Area and Perimeter of Polygons 2
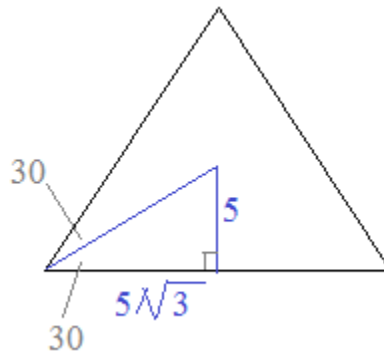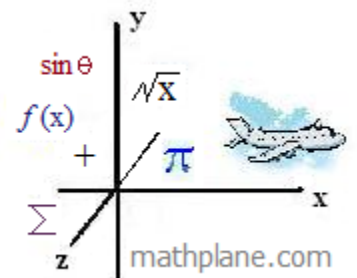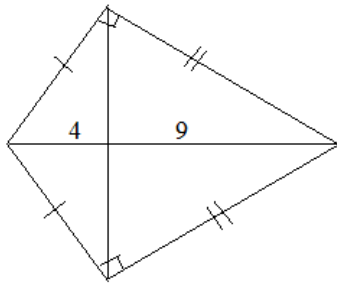

Examples and Practice Exercises (with solutions)

Example: Find the area and perimeter of the figure.

Since the quadrilateral has 2 pairs of adjacent sides that are congruent, it's a kite.
And, the diagonals of a kite are perpendicular.

We can find the geometric mean
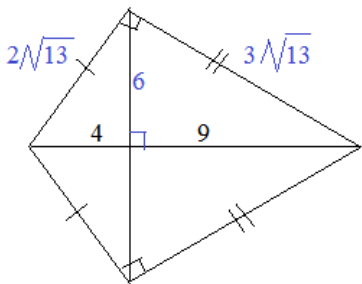(altitude to hypotenuse) to find the length of the other diagonal!

$$\frac{4}{x} = \frac{x}{9} \qquad x = 6, \text{ so the diagonal is } 12$$

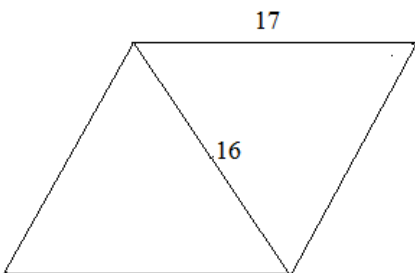The area of the kite is $\frac{1}{2}$ (diagonal 1)(diagonal 2) =

$$\frac{1}{2}(12)(13) = 78$$

Since the diagonals form 4 right triangle, we can use Pythagorean Theorem to find the sides of the kite...

The perimeter is $10\sqrt{13}$
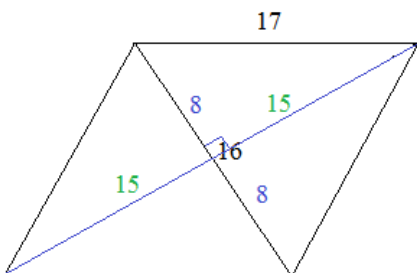

Example: Find the perimeter and area of the rhombus

Since all sides of a rhombus are congruent, the perimeter = 4 x 17 = 68 units

To find the area, the most direct approach is finding the length of the other diagonal.

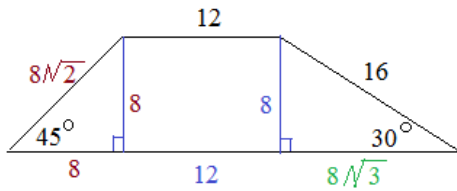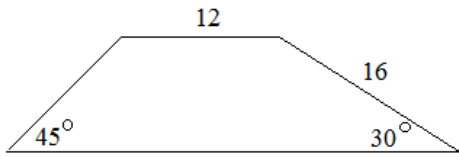***The diagonals of a rhombus are perpendicular bisectors!

Then, recognizing the 8-15-17 Pythagorean Triple.. the other diagonal is 30

$$\text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{2}(16)(30) = 240 \text{ sq. units}$$

Example: What is the area and perimeter of the trapezoid?

To find the area, we'll need to determine the altitude (height).

Fortunately, the altitudes form special right triangles.

30-60-90 right triangle: hypotenuse is 2x the small leg..
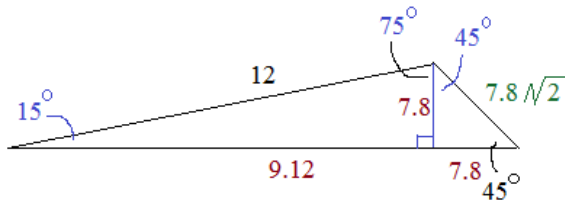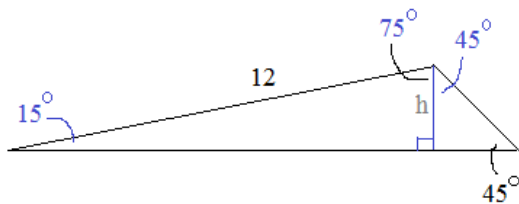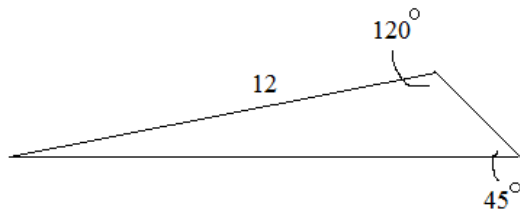
So, the height is 8...

Then, the other leg is $8\sqrt{3}$

And, since the height is 8, the other part of the base is 8... 45-45-90 right triangle...

Perimeter is the sum of all the sides: $48 + 8\sqrt{2} + 8\sqrt{3}$

Area $= \frac{1}{2}$(base1 + base2)(height) $= \frac{1}{2}(32 + 8\sqrt{3})(8)$

$= 128 + 32\sqrt{3}$

Example: Find the area and perimeter of the triangle.

First, we drop an altitude and see that we can use trigonometry and geometry tools to find the other parts...

$\sin(15°) = \dfrac{\text{height}}{12}$ height = 7.8   (approximately)

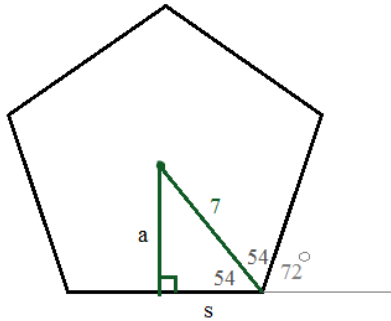Since the right part is 45-45-90 right triangle, the right part of the base is 7.8

Then, using the Pythagorean Theorem the left part of the base is 9.12

$9.12^2 + 7.8^2 = 12^2$

Perimeter of triangle $= 12 + 16.92 + 11.03 = 39.95$
                                                approximately

Area $= \frac{1}{2}(16.92)(7.8) = 66$ approximately

mathplane.com

Example: What is the perimeter and area of
a regular pentagon with radius of 7?

Method 1: Area = $\frac{1}{2}$ (apothem)(perimeter)

From the diagram,   Exterior angle = $\frac{360}{5}$ = 72°

Therefore, each interior angle is $180 - 72 = 108^{\circ}$

$$\sin(54) = \frac{a}{7}$$

apothem a = $7\sin(54) = 5.66$

$$\cos(54) = \frac{s}{7}$$

1/2 of each side = s = $7\cos(54) = 4.11$

apothem = 5.66
perimeter = 10 x 4.11 = 41.1

Area = $\frac{1}{2}$ (5.66)(41.1) = 116.3 (approximately)

Method 2: Find area of triangle then,

$$Area_{pentagon} = (\frac{1}{2}(base)(height)) \times 10 \text{ triangles})$$
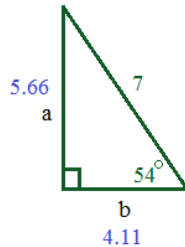
$$\sin(54) = \frac{a}{7}$$

a = $7\sin(54) = 5.66$

b = $7\cos(54) = 4.11$

Area of triangle: $\frac{1}{2}(4.11)(5.66) = 11.63$

Area of pentagon: 10 x 11.63 = 116.3

note: to check, use
pythagorean theorem:

$4.11^2 + 5.66^2 = 7^2$

approximately

Example: Find the area of 15 sided regular n-gon with perimeter 180 feet.

To find the area, we want the apothem and perimeter...
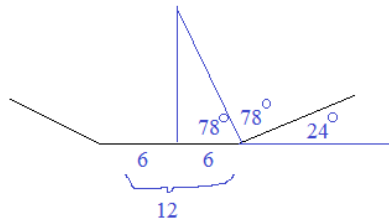
The perimeter is given: 180 feet...

Area = $\frac{1}{2}$ (apothem)(perimeter)

To find the measure of the apothem, draw a diagram:

Sum of the exterior angles is 360°.
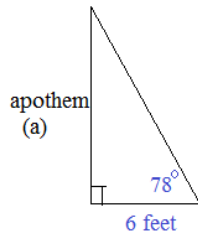So, each exterior angle measure
is 360/15 = 24 degrees...

therefore, each interior angle
is 180 - 24 = 156 degrees....

And, the radius bisects
the angle into 78 degree angles..

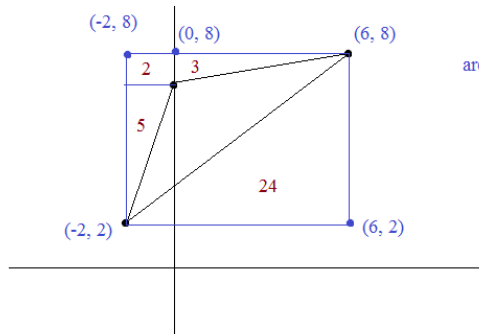Area = $\frac{1}{2}$ (28.23 feet)(180 feet) = 2540 square feet
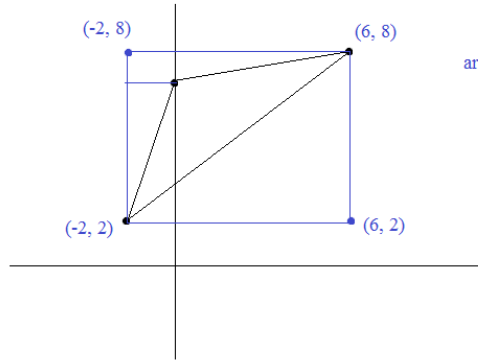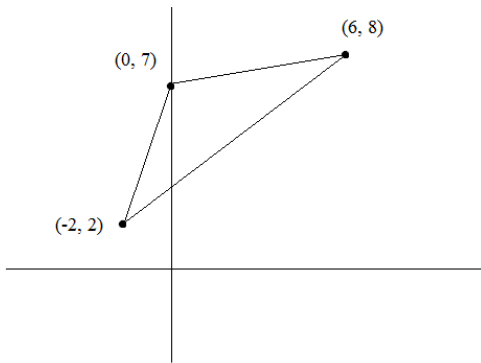
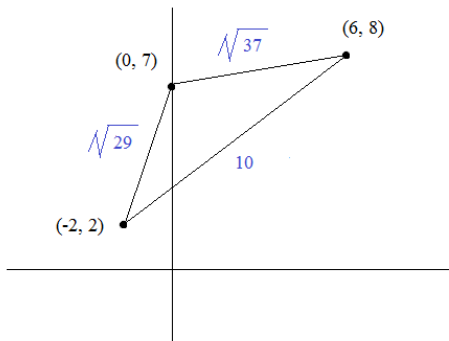$\frac{180}{15}$ = 12 feet/side

$$\tan(78) = \frac{a}{6}$$

apothem a = $6\tan(78) = 28.23$

Example: The vertices of triangle ABC are   A (-2, 2)   B (0, 7)   C (6, 8)

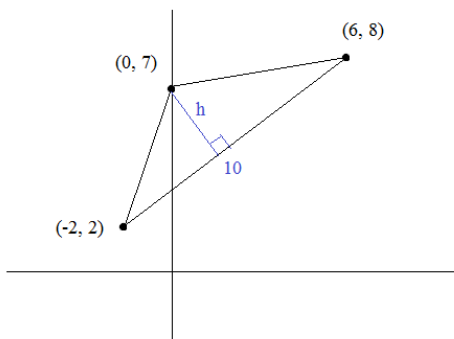Find the area of the triangle.

Method 1: Encasement

(6, 8)
(0, 7)
(-2, 2)

(-2, 8)   (6, 8)
(-2, 2)   (6, 2)

area of rectangle: 48

(-2, 8)   (0, 8)   (6, 8)

2 | 3

5

24

(-2, 2)   (6, 2)

area of rectangle: 48

minus 34...

area is 14

Method 2: Hero's Formula

(6, 8)
(0, 7) $\sqrt{37}$
$\sqrt{29}$
10
(-2, 2)

semiperimeter = $\dfrac{\sqrt{37} + \sqrt{29} + 10}{2}$ = s   approximately 10.73

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{10.73(4.65)(5.35)(.73)}$

= 14

Method 3: Finding the height...

(6, 8)
(0, 7)
h
10
(-2, 2)

Area = $\dfrac{1}{2}$ (base)(height)

We need to find the height...

line segment through (-2, 2) and (6, 8) $y - 2 = \dfrac{3}{4}(x + 2)$

slope of height is -4/3

line segment through (0, 7) $y = \dfrac{-4}{3}x + 7$

find the intersection using substitution:

$\dfrac{-4}{3}x + 7 - 2 = \dfrac{3}{4}(x + 2)$

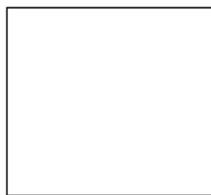$\dfrac{-4}{3}x + 5 = \dfrac{3}{4}x + \dfrac{3}{2}$

$-16x + 60 = 9x + 18$

$42 = 25x$

$x = \dfrac{42}{25}$

$y = \dfrac{-168}{75} + 7$

$y = \dfrac{357}{75}$

h = distance between (0, 7) and ($\dfrac{42}{25}$, $\dfrac{357}{75}$)

$\sqrt{2.82 + 5.02}$ = 2.8

Area = $\dfrac{1}{2}$ (base)(height) $\Longrightarrow$ $\dfrac{1}{2}$ (10)(2.8) = 14

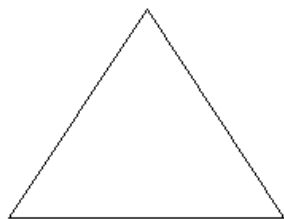Part I: Answer the following:

Square; apothem = 5 feet

What is the area?
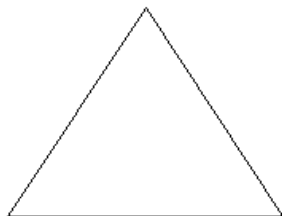
Square; diagonal = 16cm

What is the area?

What is the length of the apothem?

Equilateral triangle; sides = 7 inches

What is the area?

Equilateral triangle; apothem = 5

What is the area?

What is the length of each side?

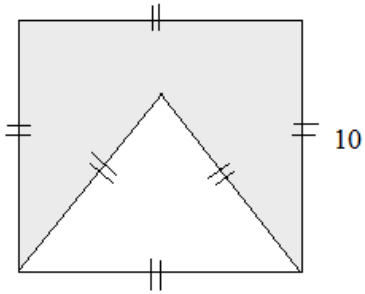Rectangle; diagonal = 26
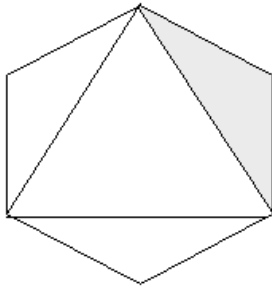   width = 10

What is the perimeter?

What is the area?

Part II: Find the area of each shaded region:
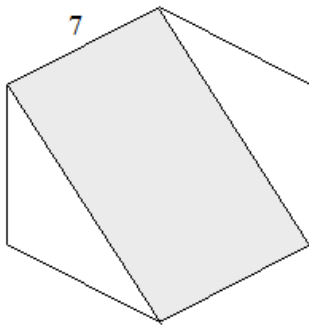
Area of shapes and regular polygons

1) Triangle inside a square:

2) A regular hexagon with sides measuring 6 feet:

3) Rectangle inscribed in a regular hexagon:

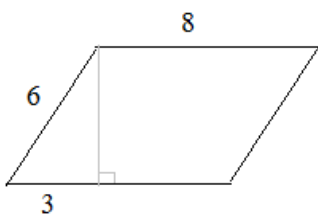Part III: Find the area of a *regular* hexagon, if

   a) the length of a side is 8

   b) the length of an apothem is 8

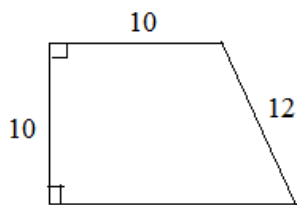   c) the length of a radius is 8

Part IV:

Parallelogram:

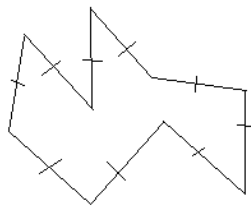   What is the perimeter?

    What is the area?

Trapezoid:

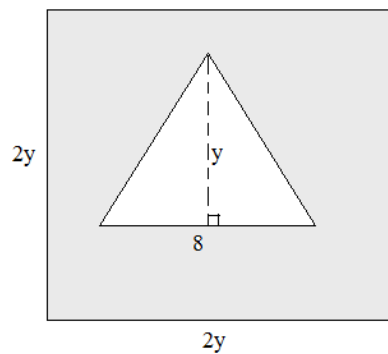   What is the perimeter?

    What is the area?

Part V:

1) Find the angles formed by

    a) 2 consecutive radii

    b) the radius and adjoining side

       in a regular   1) pentagon

            a)

            b)

        2) hexagon

            a)

            b)

        3) octagon

            a)

            b)

        4) decagon

            a)

            b)
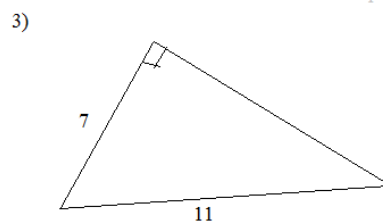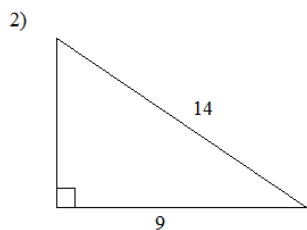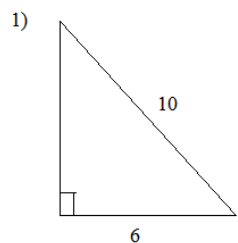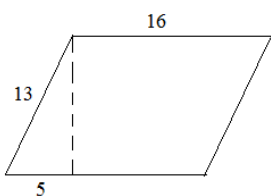
2) Is this a regular 9 sided polygon?

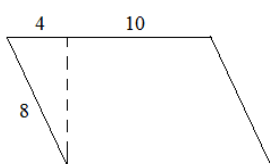3) If the area of the shaded region is 168 square units, what is y?
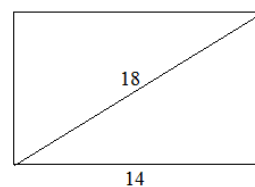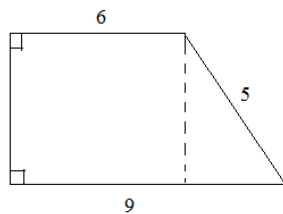
Find the area and perimeter of each figure.
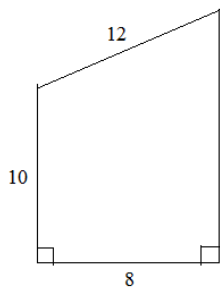
1)

10

6

2)

14

9

3)

7

11

4) (parallelogram)

16

13

5

5) (parallelogram)

4   10

8

6) (rectangle)

18

14

7)

6

5

9

8)

12

10

8

9) (kite)

5

4

7

10) (circle)

8

6

11)

13   12

10
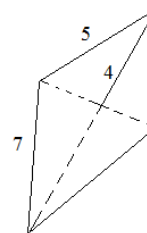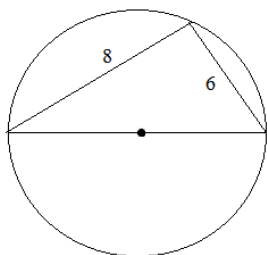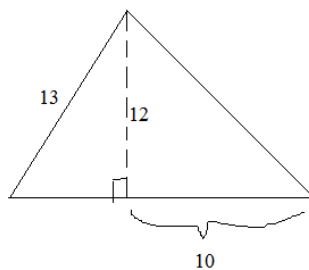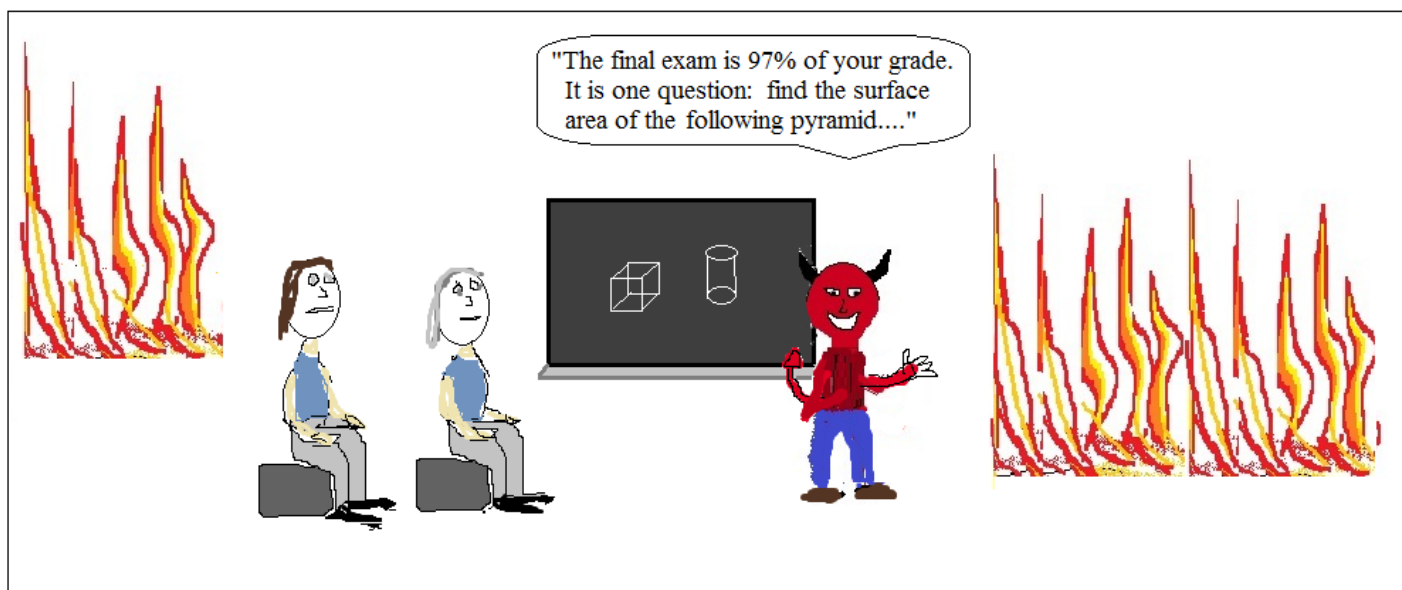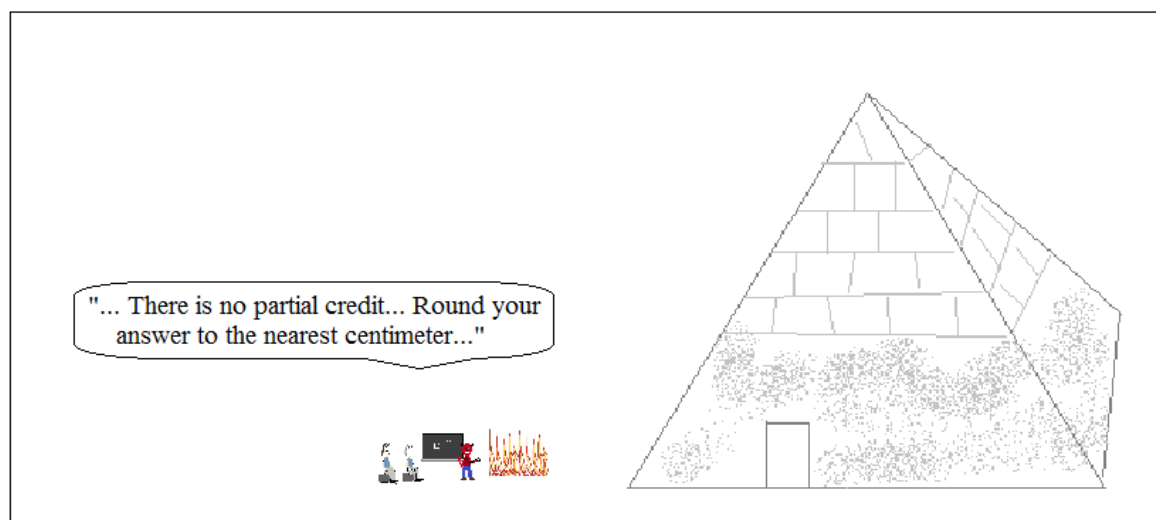
Solutions -→

Part I: Answer the following: **SOLUTIONS** | Area of shapes and regular polygons |
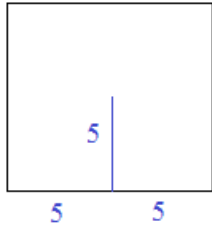
Square; apothem = 5 feet

What is the area?

The apothem connects the center to the midpoint of a side..
(If the apothem is 5', then 1/2 of each side is 5')

$$area = (side)(side) = (10ft)(10ft) = \boxed{100 \text{ square feet}}$$

Square; diagonal = 16cm

What is the area?

What is the length of the apothem?

Since the diagonal divides the square into two 45-45-90 triangles, the sides are
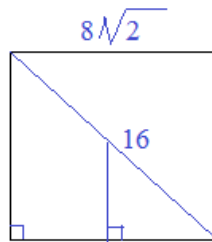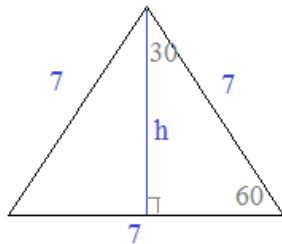$$\frac{16}{\sqrt{2}} = 8\sqrt{2}$$

$$Area = (side)^2 = \boxed{128 \text{ square cm}}$$

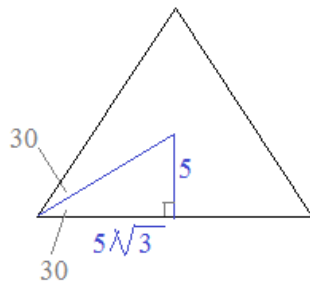Then, the apothem is 1/2 of each side: $\boxed{4\sqrt{2} \text{ cm}}$

Equilateral triangle; sides = 7 inches

What is the area?

$$area = \frac{1}{2}(base)(height)$$

Since it is an equilateral △ the altitude forms a 30-60-90 right triangle...

$$area = \frac{1}{2}(7)(\frac{7}{2}\sqrt{3}) = \boxed{\frac{49\sqrt{3}}{4} \text{ sq. units}}$$

Equilateral triangle; apothem = 5

What is the area? $\boxed{75\sqrt{3}}$

What is the length of each side?

$\boxed{\text{each side is } 10\sqrt{3}}$

Since it is an equilateral △, the apothem and radius will form a 30-60-90 right △ with small side = 5

the area of the right triangle is
$$\frac{1}{2}(base)(height) = \frac{1}{2} \cdot 5\sqrt{3} \cdot 5$$

and, since there are six right triangles inside the entire triangle, the area is $\quad 75\sqrt{3}$
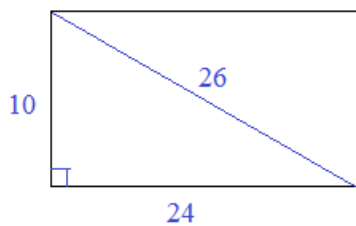
Rectangle; diagonal = 26
width = 10

What is the perimeter? $\boxed{68}$
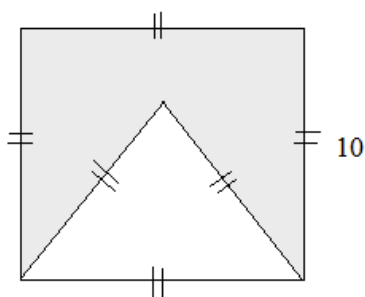
What is the area? $\boxed{120}$

The diagonal is the hypotenuse of a right triangle.. Using pythagorean theorem (or recognizing 5-12-13), the length of the rectangle is 24

Part II: Find the area of each shaded region: SOLUTIONS | Area of shapes and regular polygons |
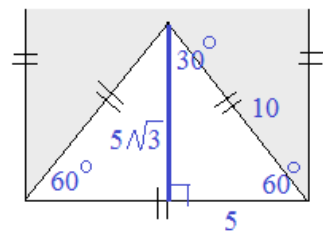
1) Triangle inside a square:

The figure is an equilateral triangle inside a square.

Area of square $= s^2 = (10)^2 = 100$

Area of triangle $= \dfrac{1}{2}$ (base)(height)

$= \dfrac{1}{2} (10)(5\sqrt{3})$

$= 25\sqrt{3}$

shaded area $= A_S - A_T$

$\boxed{= 100 - 25\sqrt{3}}$

30-60-90 right $\triangle$
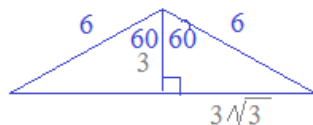
2) A regular hexagon with sides measuring 6 feet:
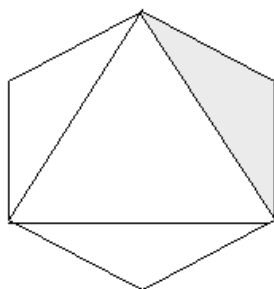
Each interior angle measures 120 degrees:

$\dfrac{(6-2) \cdot 180}{6} = 120$
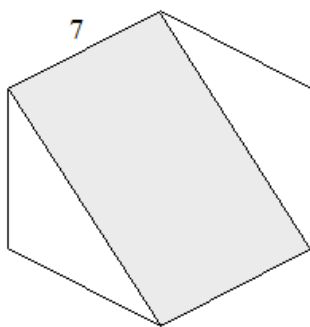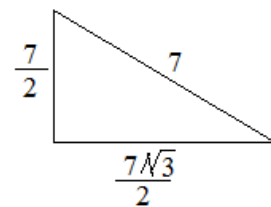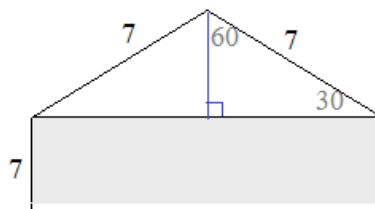
Base of the triangle $= 6\sqrt{3}$

Height of the triangle $= 3$

Area $= \dfrac{1}{2} bh = \boxed{9\sqrt{3}}$ square feet

3) Rectangle inscribed in a regular hexagon:

Area of rectangle $=$ length x width...
So, we need to find the length:

So, the length of the entire triangle base (i.e. the length of the rectangle) $= 7\sqrt{3}$

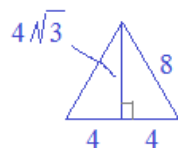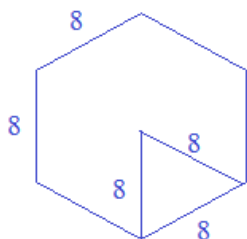Area rectangle $= 7 \times 7\sqrt{3} = \boxed{49\sqrt{3}}$

Part III: Find the area of a *regular* hexagon, if   SOLUTIONS   Area of shapes and regular polygons

a) the length of a side is 8

Each interior angle of a regular hexagon is 120 degrees...
So, there are 6 equilateral triangles inside the hexagon...
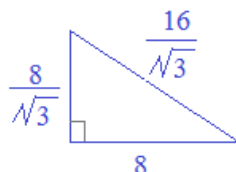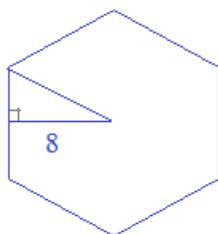
8

8   8

8

8

8

$4\sqrt{3}$ 8

4   4

Area of equilateral triangle = $16\sqrt{3}$

therefore, the area of the hexagon is
6 x (area of triangle) = $96\sqrt{3}$ 166.3

b) the length of an apothem is 8

Again, we have a 30-60-90 triangle..
The big length is 8... So, the small length is
$\dfrac{8}{\sqrt{3}}$

8

$\dfrac{8}{\sqrt{3}}$ $\dfrac{16}{\sqrt{3}}$

8

The area of this right triangle is
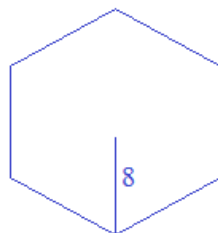
$\dfrac{1}{2}(8)(\dfrac{8}{\sqrt{3}}) = \dfrac{32}{\sqrt{3}}$

therefore, the are of the hexagon is
12 x (area of triangle) = $\dfrac{384}{\sqrt{3}}$ 221.7
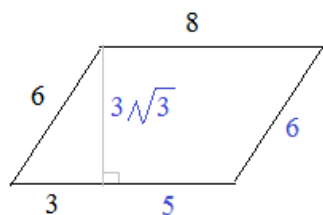
c) the length of a radius is 8

Since this is a (regular) hexagon, the length of a radius is the same as the length of a side...

8

Therefore, the area is the same as the hexagon in a)

$96\sqrt{3}$

Part IV:

8

6          $3\sqrt{3}$ 6

3   5

Parallelogram:

What is the perimeter?   opposite sides of parallelogram are congruent, so perimeter = 6 + 8 + 6 + 8 = 28

What is the area?   since the right triangle has hypotenuse 6 and base 3, it must be a 30-60-90 right triangle..
area = (base)(height) = (8)($3\sqrt{3}$)   (the hypotenuse is 2x the length).. Therefore, the height of the parallelogram is $3\sqrt{3}$

$24\sqrt{3}$

10

10   10   12

10   x
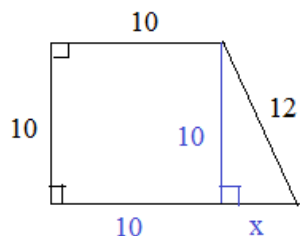
Trapezoid:

What is the perimeter?
$10 + 10 + 10 + 12 + 2\sqrt{11} =$

What is the area? $42 + 2\sqrt{11}$

Area = $\dfrac{1}{2}$(base1 + base2)(height)

= $\dfrac{1}{2}(20 + 2\sqrt{11})(10) =$ $100 + 10\sqrt{11}$

Pythagorean Theorem:

$(10)^2 + (x)^2 = (12)^2$
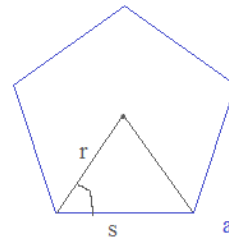
$x = \sqrt{44} = 2\sqrt{11}$

Part V:

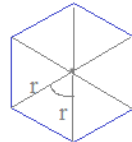Area of shapes and regular polygons

1) Find the angles formed by

   a) 2 consecutive radii

   b) the radius and adjoining side

     in a regular   1) pentagon

         a) $72°$

         b) $54°$

        2) hexagon

         a) $60°$

         b) $60°$

        3) octagon

         a) $45°$

         b) $62.5°$

        4) decagon

         a) $36°$

         b) $72°$
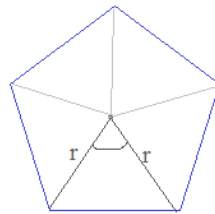
$360 \div 5 = 72°$

Since the 5 triangles are isosceles....

angle + angle + 72 = 180

angle = $54°$

$360 \div 6 = 60°$
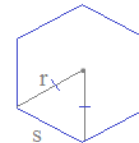
Since the radii angle is 60, the other two angles add up to 180..

Therefore, the other angles are 60 and 60..

An octagon has 8 congruent radii angles...

$360 \div 8 = 45°$

Since the radii angle is 45 degrees.. the 2 base angles must add up to 135 degrees...

62.5 and 62.5

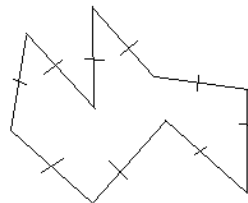Decagon has 10 sides.. 10 congruent radii angles.. 36 degrees

The radius/side angles are 72 degrees...

2) Is this a regular 9 sided polygon?

     No, it is not...

     It is 'equilateral'...

        but, it is not 'equiangular'...

3) If the area of the shaded region is 168 square units, what is y?
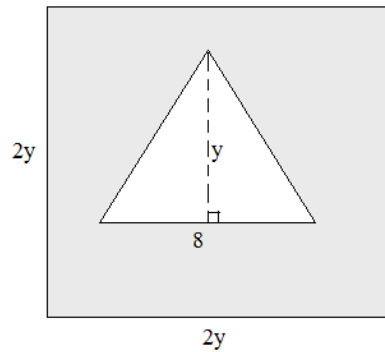
$(2y)^2 - \frac{1}{2}(8)(y) = 168$

$4y^2 - 4y = 168$

$4\left(y^2 - y - 42 \right) = 0$

$4(y + 6)(y - 7) = 0$

$y = -6$ or 7... (but, it cannot be negative...)

$y = 7$

2y

y

8

2y

Check:

Area of square: 14 x 14 = 196

Area of triangle: 1/2(8)(7) = 28

Shaded area = 196 - 28 = 168 ✓
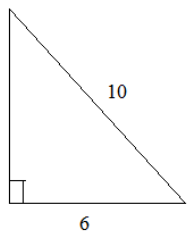
Find the area and perimeter of each figure.   mathplane.com   SOLUTIONS   Pythagorean Theorem
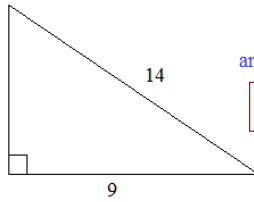
1)

$10^2 - 6^2 = b^2 \quad b = 8$

area $= \frac{1}{2}$ (base)(height)

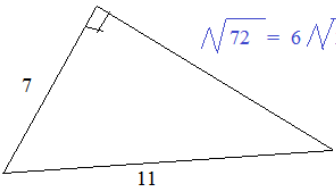$= \frac{1}{2}$ (6)(8)

$= 24$

8   10   6

perimeter: 24

2)

$14^2 - 9^2 = b^2 \quad b = \sqrt{115}$

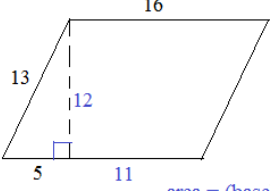area $= \frac{9}{2}\sqrt{115}$

approx 48.26

14    $\sqrt{115}$ 9

perimeter: 23 + $\sqrt{115}$

3)

area $= 21\sqrt{2}$

$\sqrt{72} = 6\sqrt{2}$

7   11

perimeter: 18 + $6\sqrt{2}$

4) (parallelogram)

16   13   12   5   11

$5^2 + 12^2 = 13^2$
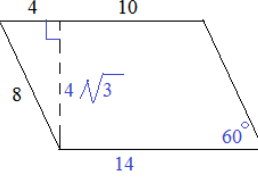
area = (base)(height)

$= (16)(12) = 192$

perimeter: 58

5) (parallelogram)

4   10   8 $4\sqrt{3}$ 14   60°
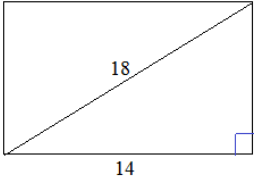
area $= 56\sqrt{3}$

perimeter: 44

6) (rectangle)

18    $\sqrt{128}$ $= 8\sqrt{2}$ 14
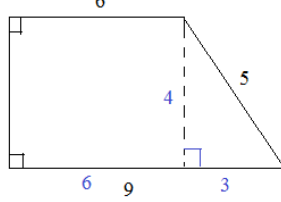
area $= 112\sqrt{2}$

perimeter $= 28 + 16\sqrt{2}$

7)

6   4   5   4   6   9   3
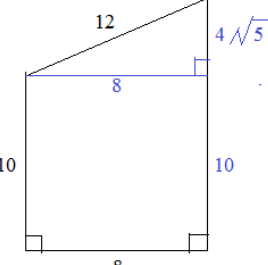
perimeter: 24

area $= \frac{1}{2}$ (base1 + base2)(height)

$= \frac{1}{2}$ (9 + 6)(4) $= 30$

8)
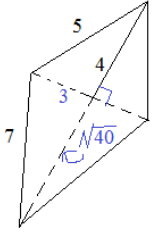
12    $4\sqrt{5}$ 8   10   10   8

area $= 80 + 16\sqrt{5}$

perimeter $= 40 + 4\sqrt{5}$

9) (kite)

5   4   3   7 $\sqrt{40}$

perimeter $= 24$

area $= \frac{1}{2}$ (diagonal 1)(diagonal 2)

$\frac{1}{2}$ (6)(4 + $2\sqrt{10}$) $\approx$ 40.97

10) (circle)

8   6   5   5
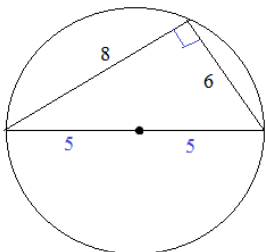
Note: triangle inscribed in semicircle is right triangle..

perimeter/circumference: 10π

area $= 25\pi$

11)

13   12 $2\sqrt{61}$ 5   10
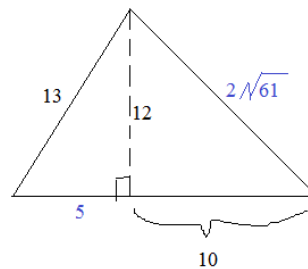
area $= (1/2)(15)(12) = 90$

perimeter $= 28 + 2\sqrt{61}$

Pythagorean Theorem: $a^2 + b^2 = c^2$

rectangle: area = (length)(width)
            perimeter = 2(length) + 2(width)

triangle: area = (1/2)(base)(height)
           perimeter = (side) + (side) + (side)

circle: area = π(radius)^2
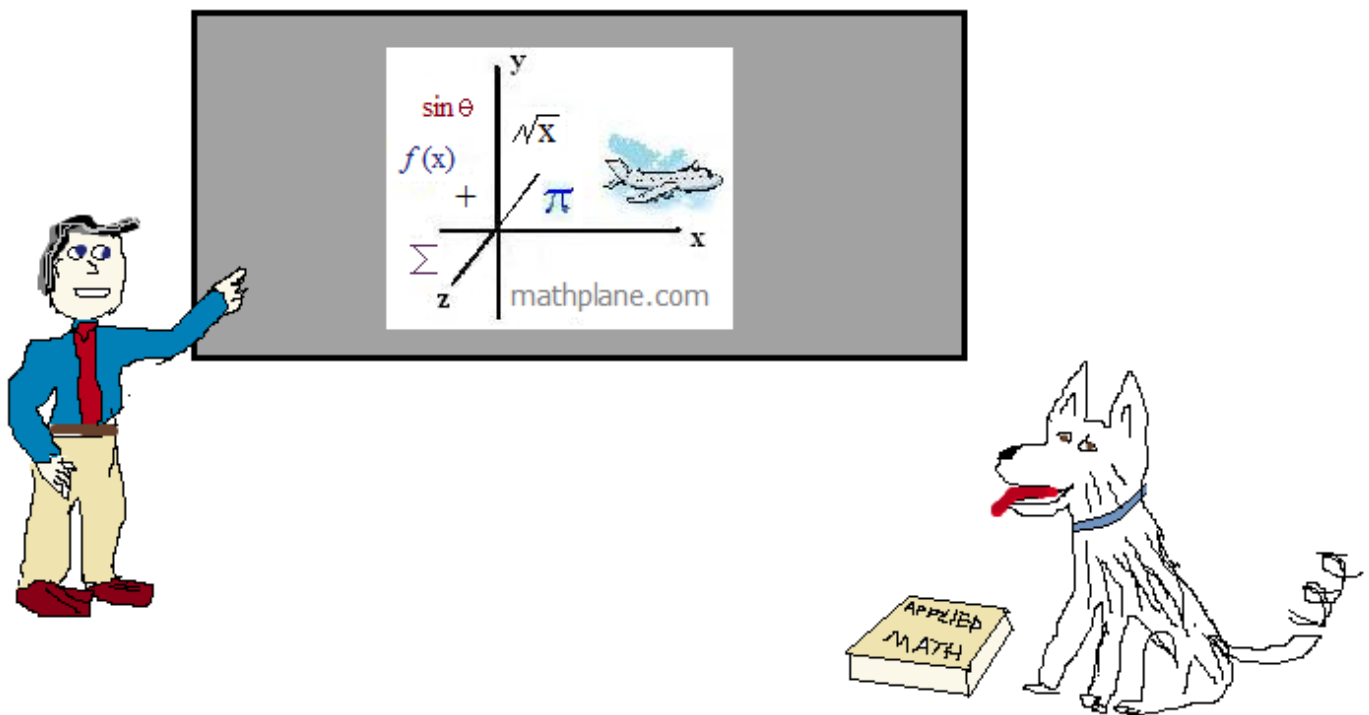
         circumference = 2π(radius)

kite: area = (1/2)(diagonal 1) (diagonal 2)
       (or, find area of each triangle)

trapezoid: area = (1/2)(base1 + base2)(height)

Thanks for visiting. (Hope it helped!)

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Enjoy