Rational Exponents and Radical Equations

Notes, Examples, and Practice Quizzes (with Answers)

Topics include exponent rules, factoring, extraneous solutions, quadratics, absolute value, and more.

Exponents & Roots

Definition of Exponent: $X^{A} = X_1 \cdot X_2 \cdot \ldots X_{A-2} \cdot X_{A-1} \cdot X_A$

Example: $4^{5} = 4 \text{ x } 4 \text{ x } 4 \text{ x } 4 \text{ x } 4 = 1024$

Rules, Examples, and Notes:

Rule #1: $X^{A} \cdot X^{B} = X^{(A+B)}$

Examples: $Y^3 \text{ x } Y^5 = Y^8$

$5^3 \cdot 5^2 = 125 \text{ x } 25 = 3125 = 5^5$

Note: $Z^2 \text{ x } Z^4 = (Z \text{ x } Z) \text{ x } (Z \text{ x } Z \text{ x } Z \text{ x } Z) = Z^6$

2 + 4 = 6 total

Rule #2: $(X^{A})^{B} = X^{(A \text{ x } B)}$

Examples: $(X^4)^3 = X^{12}$

$(4^2)^4 = 4^8 = 16^4 = 65536$

Note: $(Y^4)^3 = (Y \cdot Y \cdot Y \cdot Y) \text{ x } (Y \cdot Y \cdot Y \cdot Y) \text{ x } (Y \cdot Y \cdot Y \cdot Y) = Y^{12}$

3 groups of 4 each ----- 12 Total

Rule #3: $X^0 = 1$

Examples: $Y^0 = 1$

$8^0 = 1$

Note: $Y^4 \text{ x } Y^{-4} = \dfrac{Y \cdot Y \cdot Y \cdot Y}{Y \cdot Y \cdot Y \cdot Y} = 1$

Rule #4: $X^{(-A)} = \dfrac{1}{(X^A)}$

Example: $X^{-3} = 1/(X^3)$

$5^{-2} = 1/5^2 = \dfrac{1}{25}$

Note: $Y^{(-A)} = Y^{(-A)} \cdot \dfrac{Y^A}{Y^A} = \dfrac{Y^{(-A)} \text{ x } Y^A}{Y^A} = \dfrac{Y^0}{Y^A} = \dfrac{1}{Y^A}$

Rule #5: $X^{(1/2)} = \sqrt{X}$ (or, more generally: $X^{(m/n)} = \sqrt[n]{X^m}$)

Examples: $25^{(1/2)} = \sqrt{25} = 5$

$8^{(1/3)} = \sqrt[3]{8} = 2$

"cube root of 8"

Note: $Y^{(1/2)} \text{ x } Y^{(1/2)} = Y^1$ as $\sqrt{Y} \cdot \sqrt{Y} = Y$

$8^{(1/3)} \text{ x } 8^{(1/3)} \text{ x } 8^{(1/3)} = 8^{(1/3+1/3+1/3)} = 8^1 = 8$

$A^{(5/2)} = A^{(1/2)} \text{ x } A^5 = (\sqrt{A})^5$

Rule #6: $X^{A} \cdot Y^{A} = (XY)^{A}$

Examples: $5^3 \cdot 7^3 = 125 \text{ x } 343 = 35^3 = 42875$

$(5 \text{ x } 5 \text{ x } 5)(7 \text{ x } 7 \text{ x } 7) = (5 \text{ x } 7)(5 \text{ x } 7)(5 \text{ x } 7) = 35 \text{ x } 35 \text{ x } 35$

$4^{(1/2)} \text{ x } 16^{(1/2)} = 64^{(1/2)} = 8$

$\sqrt{4} \text{ x } \sqrt{16} = \sqrt{4 \text{ x } 16} = \sqrt{64} = 8$

Solving radical (exponent) equations

4 Steps:

1) *Isolate* radical
2) *Square* both sides
3) *Solve*
4) *Check* (for extraneous answers)

4 Steps for *fractional* exponents

1) Isolate term
2) Raise to power that eliminates the exponents
3) Solve
4) Check

Example 1:

$\sqrt{5x} + 10 = 25$ Isolate -- subtract 10 from both sides

$\sqrt{5x} = 15$ Square both sides

$5x = 225$ Solve -- divide 5 from both sides

$\boxed{x = 45}$ Check

$\sqrt{5(45)} + 10 = 25$

$25 = 25$

Example 2:

$\sqrt{3x} + 12 = 6$

$\sqrt{3x} = -6$

$3x = 36$

$x = 12$

Now, check the answer.

$\sqrt{3(12)} + 12 = 6$

$18 \neq 6$

There is no solution!

Example 3:

$\sqrt{x + 30} = x$ square both sides

$x + 30 = x^2$ solve

$x^2 - x - 30 = 0$

$(x + 5)(x - 6) = 0$

$x = \cancel{-5}, 6$ check

$\sqrt{(-5) + 30} = (-5)$

$5 \neq -5$

-5 is extraneous!

$\sqrt{(6) + 30} = (6)$

$6 = 6$

$\boxed{x = 6}$

Example 4:

$4(x - 2)^{\frac{1}{3}} - 12 = 0$ isolate the exponent

$4(x - 2)^{\frac{1}{3}} = 12$

$(x - 2)^{\frac{1}{3}} = 3$ raise to 3rd power
(to eliminate the exponent)

$x - 2 = 27$ solve

$\boxed{x = 29}$ check

$4((29) - 2)^{\frac{1}{3}} - 12 = 0$

$4(3) - 12 = 0$

$0 = 0$

Rational Exponent Equations: Negative Numbers, Absolute Values, and Eliminated Answers

Rational Exponent Equations
Domain Restrictions:
A Comparison

$y = x^{\frac{2}{3}}$ can x = -4? YES

$(-4^2)^{\frac{1}{3}}$

or $\qquad y = \sqrt[3]{16}$

$(-4^{\frac{1}{3}})^2$

$y = x^{\frac{3}{2}}$ can x = -4? NO

$(-4^3)^{\frac{1}{2}}$

or $\qquad$ NOT REAL!!

$(-4^{\frac{1}{2}})^3$

Examples:

$$2(x+4)^{\frac{2}{3}} = 8$$
$$(x+4)^{\frac{2}{3}} = 4$$
$$(x+4) = 4^{\frac{3}{2}}$$
$$x + 4 = 8 \qquad x + 4 = -8$$
$$x = 4 \qquad x = -12$$

$$2(x-3)^{\frac{2}{3}} = 50$$
$$\left((x-3)^{\frac{2}{3}}\right)^{\frac{3}{2}} = 25^{\frac{3}{2}}$$
$$x - 3 = 125 \quad \text{or} \quad x - 3 = -125$$
$$x = 128 \quad \text{or} \quad x = -122$$

$$2(x+5)^{\frac{2}{5}} = 32$$
$$(x+5)^{\frac{2}{5}} = 16$$
$$\left((x+5)^{\frac{1}{5}}\right)^2 = 16$$
$$\left|(x+5)^{\frac{1}{5}}\right| = 4$$
$$x = 1019 \quad \text{or} \quad x = -1029$$

since it is the "square root of a square", the term is absolute value

$$(x+3)^{\frac{3}{5}} = -8$$

Since it is a 1/5 root,
a negative is permitted...

$$x + 3 = (-8)^{\frac{5}{3}}$$

(if possible, "Go smaller first")

$$x + 3 = (-8^{\frac{1}{3}})^5$$
$$x + 3 = (-2)^5$$
$$x + 3 = -32$$
$$x = -35$$

(It's easier to find the cube root of 8 first, then 2 to the 5th power --- rather than 8 to the 5th power first, then the cube root of 32,768!)

$$2(x)^{\frac{3}{2}} + 21 = 13$$
$$2x^{\frac{3}{2}} = -8$$
$$x^{\frac{3}{2}} = -4$$

Since it is a 1/2 root,
a negative is NOT permitted...

$$x = (-4)^{\frac{2}{3}}$$
$$x = 16^{\frac{1}{3}}$$

But, when you check the answer:

$$2(x)^{\frac{3}{2}} + 21 = 13$$
$$2(16^{\frac{1}{3}})^{\frac{3}{2}} + 21 = 13$$
$$2(16)^{\frac{1}{2}} + 21 = 13$$
$$2(4) + 21 = 13$$
$$8 = -8$$

There is no real solution!!

General rule: If n is even, then $\sqrt[n]{x^n} = |n|$

Why do you need to include an absolute value?

Does $\sqrt{x^2} = x$?

Test points:

If x = 3: $\sqrt{3^2} = 3$, $\sqrt{9} = 3$, $3 = 3$

But, if x = -3: $\sqrt{(-3)^2} = -3$, $\sqrt{9} = -3$, $3 = -3$

However, if we include an absolute value sign:

$$\sqrt{x^2} = |x|$$

If x = 3: $\sqrt{3^2} = |3|$, $\sqrt{9} = |3|$, $3 = |3|$

But, if x = -3: $\sqrt{(-3)^2} = |-3|$, $\sqrt{9} = |-3|$, $3 = |-3|$

Practice Exercises -→

Exponents, Roots, & Addition Exercise

Solve the 15 problems below. Then, add all the solutions.
What is the total? (rounded to 3 decimal places.)

1) $(3^3)^2 =$ ________

2) $(2)^{-2} =$ ________

3) $(4)^{3/2} =$ ________

4) $\sqrt{64} - \sqrt[3]{8} =$ ________

5) $9^2 + 9^{1/2} =$ ________

6) $(.3)^3 =$ ________

7) $(32)^{2/5} =$ ________

8) $(1/3)^{-2} =$ ________

9) $(-5)^3 =$ ________

10) $\sqrt{(3)^4} =$ ________

11) $\sqrt{2} \text{ x } \sqrt{50} =$ ________

12) $1^2 - 2^3 + 3^4 =$ ________

13) $(1/2)^3 =$ ________

14) $8^{1/3} \cdot 8^{2/3} =$ ________

15) $\sqrt[3]{(-8)} - \sqrt[3]{27} =$ ________

Now Add them up! The Total of ALL 15 solutions is ________

(rounded to 3 decimal places)

Radicals and higher roots

Simplify the following expressions.

1) $3\sqrt{24} + 5\sqrt{54}$

2) $\sqrt[3]{54} + 2\sqrt[3]{16}$

3) $\sqrt[4]{162} + 5\sqrt[4]{32}$

4) $7\sqrt[4]{64} - 2\sqrt[4]{4}$

5) $\sqrt[3]{375} - \sqrt[3]{81} + \sqrt[3]{24}$

6) $4\sqrt[5]{x^9 y^3 z^6} + 3\sqrt[5]{x^4 y^3 z^{11}}$

7) $4(50)^{\frac{1}{2}} + 6(200)^{\frac{1}{2}}$

8) $8(80)^{\frac{1}{3}} - 5(10)^{\frac{1}{3}}$

Rational Exponents and Radical Equations

I. Evaluate

a) $9^{\frac{1}{2}}$ b) $9^{\frac{-1}{2}}$ c) 1^{0} d) $27^{\frac{2}{3}}$ e) $81^{\frac{-1}{4}}$

f) $25^{1.5}$ g) $16^{.25}$ h) $4^{3.5}$ i) $64^{-.5}$ j) $9^{-2.5}$

II. Simplify the expressions

a) $\sqrt{8} \cdot \sqrt{40}$ b) $6^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$ c) $\sqrt[4]{16} + \sqrt[3]{8}$

d) $\left(5\sqrt{3}\right)^{2}$ e) $(81)^{\frac{1}{4}} \cdot (81)^{\frac{1}{2}}$ f) $\sqrt[3]{\sqrt{64}}$

g) $(9m^{4})^{\frac{1}{2}}$ h) $\left(\frac{1}{4}\right)^{\frac{-1}{2}}$ i) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

Rational Exponents and Radical Equations

III. Solve the following.

a) $\sqrt{4x - 27} - 1 = 4$

b) $5\sqrt{x} + 7 = 8$

c) $2 + (4 - x)^{\frac{3}{2}} = 10$

d) $\sqrt{3x} = \sqrt{x + 4}$

e) $(x + 4)^{\frac{3}{4}} = 27$

f) $\sqrt{(x + 1)^3} - 1 = 7$

IV. Solve. (Identify any extraneous solutions)

a) $\sqrt{x + 7} + 5 = x$

b) $\sqrt{x + 2} = x$

c) $(5x + 4)^{\frac{1}{2}} - 3x = 0$

d) $\sqrt{4x - 5} = 3\sqrt{x - 5}$

e) $(x - 9)^{\frac{1}{2}} + 1 = x^{\frac{1}{2}}$

f) $(x + 5)^{\frac{1}{2}} - (5 - 2x)^{\frac{1}{4}} = 0$

V. Simplify (or factor) the following.

a) $(\sqrt{b^2+1}-1)(\sqrt{b^2+1}+1)$

b) $y^{5/2}-y^{1/2}$

c) $x^{-3/2}-2x^{-1/2}+x^{1/2}$

d) $6x^{-1/2}+8x^{1/2}+2x^{3/2}$

e) $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}$

f) $\dfrac{2(a+1)^{1/2}-a(1+a)^{-1/2}}{a+1}$

VI. More rational exponent equations

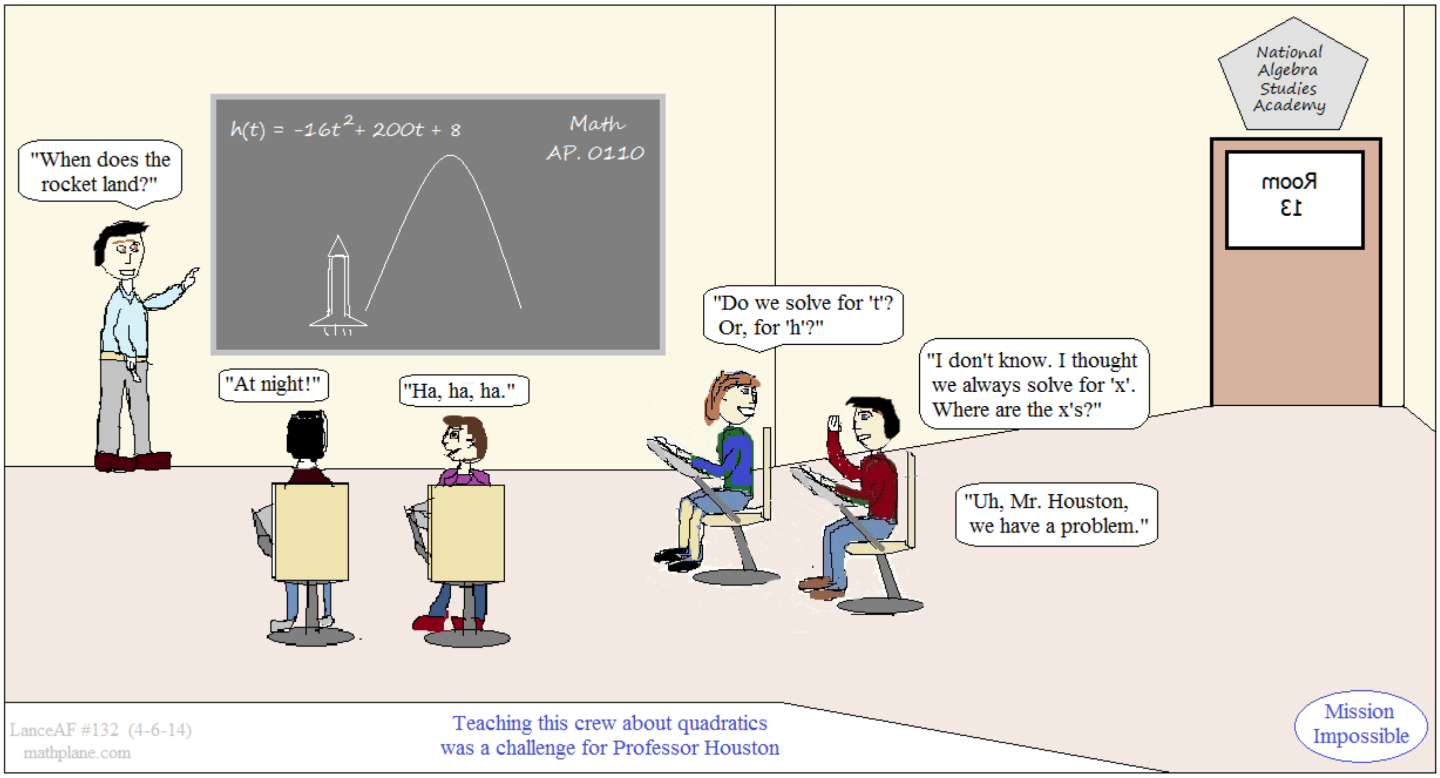
a) $2(x+5)^{\frac{3}{2}} + 128 = 0$

b) $y = 6 + \sqrt{y}$

c) $\sqrt{3-x} = \sqrt{7-2x}$

d) $3(x+5)^{\frac{2}{3}} + 2 = 50$

e) $2(x-1)^{\frac{3}{2}} - 7 = 23$



"When does the rocket land?"
$h(t) = -16t^2 + 200t + 8$
Math AP. 0110
National Algebra Studies Academy
Room 13
"At night!"
"Ha, ha, ha."
"Do we solve for 't'? Or, for 'h'?"
"I don't know. I thought we always solve for 'x'. Where are the x's?"
"Uh, Mr. Houston, we have a problem."
LanceAF #132 (4-6-14)
mathplane.com
Teaching this crew about quadratics was a challenge for Professor Houston
Mission Impossible

Solutions →

Exponents, Roots, & Addition Exercise

Solve the 15 problems below. Then, add all the solutions.
What is the total? (rounded to 3 decimal places)

SOLUTIONS

1) $(3^3)^2$ = (3 x 3 x 3)(3 x 3 x 3) = 27 x 27 = 729 — **729**

2) $(2)^{-2}$ = 2^2 = 4.. therefore, 2^{-2} = 1/4 or .25 — **.25**

3) $(4)^{3/2}$ = 4^3 is 64, and $64^{1/2}$ = 8 — **8**

4) $\sqrt{64} - \sqrt[3]{8}$ = 8 - 2 = 6 — **6**

5) $9^2 + 9^{1/2}$ = 81 + 3 = 84 — **84**

827.25

6) $(.3)^3$ = .3 x .3 x .3 = .09 x .3 = .027 — **.027**

7) $(32)^{2/5}$ = $32^{1/5}$ x $32^{1/5}$ = 2 x 2 = 4 — **4**

8) $(1/3)^{-2}$ = $(1/3)^2$ = 1/9.. therefore, $(1/3)^{-2}$ = 9 (the reciprocal of 1/9) — **9**

9) $(-5)^3$ = -5 x -5 x -5 = -125 — **-125**

10) $\sqrt{(3)^4}$ = $(3)^4$ = 81 and $\sqrt{81}$ = 9 — **9**

-102.973

11) $\sqrt{2}$ x $\sqrt{50}$ = $\sqrt{100}$ = 10 — **10**

12) $1^2 - 2^3 + 3^4$ = 1 - 8 + 81 = 74 — **74**

13) $(1/2)^3$ = 1/2 x 1/2 x 1/2 = 1/8 = .125 — **.125**

14) $8^{1/3} \cdot 8^{2/3}$ = 8^1 = 8 — **8**

15) $\sqrt[3]{(-8)} - \sqrt[3]{27}$ = -2 - 3 = -5 — **-5**

87.125

Now Add them up! The Total of ALL 15 solutions is $\boxed{811.402}$

(rounded to 3 decimal places)

Radicals and higher roots

Simplify the following expressions.

SOLUTIONS

1) $3\sqrt{24} + 5\sqrt{54}$

$3\cdot 2\sqrt{6} + 5\cdot 3\sqrt{6}$

$6\sqrt{6} + 15\sqrt{6} = \boxed{21\sqrt{6}}$

2) $\sqrt[3]{54} + 2\sqrt[3]{16}$

$\sqrt[3]{27\cdot 2} + 2\sqrt[3]{8\cdot 2}$

$3\sqrt[3]{2} + 2\cdot 2\sqrt[3]{2} = \boxed{7\sqrt[3]{2}}$

3) $\sqrt[4]{162} + 5\sqrt[4]{32}$

$\sqrt[4]{81\cdot 2} + 5\sqrt[4]{16\cdot 2}$

$3\sqrt[4]{2} + 5\cdot 2\sqrt[4]{2} = \boxed{13\sqrt[4]{2}}$

4) $7\sqrt[4]{64} - 2\sqrt[4]{4}$

$7\sqrt[4]{2\cdot 2\cdot 2\cdot 2\cdot 4} - 2\sqrt[4]{4}$

$7\cdot 2\sqrt[4]{4} - 2\sqrt[4]{4} = \boxed{12\sqrt[4]{4}}$

5) $\sqrt[3]{375} - \sqrt[3]{81} + \sqrt[3]{24}$

$\sqrt[3]{5\cdot 5\cdot 5\cdot 3} - \sqrt[3]{3\cdot 3\cdot 3\cdot 3} + \sqrt[3]{2\cdot 2\cdot 2\cdot 3}$

$5\sqrt[3]{3} - 3\sqrt[3]{3} + 2\sqrt[3]{3} = \boxed{4\sqrt[3]{3}}$

6) $4\sqrt[5]{x^9 y^3 z^6} + 3\sqrt[5]{x^4 y^3 z^{11}}$

$4xz\sqrt[5]{x^4 y^3 z} + 3z^2\sqrt[5]{x^4 y^3 z}$

$\boxed{(4x + 3z)z\sqrt[5]{x^4 y^3 z}}$

7) $4(50)^{\frac{1}{2}} + 6(200)^{\frac{1}{2}}$

$4\cdot\sqrt{50} + 6\cdot\sqrt{200}$

$4\cdot 5\sqrt{2} + 6\cdot 10\sqrt{2}$

$\boxed{80\sqrt{2}}$

8) $8(80)^{\frac{1}{3}} - 5(10)^{\frac{1}{3}}$

$8\sqrt[3]{8\cdot 10} - 5\sqrt[3]{10}$

$16\sqrt[3]{10} - 5\sqrt[3]{10}$

$\boxed{11\sqrt[3]{10}}$

Rational Exponents and Radical Equations

SOLUTIONS

I. Evaluate

a) $9^{\frac{1}{2}}$

3

b) $9^{\frac{-1}{2}}$

$\frac{1}{3}$

c) 1^{0}

1

d) $27^{\frac{2}{3}}$

$\left(27^{\frac{1}{3}}\right)^2$

$3^2 = \boxed{9}$

e) $81^{\frac{-1}{4}}$

$81^{\frac{1}{4}} = 3$

because $3 \cdot 3 \cdot 3 \cdot 3 = 81$

so, $81^{\frac{-1}{4}} = \boxed{-3}$

f) $25^{1.5}$

$25^{\frac{3}{2}} = \sqrt{25}^{\,3}$

$\boxed{= 125}$

g) $16^{.25}$

$\boxed{2}$

$16^{.25} \cdot 16^{.25} \cdot 16^{.25} \cdot 16^{.25} = 16^{1}$

$2 \text{ x } 2 \text{ x } 2 \text{ x } 2 = 16$

h) $4^{3.5}$

$4^{\frac{7}{2}}$

$2^7 = \boxed{128}$

i) $64^{-.5}$

$\frac{1}{\sqrt{64}} = \boxed{\frac{1}{8}}$

j) $9^{-2.5}$

$\frac{1}{9^{2.5}} = \frac{1}{9^{\frac{5}{2}}} = \frac{1}{3^5}$

$\boxed{= \frac{1}{243}}$

II. Simplify the expressions

a) $\sqrt{8} \cdot \sqrt{40}$

$\sqrt{48}$

$\sqrt{16 \cdot 3} = \boxed{4\sqrt{3}}$

b) $6^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$

$\sqrt{6} \cdot \sqrt{12}$

$\sqrt{72} = \boxed{6\sqrt{2}}$

c) $\sqrt[4]{16} + \sqrt[3]{8}$

$2 + 2 = \boxed{4}$

d) $\left(5\sqrt{3}\right)^2$

$5\sqrt{3} \cdot 5\sqrt{3} =$

$25 \cdot 3 =$

$\boxed{75}$

e) $(81)^{\frac{1}{4}} \cdot (81)^{\frac{1}{2}}$

$3 \cdot 9 = \boxed{27}$

$81^{\left(\frac{1}{4} + \frac{1}{2}\right)} = 81^{\frac{3}{4}} = 27$

f) $\sqrt[3]{\sqrt{64}}$

$\sqrt[3]{8}$ $\boxed{= 2}$

g) $(9m^4)^{\frac{1}{2}}$

$9^{\frac{1}{2}} \cdot m^{\frac{4}{2}}$

$\boxed{3m^2}$

h) $\left(\frac{1}{4}\right)^{\frac{-1}{2}}$

$\frac{1}{\left(\frac{1}{4}\right)^{\frac{1}{2}}} = \frac{1}{\frac{1}{2}}$ $\boxed{= 2}$

i) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

$\left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$

and $\left(\frac{3}{4}\right)^3 = \boxed{\frac{27}{64}}$

Rational Exponents and Radical Equations

SOLUTIONS

III. Solve the following.

a) $\sqrt{4x-27} - 1 = 4$

$\sqrt{4x-27} = 5$

(square both sides)

$4x - 27 = 25$

$4x = 52$

$\boxed{x = 13}$

b) $5\sqrt{x} + 7 = 8$

(isolate the radical)

$5\sqrt{x} = 1$

$\sqrt{x} = \frac{1}{5}$

(square both sides)

$\boxed{x = \frac{1}{25}}$

c) $2 + (4-x)^{\frac{3}{2}} = 10$

$(4-x)^{\frac{3}{2}} = 8$

$(4-x)^{1} = 8^{\frac{2}{3}}$

$4 - x = 4$

$\boxed{x = 0}$

d) $\sqrt{3x} = \sqrt{x+4}$

$3x = x + 4$

$2x = 4$

$\boxed{x = 2}$

To check answer, substitute into original problem: $\sqrt{3(2)} = \sqrt{(2)+4}$

$\sqrt{6} = \sqrt{6}$ ✓

e) $(x+4)^{\frac{3}{4}} = 27$

$x + 4 = 27^{\frac{4}{3}}$

$x + 4 = 81$

$\boxed{x = 77}$

f) $\sqrt{(x+1)^3} - 1 = 7$

$\sqrt{(x+1)^3} = 8$

$(x+1)^3 = 64$

$x + 1 = 4$

$\boxed{x = 3}$

IV. Solve. (Identify any extraneous solutions)

a) $\sqrt{x+7} + 5 = x$

(isolate radical)

$\sqrt{x+7} = x - 5$

(square both sides)

$x + 7 = x^2 - 10x + 25$

$x^2 - 11x + 18 = 0$

$(x-2)(x-9) = 0$

$x = \cancel{2}, 9$

(check answers)

$\sqrt{(2)+7} + 5 = (2)$ ✗

$3 + 5 \neq 2$

$\sqrt{(9)+7} + 5 = (9)$ ✓

$4 + 5 = 9$

$\boxed{x = 9}$

b) $\sqrt{x+2} = x$

$x + 2 = x^2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -\cancel{1}, 2$ (check answers)

$\sqrt{(-1)+2} = (-1)$ NO (extraneous)

$\sqrt{(2)+2} = (2)$ YES

$\boxed{x = 2}$

c) $(5x+4)^{\frac{1}{2}} - 3x = 0$

$\sqrt{(5x+4)} = 3x$

$5x + 4 = 9x^2$

$9x^2 - 5x - 4 = 0$

$(9x+4)(x-1) = 0$

$x = 1, \cancel{-4/9}$

$\boxed{x = 1}$

(check answers)

$(5(1)+4)^{\frac{1}{2}} - 3(1) = 0$ ✓

$(5(\frac{-4}{9})+4)^{\frac{1}{2}} - 3(\frac{-4}{9}) = \frac{8}{3}$ ✗

d) $\sqrt{4x-5} = 3\sqrt{x-5}$

(square both sides)

$4x - 5 = 9(x-5)$

$4x - 5 = 9x - 45$

$40 = 5x$

$\boxed{x = 8}$

e) $(x-9)^{\frac{1}{2}} + 1 = x^{\frac{1}{2}}$

$\sqrt{(x-9)} = \sqrt{x} - 1$

(square both sides)

$x - 9 = x - 2\sqrt{x} + 1$

(isolate the radical)

$2\sqrt{x} = 10$

$\sqrt{x} = 5$

$\boxed{x = 25}$

f) $(x+5)^{\frac{1}{2}} - (5-2x)^{\frac{1}{4}} = 0$

$(x+5)^{\frac{1}{2}} = (5-2x)^{\frac{1}{4}}$

(Remove the exponents by taking the '4th power' of each side -- or, squaring each side twice)

$(x+5)^2 = 5 - 2x$

$x^2 + 10x + 25 = 5 - 2x$

$x^2 + 12x + 20 = 0$

$(x+2)(x+10) = 0$

$x = -2, \cancel{-10}$

$\boxed{x = -2}$

(check answers)

$-2:\ 3^{\frac{1}{2}} = 9^{\frac{1}{4}}$ ✓

$-10:\ (-5)^{\frac{1}{2}} = (25)^{\frac{1}{4}}$ ✗

extraneous!

V. Simplify (or factor) the following. SOLUTIONS

Rational Exponents and Radical Equations

a) $(\sqrt{b^2+1} - 1)(\sqrt{b^2+1} + 1)$

$\left(\sqrt{b^2+1}\right)^2 - 1$ (FOIL (the conjugates))

$b^2 + 1 - 1$

b^2

b) $y^{5/2} - y^{1/2}$ (GCF: the lowest exponent)

$y^{1/2}(y^2 - 1)$ (factor)

$y^{1/2}(y+1)(y-1)$

c) $x^{-3/2} - 2x^{-1/2} + x^{1/2}$ (factor out the lowest exponent)

$x^{-3/2}(1 - 2x + x^2)$ (factor the quadratic)

$x^{-3/2}(x-1)(x-1)$

$x^{-3/2}(x-1)^2$

d) $6x^{-1/2} + 8x^{1/2} + 2x^{3/2}$ (take out greatest common factor and "smallest exponent")

$2x^{-1/2}(3 + 4x + x^2)$

(factor quadratic)

$2x^{-1/2}(x+1)(x+3)$

$\dfrac{2(x+1)(x+3)}{x^{1/2}}$ or $\dfrac{2x^{1/2}(x+1)(x+3)}{x}$

e) $\dfrac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}}$ (re-write the negative exponents)

$\dfrac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}}$ (add numerator terms; add denominator terms)

$\dfrac{\frac{y^2 - x^2}{x^2y^2}}{\frac{y+x}{xy}}$ (invert and multiply)

$\dfrac{y^2 - x^2}{x^2y^2} \cdot \dfrac{xy}{y+x}$ (factor, cancel, and simplify)

$\dfrac{(y-x)(y+x)}{x^2y^2} \cdot \dfrac{xy}{y+x}$

$\dfrac{(y-x)}{xy}$

f) $\dfrac{2(a+1)^{1/2} - a(1+a)^{-1/2}}{a+1}$

$\dfrac{(a+1)^{-1/2}[2(a+1)^1 - a]}{a+1}$ factor out term with lowest exponent $(a+1)^{-1/2}$

$\dfrac{(a+1)^{-1/2}[2a + 2 - a]}{a+1}$

$\dfrac{2+a}{(a+1)^{3/2}}$

VI. More rational exponent equations

SOLUTIONS

a) $2(x+5)^{\frac{3}{2}} + 128 = 0$

$2(x+5)^{\frac{3}{2}} = -128$

$(x+5)^{\frac{3}{2}} = -64$

Note: square root isn't negative,
so there will be no solution!!

$x + 5 = (-64)^{\frac{2}{3}}$

$x + 5 = (-4)^2$

$x = 11$

$\boxed{\text{NO SOLUTION}}$

if $x = 11$,

then $2(11+5)^{\frac{3}{2}} + 128 = 0$

$128 + 128 = 0$

b) $y = 6 + \sqrt{y}$

$y - y^{\frac{1}{2}} - 6 = 0$

$(y^{\frac{1}{2}} - 3)(y^{\frac{1}{2}} + 2) = 0$

$(y^{\frac{1}{2}} - 3) = 0$ $\quad$ $(y^{\frac{1}{2}} + 2) = 0$

$\boxed{y = 9}$ $\quad$ no real solution

c) $\sqrt{3-x} = \sqrt{7-2x}$

square both sides

$3 - x = 7 - 2x$

$x = 4$

quick check:

$\sqrt{3-4} = \sqrt{7-2(4)}$

$\sqrt{-1} = \sqrt{-1}$

$\boxed{\text{NO REAL SOLUTIONS}}$

d) $3(x+5)^{\frac{2}{3}} + 2 = 50$

isolate the exponent part

$3(x+5)^{\frac{2}{3}} = 48$

$(x+5)^{\frac{2}{3}} = 16$

Since the root is 2/3, a
negative is permitted!

$\left((x+5)^{\frac{1}{3}}\right)^2 = 16$

$(x+5)^{\frac{1}{3}} = \pm 4$

$x + 5 = \pm 64$

$\boxed{x = 59 \text{ or } -69}$

e) $2(x-1)^{\frac{3}{2}} - 7 = 23$

$2(x-1)^{\frac{3}{2}} = 30$

$(x-1)^{\frac{3}{2}} = 15$

Since it is a 1/2 root,
a negative is NOT permitted...

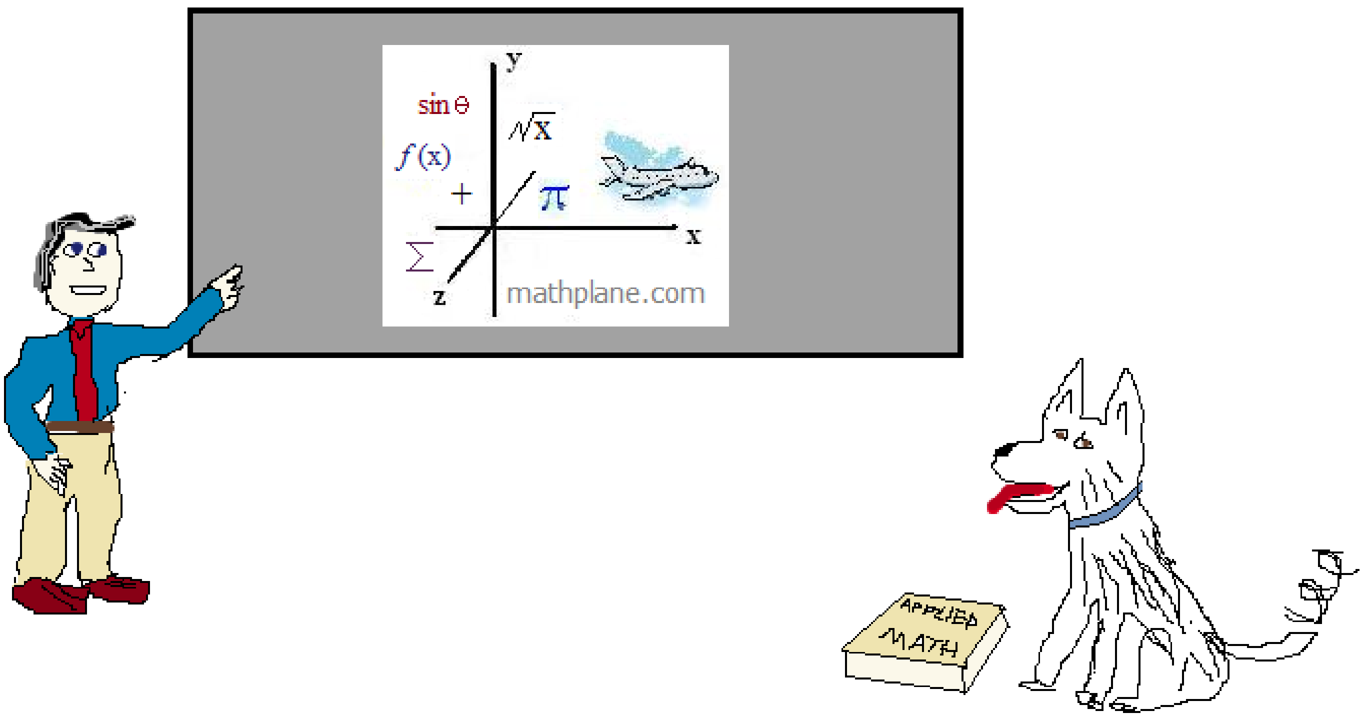
$(x-1) = (15)^{\frac{2}{3}}$

$\boxed{x = \sqrt[3]{225} + 1}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy.



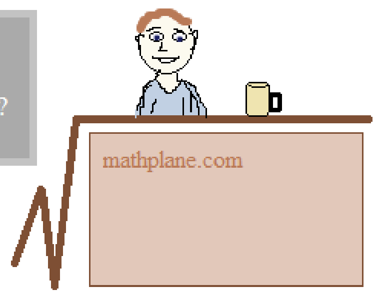
Also, at TeachersPayTeachers

And, *Mathplane Express* for mobile and tablets at Mathplane.ORG

ONE MORE EXERCISE! -→

Hidden Message

Hint:
A math beverage?



Letter/Number Key

A	B	E	O	P	Q	R	S	T	U
1	2	3	4	5	6	7	8	9	0

Solve the 14 equations.
Then, convert the numbers into letters to reveal the answer!

Find X:

1) $\sqrt[3]{X} = 2$

2) $X^3 = 216$

3) $N^X = 1$

4) $4 \cdot 2^{-2} = X$

5) $3^3 = X$

6) $(27)^{\frac{1}{3}} = X$

7) $\left(\frac{1}{49}\right)^{\frac{-1}{2}} = X$

8) $2^{X-2} = 4$

9) $(32)^{\frac{2}{5}} = X$

10) $3\sqrt[4]{81} = X$

11) $(125)^{\frac{-1}{3}} = X$ (express as decimal)

12) $\sqrt{49} - \sqrt{16} = X$

13) $3^{(X+3)} = 27^2$

14) $\sqrt[3]{(7)^3} = X$

$\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

2 $\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

0. $\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

$\square \rightarrow$ ___

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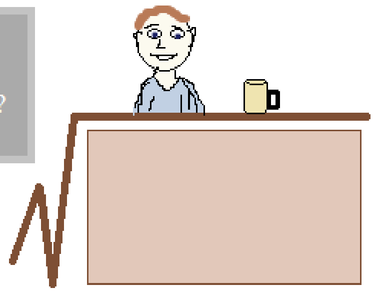
Hint:
A math beverage?



Letter/Number Key

A	B	E	O	P	Q	R	S	T	U
1	2	3	4	5	6	7	8	9	0

Solve the 14 equations.
Then, convert the numbers into letters to reveal the answer!

Find X:

SOLUTIONS

1) $\sqrt[3]{X} = 2$ $\quad X = 2^3 = 8$

2) $X^3 = 216$ $\quad X = \sqrt[3]{216} = 6$

3) $N^X = 1$ $\quad X = 0$

4) $4 \cdot 2^{-2} = X$ $\quad 4 \cdot \frac{1}{4} = 1$

5) $3^3 = X$ $\quad 3 \cdot 3 \cdot 3 = 27$

6) $(27)^{\frac{1}{3}} = X$ $\quad \sqrt[3]{27} = 3$

7) $\left(\frac{1}{49}\right)^{\frac{-1}{2}} = X$ $\quad \left(\frac{49}{1}\right)^{\frac{1}{2}} = 7$

8) $2^{X-2} = 4$ $\quad 2^{X-2} = 2^2$ then, $X - 2 = 2$ $\quad X = 4$

9) $(32)^{\frac{2}{5}} = X$ $\quad (32^{\frac{1}{5}})^2 = X$ $\quad (2)^2 = X$ $\quad X = 4$

10) $3\sqrt[4]{81} = X$ $\quad 3(3) = 9$

11) $(125)^{\frac{-1}{3}} = X$ (express as decimal) $\quad \left(\frac{1}{125}\right)^{\frac{1}{3}} = \frac{1}{5} = .2$

12) $\sqrt{49} - \sqrt{16} = X$ $\quad 7 - 4 = 3$

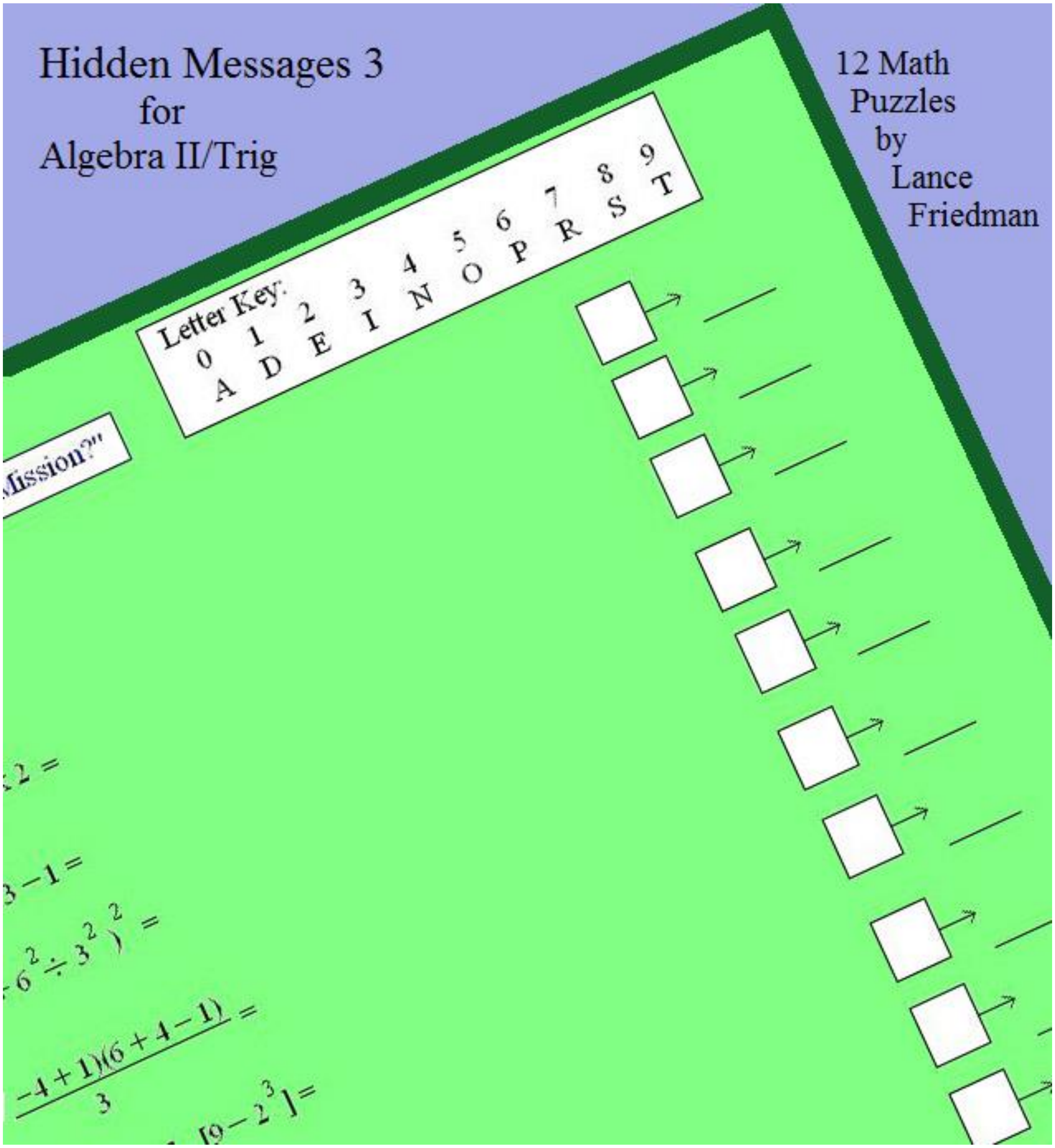
13) $3^{(X+3)} = 27^2$ $\quad 3^{(X+3)} = (3^3)^2$ $\quad 3^{(X+3)} = 3^6$ $\quad X + 3 = 6$, $X = 3$

14) $\sqrt[3]{(7)^3} = X$ $\quad (7^3)^{\frac{1}{3}} = 7^1 = 7$

A math beverage?

Square root beer!

	8	S
	6	Q
	0	U
	1	A
2	7	R
	3	E
	7	R
	4	O
	4	O
	9	T
0.	2	B
	3	E
	3	E
	7	R



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