lower (or, the same if the largest value has more than one term)

7) A sample of 100 households were asked "how many cars do you own?" The table shows the response.

SOLUTIONS

# of cars	Frequency	
0	12	0 x 12 = 0
1	39	1 x 39 = 39
2	31	2 x 31 = 62
3	15	3 x 15 = 45
4	3	4 x 3 = 12
		158

Determine the
a) mean 15.8
b) median 1
c) mode 1
d) range 4

8) For the set of values, 23, 28, 34, 37,

a) what value would increase the range to 20?

There are 2 answers!

17 on the lower side or 43 on the upper side

b) What values would increase the median to 34?

Any value $\geq$ 34

9) Create data set for n = 11 (i.e. 11 terms)
median 15
bi-modal with modes 4 and 7

There are many answers... here is one:

4, 4, 7, 7, 8, 15, 20, 21, 22, 23, 24

10) If the first 5 scores are 38, 51, 42, 50, and 45,

a) what score is needed to have a range of 15?

53 OR 36

b) what score is needed to have a mean of 50?

If the 6 scores have a mean of 50, then the total will be 6 x 50 = 300...

Right now, the total is 38 + 51 + 42 + 50 + 45 = 226

Therefore, the 6th score must be 74...

Statistics Review

My dog ate part of the teacher's grade book.
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SOLUTIONS

Answer the following:

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Randi	85	
Sam		
Tim		

1) Sample size

We can see 12 grade averages, so sample size n = 12

2) Mean, Median, Mode, and Range

62 71 78 82 85 85 86 88 88 90 91 98

Mean: Total = 1004 1004/12 = 83.67

Median: middle terms are 85 and 86 ---> median = 85.5

Mode: 85 and 88 are shown twice, so bimodal: 85, 88

Range: 98 - 62 = 36

3) Variance and Standard Deviation

83.67 is the mean $\bar{x}$

$62 - 83.67 = 21.67 \longrightarrow 469.6$

$71 - 83.67 = 12.67 \longrightarrow 160.5$

etc...

$$\text{variance} = \frac{\sum_{i=1}^{12} (x_i - \bar{x})^2}{12} = \frac{1010.67}{12} = 84.22$$

then, standard deviation of a *sample* $\sqrt{\frac{1010.67}{11}} = 9.585$

(note: the standard deviation of a *population* is the square root of the variance) 9.177

Sketch the following:

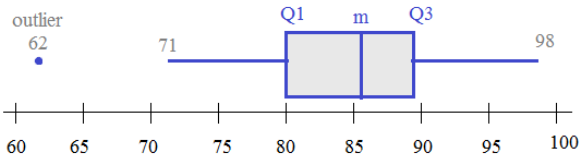
a) boxplot ("box and whiskers") -- identify the quartiles, IQR (Interquartile Range), and any outliers

- Q1 is median of bottom half
- Q3 is median of top half
- Outliers are beyond 1.5 of IQR

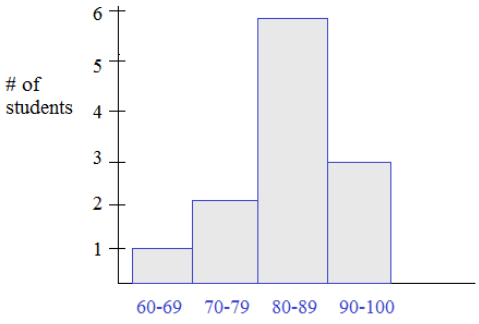
median: 85.5
Q1: 80
Q3: 89

IQR: 9

outlier: 1.5 x 9 = 13.5
so, under 66.5 or over 102.5



b) histogram showing grade distribution



c) stem and leaf diagram

Stem	*Leaves*
6	2
7	1, 8
8	2, 5, 5, 6, 8, 8
9	0, 1, 8

A) Sketch the boxplot ('box and whiskers') and find the outliers in the set:

14, 32, 40, 41, 42, 44, 46, 54, 70

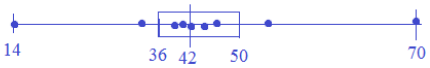
SOLUTIONS

For the boxplot, we need:

median: 42

Q1: (the median of lower half 14, 32, 40, 41) -- 36
Q3: (the median of upper half 44, 46, 54, 70) -- 50

minimum: 14 maximum: 70



To find the outliers, we identify the IQR (InterQuartile Range)...
Q1 = 36
Q3 = 50
IQR = 14

Then, we find 1.5 x IQR = 1.5 x 14 = 21

And, finally, add 21 to the Q3 and subtract 21 from the Q1

36 - 21 = 15
50 + 21 = 71

These are the boundaries...

Since 14 is outside the boundary, it is an outlier!

B) Find the standard deviation of 7, 11, 13, 16, 18. (NO CALCULATOR)

Step 1: Find the mean $7 + 11 + 13 + 16 + 18 = 65 / 5 = 13$

Step 2: Find squared differences and add them... $6^2 + 2^2 + 0^2 + 3^2 + 5^2 = 74$

Step 3: Since this is a sample, divide by (n - 1)... $74/4 = 18.5$

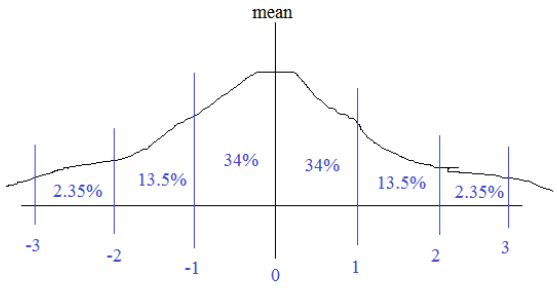
Step 4: Square root ... $\sqrt{18.5} = 4.30$

C) In a normal distribution, what percentage is

a) above the mean 50%

b) is within 1 standard deviation of the mean 68%

c) is less than the value marking 1 standard deviation below the mean 16%



D) Given the sample set { 3, 7, 6, 4, 10, 4, 1 }

Find the following: mean, median, mode, range

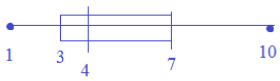
box and whiskers plot

Interquartile range. outliers?

standard deviation

z-score of x = 6

mean $(\bar{X}) = \frac{35}{7} = 5$ median ('middle') = 4 mode ('most often') = 4 range = 10 - 1 = 9



standard deviation: 2.94

IQR = (7 - 3) = 4
1.5 x 4 = 6
There are no outliers... (outside of -3 or 13)

z-score for x = 6 $\frac{6 - 5}{2.94} = .34$

value mean

std dev

E) Here is a stem and leaf diagram...

Construct a box plot (box and whiskers).
Label the median, quartiles, and any outliers

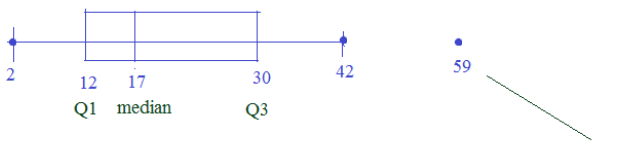
0	2, 4, 5
1	2, 4, 4, 6, 7
2	3, 5, 5
3	0, 1
4	2
5	9

there are 15 terms, so the 8th term is the median...

2, 4, 5, 12, 14, 14, 16, 17

then, to find the lower quartile, find the median of the lower 7 terms.. 12

And, to find the upper quartile, find the median of the upper 7 terms.. 30



Interquartile range (IQR) = 30 - 12 = 18
so, the outliers must be outside of 1.5 x IQR of each quartile... 1.5 x 18 = 27

12 − 27 = -25 30 + 27 = 57

Probability and Counting Questions

SOLUTIONS

1) Two teams meet in a 7 game world series.
If the teams are evenly matched (i.e. the chance of either winning a game is 1/2),
then what is the probability the series goes 7 games?

Since the teams are evening matched, we can ignore part of the binomial distribution...
In other words the probability of A, A, A, A is the same as A, B, A, B

$(.5)(.5)(.5)(.5) = (.5)(.5)(.5)(.5)$

So, the p(7 game series) = $\frac{\text{different ways world series goes 7 games}}{\text{total ways world series can play out}}$

	Team A wins series		Team B wins series	
4 games	${}_4C_4$	+	${}_4C_0$	= 2
	A wins 4 games		A wins 0 games	
5 games	${}_5C_4$	+	${}_5C_1$	= 10
	A wins 4 games (loses 1)		A wins 1 game (loses 4)	
6 games	${}_6C_4$	+	${}_6C_2$	= 15 + 15 = 30
7 games	${}_7C_4$	+	${}_7C_3$	= 35 + 35 = 70

Total ways the series can go: $2 + 10 + 30 + 70 = 112$

Total ways the series goes 7 games: $= 70$

P(7 games series) = $\frac{70}{112}$

Again, if the teams were NOT evenly matched, the probability would
be different (because the series/games outcomes would be weighted)

for example: suppose team A is favored and expected to win 3/4 of the time....

P(4 games) = ${}_4C_4\,(.75)^4\,(.25)^0 + {}_4C_0\,(.75)^0\,(.25)^4 = .316 + .004 = .320$

instead of ${}_4C_4\,(.5)^4\,(.5)^0 + {}_4C_0\,(.5)^0\,(.5)^4 = .125$

2) A game: Flip 5 coins simultaneously... If 1, 2, 3, or 4 coins are heads, you win 1 dollar...
If 0 or all 5 coins are heads, then you lose 12 dollars...

Is it a 'good' bet?

Heads	Probability	Expected gain
0	$\frac{1}{32}$	$-12 \cdot \frac{1}{32}$
1	$\frac{5}{32}$	$1 \cdot \frac{5}{32}$
2	$\frac{10}{32}$	$1 \cdot \frac{10}{32}$
3	$\frac{10}{32}$	$1 \cdot \frac{10}{32}$
4	$\frac{5}{32}$	$1 \cdot \frac{5}{32}$
5	$\frac{1}{32}$	$-12 \cdot \frac{1}{32}$

Yes, it's a good bet...

the sum is $\frac{6}{32}$

Each play the expected gain is approximately 18.75 cents

3) Which odds are better: A - drawing an Ace or a Club
B - rolling a 2, 3, 7, or 11 with two ordinary dice ?

SOLUTIONS

A: P(drawing an Ace or a Club) = P(Ace) + P(Club) − P(Ace & Club)

Ace and Club are NOT mutually exclusive $= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = .308$

B: P(2, 3, 7, or 11) = P(2) + P(3) + P(7) + P(11)

2, 3, 7, and 11 are mutually exclusive $= \frac{1}{36} + \frac{2}{36} + \frac{6}{36} + \frac{2}{36} = \frac{11}{36} = .306$

A is slightly better!

4) Box #1 has 2 red blocks and 1 white block

Box #2 has 3 red blocks and 2 white blocks

A person randomly picks a block from Box #1 and places it into Box #2...
Then, if a block is randomly picked from Box #2, what is the probability it is red?

Case #1: Red first...

P(Red block then Red block): $\frac{2}{3} \cdot \frac{4}{6} = \frac{4}{9}$

Box #1 Box #2

Case #2: White first...

P(White block then Red block): $\frac{1}{3} \cdot \frac{3}{6} = \frac{1}{6}$

Box #1 Box #2

$\frac{4}{9} + \frac{1}{6} = \boxed{\frac{11}{18}}$

5) Joe is batting .350 and Carl is batting .250...

What is the P(Carl goes 1 for 4)?

What is the P(Joe goes 1 for 4)?

What is the P(both go 1 for 4)?

What is the P(EITHER goes 3 for 4)?

Binary probability....

Carl: 1 for 4 $\binom{4}{1}(.25)^1(.75)^3 = 4 \cdot .105 = .422$

Joe: 1 for 4 $\binom{4}{1}(.35)^1(.65)^3 = 4 \cdot .096 = .384$

P(BOTH go 1 for 4) = .422 x .384 = .162 (approx.)

P(either goes 3 for 4) = P(Carl goes 3 for 4 only) + P(Joe goes 3 for 4 only) + P(BOTH go 3 for 4)

Carl: 3 for 4 $\binom{4}{3}(.25)^3(.75)^1 = 4 \cdot .0117 = .047$

Joe: 3 for 4 $\binom{4}{3}(.35)^3(.65)^1 = 4 \cdot .0279 = .111$

P(Carl/not Joe) = .047 • .889 = .042

P(Joe/not Carl) = .111 • .953 = .106

P(Joe and Carl) = .047 • .111 = .005

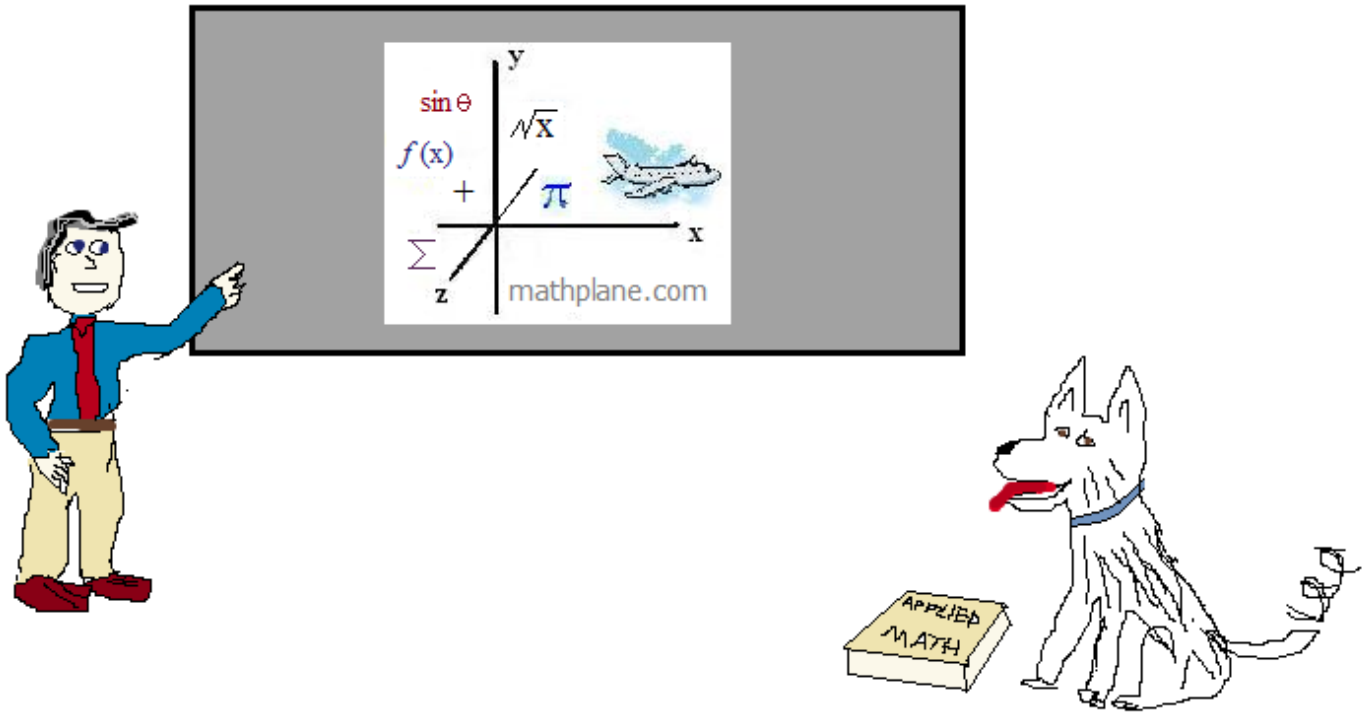
approximately: .042 + .106 + .005 = .153

approx 15%

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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