Absolute Value and Inequalities
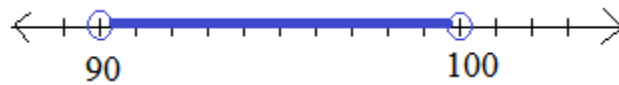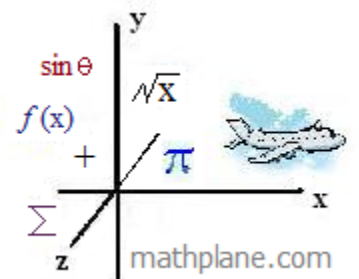
Notes, strategies, examples, and practice test (w/solutions)

1) Fill in the blank with $<$, $>$, or $=$

$$-(-5) \_\_\_\_ |-5|$$

$$(-5) \_\_\_\_ |-5|$$

$$|4-2| \_\_\_\_ |2-4|$$

$$-2|4| \_\_\_\_ -4|2|$$

$$|7-(-9)| \_\_\_\_ 7-|9|$$

$$|-3| \_\_\_\_ 2$$

$$-|2| \_\_\_\_ -(-2)$$

$$|-4| \_\_\_\_ -4$$

2) $|-5+3| - |-8| =$

    a) -6

    b) 0

    c) 6

    d) 10

    e) 16

3) $\dfrac{|-8|}{4} - \dfrac{|14|}{|2|} =$

$\dfrac{|-9|}{-3} - \dfrac{-9}{|-3|} =$

$-|8| - |-10| =$

$-|-8| - |10| =$

$|-8| - |-10| =$

Answers-→

1) Fill in the blank with $<$, $>$, or $=$ SOLUTIONS   Absolute Value Quick Review

$$-(-5) \underline{\ =\ } |-5|$$ $5 = 5$

$$(-5) \underline{\ <\ } |-5|$$ $-5 < 5$

$$|4 - 2| \underline{\ =\ } |2 - 4|$$ $2 = 2$

$$-2|4| \underline{\ =\ } -4|2|$$ $-8 = -8$

$$|7 - (-9)| \underline{\ >\ } 7 - |9|$$ $16 > -2$

$$|-3| \underline{\ >\ } 2$$ $3 > 2$

$$-|2| \underline{\ <\ } -(-2)$$ $-2 < 2$

$$|-4| \underline{\ >\ } -4$$ $4 > -4$

2) $|-5 + 3| - |-8| =$

       $|-2| - 8 =$

   a) -6

     b) 0 $2 - 8 = -6$

     c) 6

     d) 10

     e) 16

3) $\dfrac{|-8|}{4} - \dfrac{|14|}{|2|} =$ $\dfrac{8}{4} - \dfrac{14}{2} = \boxed{-5}$

   $\dfrac{|-9|}{-3} - \dfrac{-9}{|-3|} =$ $\dfrac{9}{-3} - \dfrac{-9}{3} = -3 - (-3) = \boxed{0}$

   $-|8| - |-10| =$ $-8 - 10 = \boxed{-18}$

   $-|-8| - |10| =$ $-8 - 10 = \boxed{-18}$

   $|-8| - |-10| =$ $8 - 10 = \boxed{-2}$

Notes and Examples-→

Solving Inequalities

Solve: Determine "Critical Values"

Graph: "Open Circles" or "Closed Circles"
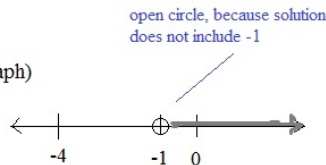
Check Answer: Test regions (Plug solutions into original equation)

Example:

$3x + 10 > -2x + 5$

(solve) $3x + 10 = -2x + 5$
$5x = -5$
$x = -1$

(graph)

open circle, because solution does not include -1

-4   -1   0

(check answer)

$3(-1) + 10 = -2(-1) + 5$
$7 = 7$ ✓

(test regions)

Test Left: Try -4

$3(-4) + 10 > -2(-4) + 5$
$-2 \not> 13$

NO

Test Right: Try 0

$3(0) + 10 > -2(0) + 5$
$10 > 5$

YES

Example: $3x > 6$ OR $\frac{-3}{2}(x + 2) \geq 4$
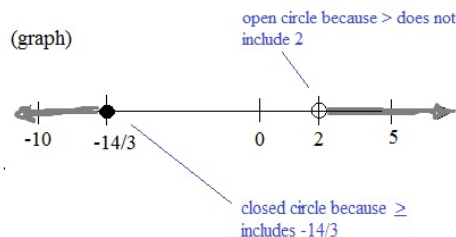
(solve) $3x = 6$
$x = 2$

$\frac{-3}{2}(x + 2) = 4$

$(x + 2) = 4\left(\frac{-2}{3}\right)$

OR $x + 2 = -8/3$

$x = -14/3$

(graph)

open circle because > does not include 2

-10   -14/3   0   2   5

closed circle because ≥ includes -14/3

(check critical points)

$3(2) = 6$ ✓

$-3/2 (-14/3 + 2) = 42/6 - 3 = 24/6$
$= 4$ ✓

(test regions)

Test Left: Try -10

$3(-10) > 6$? No

$-3/2 (-10 + 2) \geq 4$?
$12 > 4$ Yes

(Since -10 satisfies one of the "or" equations, the left side is a solution)

Test Middle: Try 0

$3(0) > 6$? No

$-3/2 (0 + 2) \geq 4$?
$-4/3 \not\geq 4$ No

(Since 0 satisfies neither equation, the middle is not a solution)

Test Right: Try 5

$3(5) > 6$? Yes

$-3/2 (5 + 2) \geq 4$?
$-21/2 \geq 4$ No

(Although 5 does not satisfy the second equation, it works in the first one. Therefore, the right region is a solution)

Example: $4y < 8$ AND $3y + 5 \geq y - 13$

(solve: find critical values)

$4y = 8$
$y = 2$

$3y + 5 = y - 13$
$2y = -18$
$y = -9$

(graph: "open and closed" circles -- test regions)

-9   0   2

("closed circle")   ("open circle")

(Quick check)

$4(2) = 8$ ✓

$3(-9) + 5 = (-9) - 13$ $-22 = -22$ ✓

Test Left with -10: $4(-10) < 8$? Yes
$3(-10) + 5 \geq -10 - 13$? No

Test Middle with 0: $4(0) < 8$? Yes
$3(0) + 5 \geq 0 - 13$? Yes

Test Right with 5: $4(5) < 8$? No
$3(5) + 5 \geq 5 - 13$? Yes

Since this is an 'AND' problem, must satisfy BOTH equations. Therefore, only the middle region works..

Solving Absolute Value Equations

Example: $|3x + 12| = 24$

(the absolute value is 'isolated' already)

"Positive" Answer: $3x + 12 = 24$
$3x = 12$
$\boxed{x = 4}$

"Negative" Answer: $3x + 12 = -24$
$3x = -36$
$\boxed{x = -12}$

Check solutions: $|3(4) + 12| = 24$
$24 = 24$ ✓

$|3(-12) + 12| = 24$
$|-24| = 24$
$24 = 24$ ✓

Steps:

1) Isolate Absolute Value

2) Solve for 'Positive Answer'

3) Solve for 'Negative Answer'

4) Check Solutions!
   (Plug into original equation)

Example: $2\left|x - 5\right| + 6 = 16$

Isolate the absolute value part $2\left|x - 5\right| + 6 = 16$

$2|x - 5| = 10$

$|x - 5| = 5$

"Positive" Answer: $x - 5 = 5$
$\boxed{x = 10}$

"Negative" Answer: $x - 5 = -5$
$\boxed{x = 0}$

Check Solutions: $2\left|(10) - 5\right| + 6 = 16$
$2 \cdot 5 + 6 = 16$
$16 = 16$ ✓

$2\left|(0) - 5\right| + 6 = 16$
$2 \cdot 5 + 6 = 16$
$16 = 16$ ✓

Example: $|3x + 4| + 6 = 2$

Isolate the absolute Value: $|3x + 4| + 6 = 2$

$|3x + 4| = -4$

"Positive" Answer: $3x + 4 = -4$
$3x = -8$
$x = -8/3$

"Negative" Answer: $3x + 4 = 4$
$3x = 0$
$x = 0$

No Solutions!
(absolute value result can never be negative)

Check Solutions!
$|3(-8/3) + 4| + 6 = 2$
$|-4| + 6 = 2$
$10 \neq 2$ ✗

$|3(0) + 4| + 6 = 2$
$4 + 6 = 2$
$10 \neq 2$ ✗

Absolute Value Equations: How many solutions?

Steps for solving absolute value equations:

1) Isolate the absolute value
2) "Split into negative and positive equations"
3) Solve
4) Check your answer(s)!

Two solutions:

$$|5x + 4| - 10 = -5$$

(isolate the absolute value)

$$|5x + 4| = 5$$

('positive equation')

$$5x + 4 = 5$$
$$5x = 1$$
$$x = \frac{1}{5}$$

('negative equation')

$$5x + 4 = -5$$
$$5x = -9$$
$$x = \frac{-9}{5}$$

(Check)

$$|5(1/5) + 4| - 10 = -5$$
$$5 \ - 10 = -5$$
$$\checkmark \quad -5 = -5$$

$$|5(-9/5) + 4| - 10 = -5$$
$$|-5| \ - 10 = -5$$
$$\checkmark \quad 5 - 10 = -5$$

One solution:

$$2|x + 3| + 8 = 8$$
$$2|x + 3| = 0$$
$$|x + 3| = 0$$
$$x + 3 = 0$$
$$x = -3$$

$$2|(-3) + 3| + 8 = 8$$
$$2|0| + 8 = 8$$
$$\checkmark \quad 8 = 8$$

After isolating the absolute value,

for $|ax + b| = c$

if $c > 0$, then 2 solutions
$c = 0$, then 1 solution
$c < 0$, then no solutions

No solutions:

$$5|3x + 2| + 20 = 10$$

(isolate the absolute value)

$$5|3x + 2| = -10$$

$$|3x + 2| = -2$$ ⬅ Absolute value output is never negative!

('positive equation')

$$3x + 2 = -2$$
$$3x = 4$$
$$x = 4/3$$

('negative equation')

$$3x + 2 = 2$$
$$3x = 0$$
$$x = 0$$

$$5|3(4/3) + 2| + 20 = 10$$
$$5|6| + 20 = 10$$
$$\times \quad 30 + 20 = 10$$

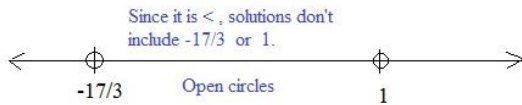$$5|3(0) + 2| + 20 = 10$$
$$5|2| + 20 = 10$$
$$\times \quad 30 = 10$$

Solving Absolute Value/Inequality Equations
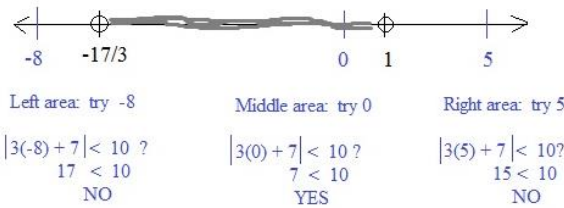
Example: $|3x + 7| < 10$

(solve) $|3x + 7| = 10$

$$3x + 7 = 10 \qquad 3x + 7 = {}^-10$$
$$3x = 3 \qquad\qquad 3x = -17$$
$$x = 1 \qquad\qquad x = {}^-17/3$$

(graph)

Since it is <, solutions don't include -17/3 or 1.

-17/3   Open circles   1

(test regions)

-8   -17/3   0   1   5

Left area: try -8   Middle area: try 0   Right area: try 5

$|3(-8) + 7| < 10$? $|3(0) + 7| < 10$? $|3(5) + 7| < 10$?
$17 < 10$ $7 < 10$ $15 < 10$
NO   YES   NO

(check answers -- "critical points")

$$|3(-17/3) + 7| = |-17 + 7| = 10 \checkmark$$
$$|3(1) + 7| \quad = |3 + 7| = 10 \checkmark$$

Example: $3|x - 7| + 4 \geq 10$

(solve)

Isolate the absolute value terms.
Then, solve equal to 'positive answer'
and equal to 'negative answer'

$$3|x - 7| + 4 = 10$$
$$3|x - 7| = 6$$
$$|x - 7| = 2$$

'positive' $x - 7 = 2$
           $x = 9$

'negative' $x - 7 = {}^-2$
           $x = 5$

(graph)

"critical values" are
$x = 5$ and $x = 9$

"closed circles" because solution
includes 5 and 9

5   9

(test regions)

0   5   7   9   12

Use:

0 (left) $3|(0) - 7| + 4 = 25 \geq 10$ YES

7 (middle) $3|(7) - 7| + 4 = 4 \ngeq 10$ NO

12 (right) $3|(12) - 7| + 4 = 19 \geq 10$ YES

(check answers)

$$3|(5) - 7| + 4 = 10 \checkmark$$
$$3|(9) - 7| + 4 = 10 \checkmark$$

(**Remember: plug values into <u>original</u> equation)

"Working Backwards": Determining the absolute value inequality

I. Absolute Value can represent distance on a number line

Example 1:
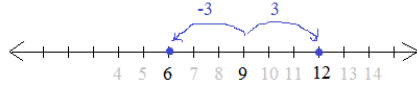
"6 is three units to the left of 9" $6 - 9 = -3$

"12 is three units to the right of 9" $12 - 9 = 3$

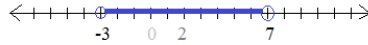The <u>absolute value</u> of both is 3, and the distance from 9 is 3.

$|X - 9| = 3$

"The distance from 9 (in either direction) is 3"

$X = 12$ $X = 6$

Example 2:

This number line graph represents a set of numbers <u>within</u> a distance.

The middle of the graph is the midpoint: 2

and, the distance to each endpoint is 5 units

$X - 2$ ("distance from 2")

$|X - 2| < 5$ ("All numbers X less than 5 units from 2")

Note: You can plug in values from the number line to verify the equation!

II. Identify the equation using absolute value inequalities

Step 1: Identify the midpoint (central value)
The midpoint of 8 and 16: $(8 + 16)/2 = 12$

Step 2: Determine the distance to each endpoint
The distance to each endpoint is 4 units.

Step 3: Construct the inequality

$|X - 12| \le 4$ "The distance between X and 12 must be less than or equal to 4"

Step 4: Check your work!

Plug in values to test endpoints and regions.

Endpoints: $|8 - 12| = 4$ $|16 - 12| = 4$ ✓

Regions: $|0 - 12| \not< 4$ $|10 - 12| < 4$ $|20 - 12| \not< 4$ ✓

III. Word Problem Applications

Example 1:   The length of a local football field must be between 90 and 100 yards. Write an *absolute value inequality* to represent all possible lengths.

Obviously, we could write the equation $90 < L < 100$ where
L is possible lengths.
But, the question asks for an absolute value inequality....
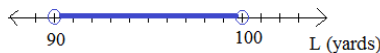
Step 1: Find midpoint.   95 yards

Step 2: Determine distance/length to each endpoint. ± 5 yards

Step 3: Construct the inequality $|L - 95| < 5$

Step 4: Check answers! (construct a number line and check values)

note: in this example, "between 90 and 100" did not include 90 and 100. There may be different interpretations.
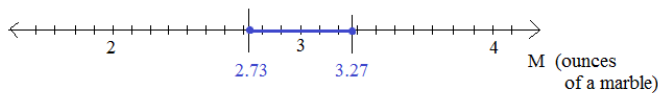
Example 2:   A marble manufacturer sets a machine to produce 3 ounce marbles. The company allows a tolerance of .27 ounces for any marble it sells. Write an absolute value inequality to represent the acceptable marbles. Then, graph on a number line.

The target value of marbles is 3 (ounces)..

The variation (tolerance) allowed is $\pm$ .27 (ounces)

$M - 3$

$|M - 3| \le .27$
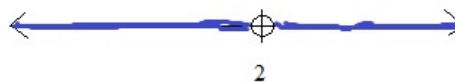
"Unusual Absolute Value Solutions"

1) $|3x - 6| - 48 > -48$

$|3x - 6| > 0$

$3x - 6 = 0$

$x = 2$

All Real Numbers <u>except</u> 2

2

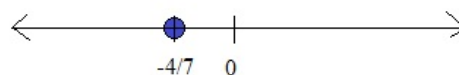2) $|2x - 4| - 10 \geq -13$

$|2x - 4| \geq -3$

All Real Numbers

3) $14 + |7x + 4| \leq 14$

$|7x + 4| \leq 0$

$7x + 4 = 0$

$x = -4/7$

Only ONE solution

-4/7   0

4) $2 + |.33756x + \sqrt{37}| < -4$

$|.33756x + \sqrt{37}| < -6$

$\emptyset$ No Solutions

$|\text{"anything"}| \not< -6$

"Slope of the Absolute Value Line"

$y = a|x - h| + k$ is the standard form where (h, k) is the vertex

Assume the vertex is on the origin (0, 0).
then,

    $y = a|x|$

        where a is the amount the parent function is
        "multiplied" or "stretched".

        (see graphs and tables)

Also, 'a' indicates the slope:

> a is the slope of the absolute value line on the
> right side of the vertex.
> And,
> -a is the slope of the absolute value line on the
> left side of the vertex.

If a < 0, then the graph 'faces down'

Slope on the left side of the vertex will be
positive. And, slope on the right side of the
vertex will be negative.

slope = -3   slope = 3

Parent function: $y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

$y = 3|x|$
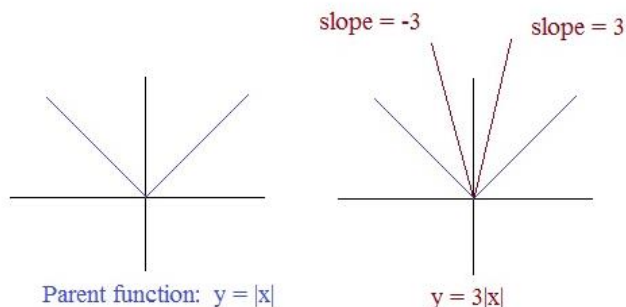
x	y
-2	6
-1	3
0	0
1	3
2	6

$y = -4|x|$

x	y
-2	-8
-1	-4
0	0
1	-4
2	-8

slope = 4   slope = -4
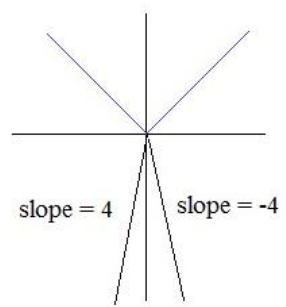
How many solutions would the equation $a|x + b| + c = d$ have if

   a) a < 0, c = d ?

   b) a < 0, c > d ?

   c) a > 0, c > d ?

First, subtract c from both sides: $a|x + b| = d - c$

Examples:

   a) If c = d, then d − c = 0 $a|x + b| = 0$

                       $|x + b| = 0$ (one solution!)

                  x must equal −b

$3|x + 7| + 8 = 8$
$3|x + 7| = 0$
$|x + 7| = 0$
$x = -7$

   b) If c > d, then $a|x + b| = $ 'negative number'
      and
      if a < 0, then $-|x + b| = $ 'negative number'   (two solutions!)

      Divide both sides by − : $|x + b| = $ 'negative'/'negative' = 'positive'

$-4|x + 4| + 12 = 8$
$-4|x + 4| = -4$
$|x + 4| = 1$
$x = -3$ or -5

   c) If c > d, then $a|x + b| = $ 'negative number'
      and   (NO solutions!)
      if a > 0, then $a|x + b|$ must be positive..

$6|x + 2| + 10 = 4$
$6|x + 2| = -6$
$|x + 2| = -1$
Impossible!

Variables inside and outside the absolute value sign

A) $5|4 - 3x| = 10x - 30$

Isolate the absolute value:

$$|4 - 3x| = 2x - 6$$

Split the equations:

$4 - 3x = 2x - 6$	or	$4 - 3x = -(2x - 6)$
$10 = 5x$		$4 - 3x = 6 - 2x$

Solve:

$x = 2$ $x = -2$

Check for extraneous:

$5|4 - 3(2)| = 10(2) - 30$ $\qquad$ $5|4 - 3(-2)| = 10(-2) - 30$

$5|-2| = 20 - 30$ $\qquad\qquad$ $5|10| = -20 - 30$

$10 = -10$? NO $\qquad\qquad$ $50 = -50$? NO

No solutions

B) $\frac{1}{3}|3x + 7| = 6x + 12$

$$|3x + 7| = 18x + 36$$

Isolate the absolute value:

$3x + 7 = 18x + 36$ $\qquad$ or $\qquad$ $3x + 7 = -(18x + 36)$

Split the equations:

$-29 = 15x$ $\qquad\qquad\qquad$ $21x = -43$

$\boxed{x = -29/15}$ $\qquad\qquad$ $x = -43/21$

Solve:

$\frac{1}{3}|3(-29/15) + 7| = 6(-29/15) + 12$ $\qquad$ $\frac{1}{3}|3(-43/21) + 7| = 6(-43/21) + 12$

Check for extraneous:

$\frac{1}{3}|-29/5 + 35/5| = -58/5 + 60/5$ $\qquad$ positive answer $\qquad$ negative answer

$2/5 = 2/5$ ✓ $\qquad\qquad\qquad\qquad$ Extraneous!

A Product of Sweden's Math Department

ABSOLUTE
Mathematics
VALUE

This superb operator was instilled in numerous math books.

Over 200 years of usage throughout the world.

Symbolized by $|X|$...

(100% proof) - it is IMPORTANT
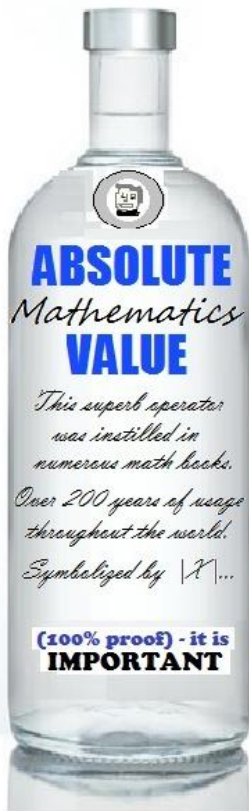
Absolutely, the best there is...

Serve with sub-zero temperatures..

Mix with any ingredient to bring out a positive result..

Remember to drink and calculate responsibly.

Practice Test and Solutions-→

Solve the following:

a) $|x + 2| = 14$

b) $|2x + 6| = 4x$

c) $|x + 5| = -x + 3$

d) $|x - 2| = |5 + x|$

e) $2|x + 1| = 5|x + 1| - 12$

f) $\dfrac{2|x + 7|}{5} = 12$

g) $|3x + 9| = x + 1$

h) $3 - 10|x - 5| = -17$

i) $|x + 6| = -|2x + 10| + 20$

mathplane.com

Quiz: Absolute Value, Inequalities, and the Number Line

I. Graph the following on a number line

a) $x > 5$
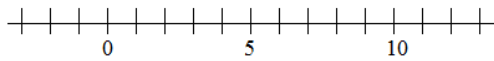
b) $2x \leq 4$
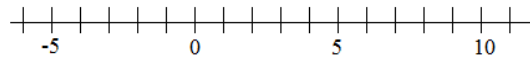
c) $3x + 2 < 9$
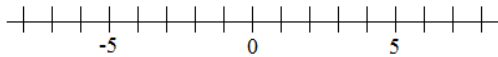
d) $x = -4$
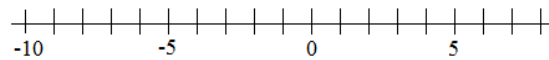
e) $-3x > 6$
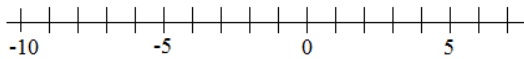
f) $-5 < x \leq 1$
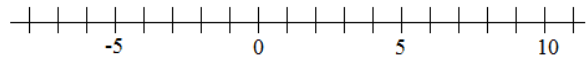
II. Absolute Values: Solve and graph the following

a) $|x + 3| < 6$
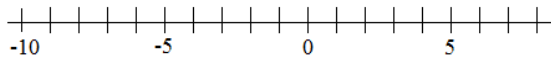
b) $|3x| < 18$
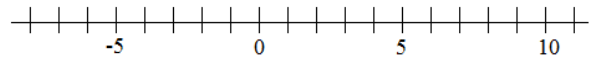
c) $2|x + 1| \geq 6$
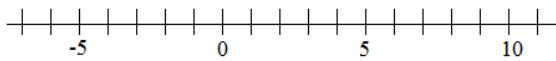
d) $|2x| + 8 < 4$
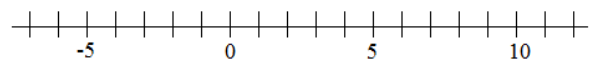
e) $|x - 3| + 4 \geq 9$
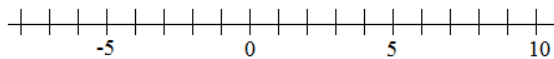
f) $|x + 7| = 1$
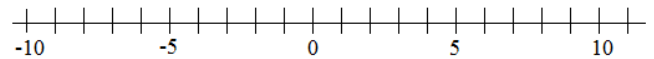
III. Solve and Graph

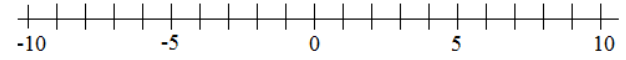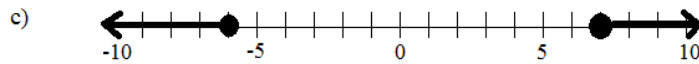a) $x > 4$ or $x < 0$
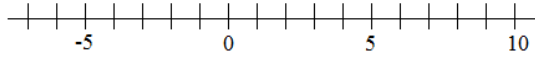
b) $x > 5$ and $x \leq 9$
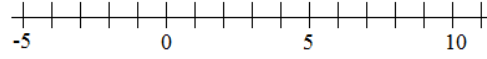
c) $|x + 4| > 0$
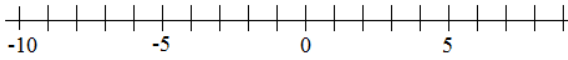
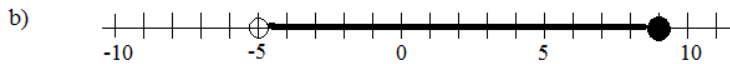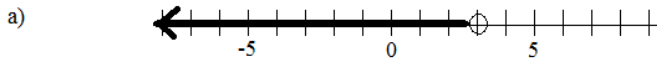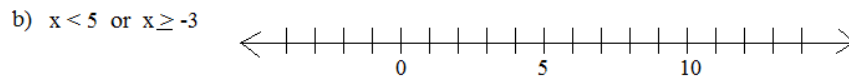d) $2|x + 1| + 6 > 10$
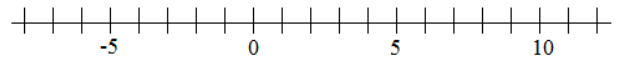
e) $-|x +3| > -5$
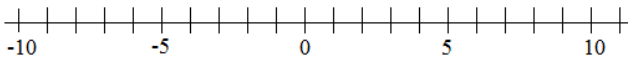
f) $|x + 6| \leq 0$
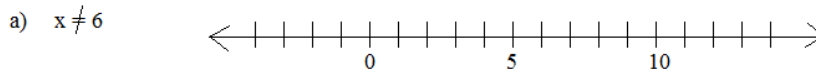

IV: Describe each graph with an equation

a)

b)
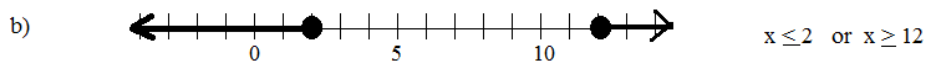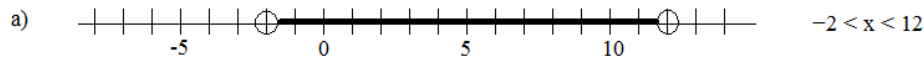

c)

BONUS:

1) Graph the following on a number line:

a) $x \neq 6$

b) $x < 5$ or $x \geq -3$

2) Use <u>Absolute Value Inequalities</u> to describe the following number line graphs:

a)

     $-2 < x < 12$

b)

     $x \leq 2$ or $x \geq 12$

V. More concept questions

a) $|x + 8| < 3x$

b) 'What is wrong with this?'

1) $|x - 4| + 5 > 7$

    $|x| + 5 > 11$

    $|x| > 6$

    $x > 6$ or $x < -6$

2) $3|x + 9| + 7 > 9$

    $|x + 9| + 7 > 3$

    $|x + 9| > -4$

    all real numbers

c) $8|2x + 4| - 3|2x + 4| > 5$

$7|x - 1| + 4|x - 1| = 44$

d) What is the solution of $2x < 14$ or $-3x < -9$?

    a) $3 < x < 7$

    b) $-7 < x < -3$

    c) $x < 3$ or $x > 7$

    d) all real numbers

e) *ALWAYS/SOMETIMES/NEVER*

For any real number N,

$|x - 3| = N$ will have 2 solutions

f) Solve $2\sqrt{3x - 4} < 16$

Solve the following:

Absolute Value Equations

a) $|x + 2| = 14$

"split the absolute value"

$x + 2 = 14$ OR $x + 2 = -14$

$x = 12$ $x = -16$

b) $|2x + 6| = 4x$

"Split the absolute value"

$2x + 6 = 4x$ $2x + 6 = -4x$

$x = 3$ $x = -1$

"Check for extraneous solutions"

If x = 3:   If x = -1:

$|2(3) + 6| = 4(3)$ $|2(-1) + 6| = 4(-1)$

$12 = 12$ ✓ $4 \neq -4$

c) $|x + 5| = -x + 3$

$x + 5 = -x + 3$ OR $x + 5 = -(-x + 3)$

$2x = -2$ $x + 5 = x - 3$

$x = -1$ $5 \neq -3$

Check answer:

$|(-1) + 5| = -(-1) + 3$

$4 = 4$ ✓

d) $|x - 2| = |5 + x|$

+ + $x - 2 = 5 + x$

$-2 = 5$ NO solution

+ − $x - 2 = -(5 + x)$

$3 = -2x$ $x = -3/2$

− + $-(x - 2) = 5 + x$

$-2x = 3$ $x = -3/2$

− − $-(x - 2) = -(5 + x)$

$-x + 2 = -x - 5$ NO solution

e) $2|x + 1| = 5|x + 1| - 12$

"Collect 'like' terms"

$-3|x + 1| = -12$

$|x + 1| = 4$

"Split and solve"

$x + 1 = 4$ $x + 1 = -4$

$x = 3$ or -5

f) $\dfrac{2|x + 7|}{5} = 12$

$2|x + 7| = 60$

$|x + 7| = 30$

$x + 7 = 30$ $x + 7 = -30$

$x = 23$ OR $x = -37$

(If you check both answers, you'll
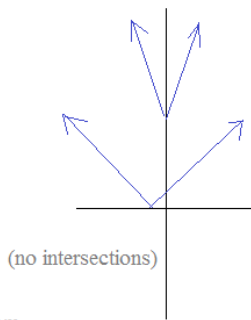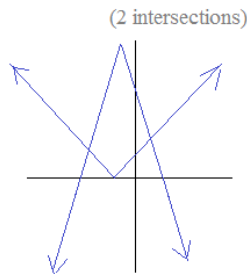see that they both work!)

g) $|3x + 9| = x + 1$

$3x + 9 = x + 1$ $3x + 9 = -(x + 1)$

$x = -4$ $3x + 9 = -x - 1$

Both solutions
are extraneous! $x = -\dfrac{5}{2}$

NO SOLUTIONS

(no intersections)

h) $3 - 10|x - 5| = -17$

$-10|x - 5| = -20$

$|x - 5| = 2$

$x = 7$ or 3

(2 intersections)

i) $|x + 6| = -|2x + 10| + 20$

+ + $x + 6 = -(2x + 10) + 20$

$x + 6 = -2x + 10$

$x = 4/3$

+ − $x + 6 = -(-2x - 10) + 20$

$x + 6 = 2x + 30$ extraneous

$x = -24$

− + $-(x + 6) = -2x - 10 + 20$

$-x - 6 = -2x + 10$ extraneous

$x = 16$
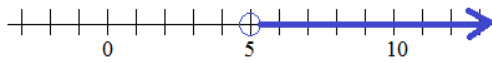
− − $-(x + 6) = -(-2x - 10) + 20$

$-x - 6 = 2x + 30$

$x = -12$

Quiz: Absolute Value, Inequalities, and the Number Line
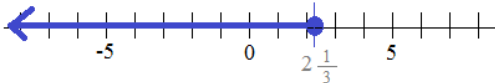
SOLUTIONS
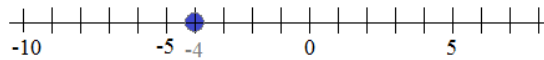
I. Graph the following on a number line
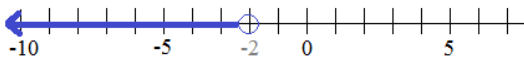
a) $x > 5$

b) $2x \le 4$ $x \le 2$

c) $3x + 2 < 9$ subtract 2 from each side:
$3x < 7$
divide both sides by 3:
$x < 7/3$

d) $x = -4$

e) $-3x > 6$ divide both sides by -3
(**flip the sign!)

$x < -2$
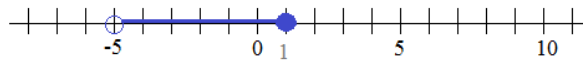
test points to check your answer!
try -5: -3(-5) > 6   15 > 6   yes
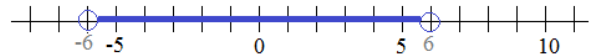try 0: -3(0) > 6   0 ≯ 6   no

f) $-5 < x \le 1$

II. Absolute Values: Solve and graph the following

a) $|x + 3| < 6$ $x + 3 = 6$ $\boxed{x = 3}$ test pts: -10; |-10 + 3| < 6 no
$x + 3 = -6$ $x = -9$ 0; |0 + 3| < 6   yes
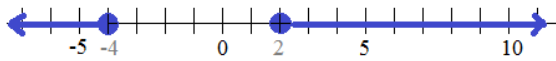   5; |5 + 3| < 6   no
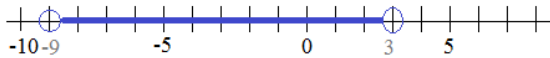
b) $|3x| < 18$ $3x = 18$ $x = 6$ 6 and -6 are critical points
$3x = -18$ $x = -6$ since it is <, we use open circles.
(then, test a point in each region)

c) $2|x + 1| \ge 6$ isolate the absolute value: (divide by 2)
$|x + 1| \ge 3$ then, find $x + 1 = 3$
$x + 1 = -3$
2 and -4

d) $|2x| + 8 < 4$ to isolate the absolute value, subtract 8
from both sides: $|2x| < -4$ $\boxed{\text{NO SOLUTION!!}}$
no absolue value is negative!
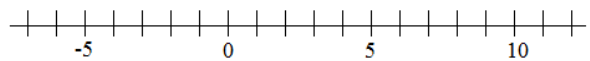
e) $|x - 3| + 4 \ge 9$
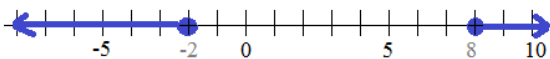
isolate the absolute value to solve:

$|x - 3| \ge 5$ $x - 3 \ge 5$ $x \ge 8$ test pts to check answer:
or   -5: |-5 - 3| + 4 ≥ 9 yes
$x - 3 \le -5$ $x \le -2$ 0: |0 - 3| + 4 ≱ 9 no
10: |10 - 3| + 4 ≥ 9 yes

f) $|x + 7| = 1$ the points are
$x = -6$ or -8
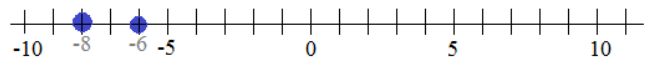
III. Solve and Graph   SOLUTIONS

a) x > 4 or x < 0   note: all shaded values are EITHER greater
                        than 4 OR less than 0

b) x > 5 and x ≤ 9   all shaded values are BOTH
                         greater than 5 AND less than/equal
                         to 9

c) |x + 4| > 0   at any value EXCEPT -4, the solution is
                   greater than 0.. (at -4, it equals 0)

d) 2|x + 1| + 6 > 10   isolate the absolute value: 2|x + 1| > 4
                                                       |x + 1| > 2
                         then, solve...

                         (test points in each region
                         to check your answer)

e) −|x +3| > −5 |x + 3| < 5   x < 2
                                    and
   (divide by -1; flip sign)   x > -8

f) |x + 6| ≤ 0

At -6, the output is = 0;
at every other point, the absolute value
is greater than 0..
therefore, there is only one solution..

IV: Describe each graph with an equation

a)

x < 3

b)

-5 < x ≤ 9

c)

x ≥ 7 or x ≤ -6

Also, an alternative answer
(using absolute values): |x − 1/2| ≥ 6 1/2

BONUS:

1) Graph the following on a number line:

a)   x ≠ 6

b) x < 5 or x ≥ -3

Any real number is either less than 5
or greater than -3!!

2) Use Absolute Value Inequalities to describe the following number line graphs:

a) |x − 5| < 7

−2 < x < 12

midpoint of -2 and 12: 5
distance from 5 to either endpoint:
7 units
|x − 5| < 7

"the distance between x and 5 will
be less than 7 units"

b)

x ≤ 2 or x ≥ 12

|x − 7| ≥ 5

midpoint of 2 and 12: 7
distance from each endpoint: 5
"closed circles", arrows outward: ≥

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V. More concept questions

a) $|x + 8| < 3x$

$x + 8 < 3x$ $\boxed{x > 4}$

$x + 8 = -3x$ $x = -2$

but, x cannot be negative!

SOLUTIONS

b) 'What is wrong with this?'

1) $|x - 4| + 5 > 7$ ⇐ cannot add 4 to
both sides when
inside absolute value

$|x| + 5 > 11$

$|x| > 6$ should be $|x - 4| > 2$

$x > 6$ or $x < -6$ $x - 4 > 2$ or $x - 4 < -2$

$x > 6$ or $x < 2$

2) $3|x + 9| + 7 > 9$ ⇐ must divide ALL terms by 3

$|x + 9| + 7 > 3$ should be $|x + 9| + 7/3 > 3$

$|x + 9| > -4$ $|x + 9| > 2/3$

all real numbers $x + 9 > 2/3$ or $x + 9 < -2/3$

$x > -8\ 1/3$ or $x < -9\ 2/3$

c) $8|2x + 4| - 3|2x + 4| > 5$

combine $5|2x + 4| > 5$
like terms...
                    $|2x + 4| > 1$

$2x + 4 > 1$ or $2x + 4 < -1$

$\boxed{x > -3/2 \quad \text{or} \quad x < -5/2}$

$7|x - 1| + 4|x - 1| = 44$

$7|x - 1| + 4|x - 1| = 44$

$11|x - 1| = 44$

$|x - 1| = 4$ $\boxed{x = 5 \text{ or } -3}$

d) What is the solution of $2x < 14$ or $-3x < -9$?

a) $3 < x < 7$

b) $-7 < x < -3$

c) $x < 3$ or $x > 7$

$\boxed{\text{d) all real numbers}}$

$x < 7$ or $x > 3$

all real numbers, since every number is
either less than 7 OR greater than 3

e) *ALWAYS/SOMETIMES/NEVER*

For any real number N,

$|x - 3| = N$ will have 2 solutions

$\boxed{\text{SOMETIMES}}$

If N = 0, then x will have 1 solution
if N < 0, then x will have 0 solutions
if N > 0, then x will have 2 solutions

f) Solve $2\sqrt{3x - 4} < 16$

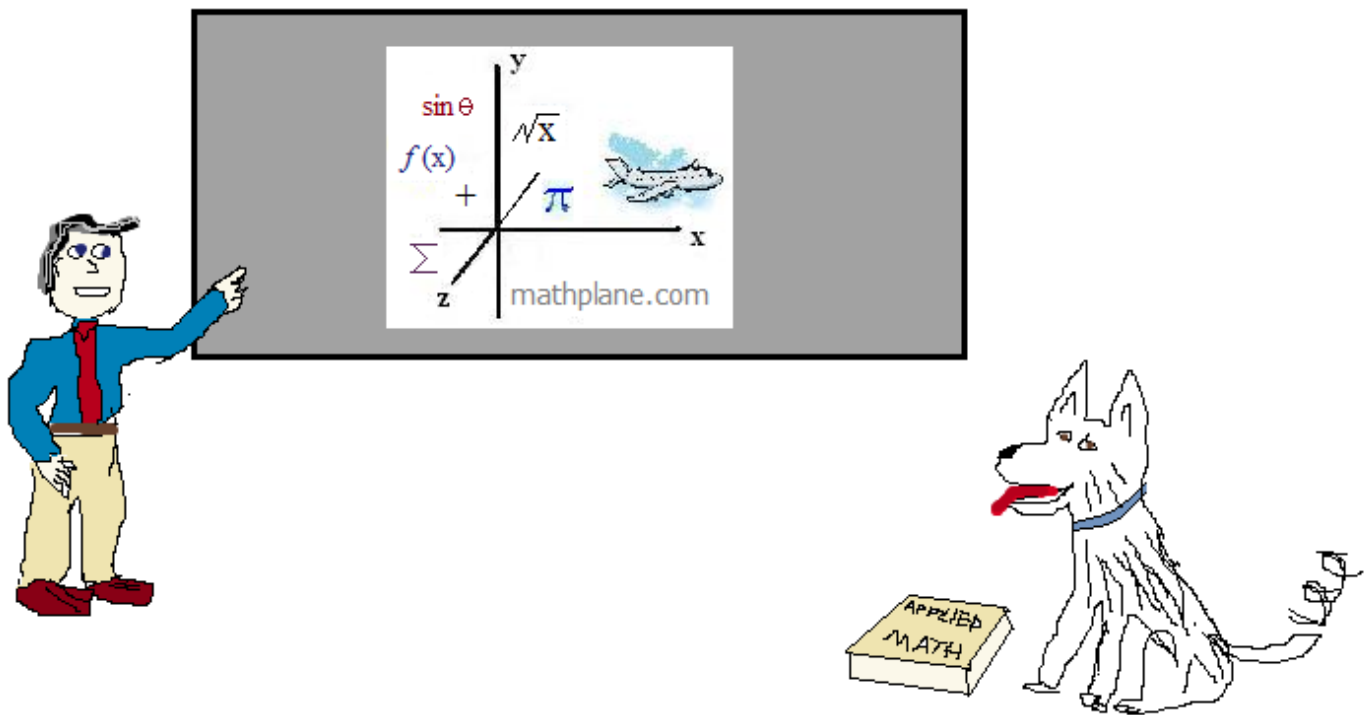$2\sqrt{3x - 4} < 16$ $x = 68/3$

$\sqrt{3x - 4} = 8$

$3x - 4 = 64$

not in
the domain!

$\boxed{[\ 4/3\ ,\ 68/3\ )}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers

Also, at TeachersPayTeachers, TES, and Mathplane.ORG