Area of Shapes (Honors)

Examples and Practice Test (with solutions)

Topics include ratios, properties of circles, trigonometry, triangles, quadrilaterals, and more.

Mathplane.com

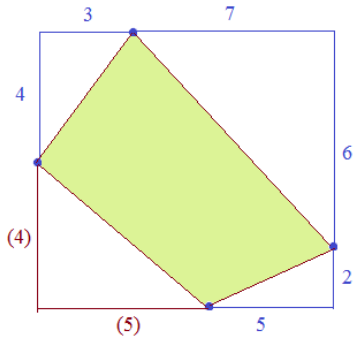
Example. A city has vertical and horizontal streets that form a city shaped in equal square grids/blocks.

Inside there is a park that is not aligned with the square grids. However, you can walk to the 4 corners of the park.

If you start at the first corner, you can walk 4 blocks north and 3 blocks east to reach the second corner.
Then, you can continue 7 blocks east and 6 blocks south to reach the third corner.
And, finally, 2 blocks south and 5 blocks west to reach the fourth corner.

What is the area of the park?

Encasement application



area of rectangle: 8 x 10 = 80

area of right triangles:

(1/2)(3)(4) = 6

(1/2)(7)(6) = 21

total: 42

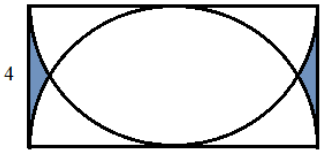
(1/2)(2)(5) = 5

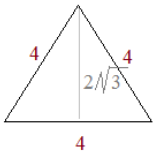
(1/2)(5)(4) = 10

area of the park: 80 − 42 = 38

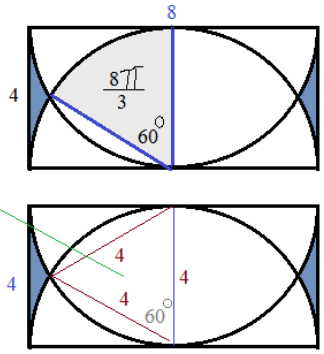
Example: 2 semicircles are inscribed inside the rectangle...

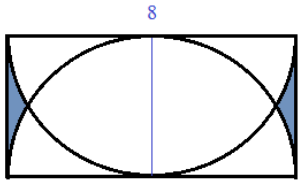
What is the shaded area?



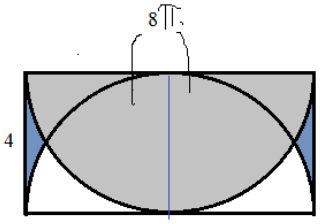


area of triangle is $4\sqrt{3}$

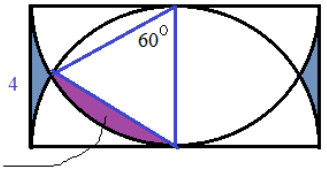




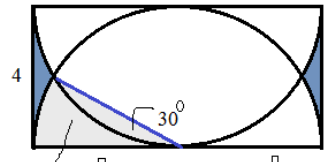
so, area of rectangle is 32



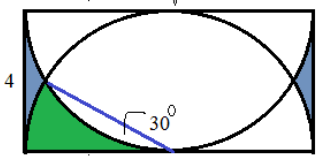
the area of the semicircle is 8π



$\frac{8\pi}{3} - 4\sqrt{3}$



$\frac{4\pi}{3}$



$\frac{4\pi}{3} - \left(\frac{8\pi}{3} - 4\sqrt{3}\right)$

$= 4\sqrt{3} - \frac{4\pi}{3}$

area of green section

SUMMARY:

Blue area = 32 (rectangle) − 8π (one semicircle) − $2\left(4\sqrt{3} - \frac{4\pi}{3}\right)$ (2 green sections)

$32 - 8\sqrt{3} - \frac{16\pi}{3}$ = 1.39 (approximately)

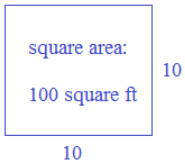
Example: You're looking to construct an outdoor enclosed area.
There are 2 choices: a square area or a circular area.

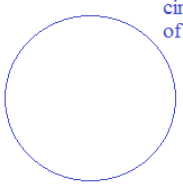
The material needed for the enclosure costs $3 per foot for straight borders and $4 per foot for bendable curved borders.

a) If you plan to use 40 feet of border material, which area will be larger?

b) If you plan to spend $180, which area would be larger?

a) If you use 40 feet of material....





circumference of 40 feet..

$C = 2\pi(\text{radius})$

$40 = 2\pi(\text{radius})$

$\text{radius} = 6.37$

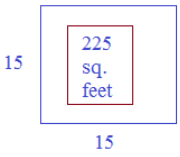
$\text{area} = \pi(\text{radius})^2$

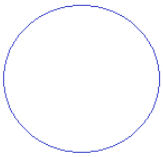
area = 127 feet (approx)

b) if you plan to spend $180.....

at $3 per square foot,

60 feet of border...





at $4 per square foot,

can afford 45 feet of bendable material...

$C = 2\pi(\text{radius})$

$45 = 2\pi(\text{radius})$

$\text{radius} = 7.16$

$\text{area} = \pi(\text{radius})^2$

area = 161 feet (approx)

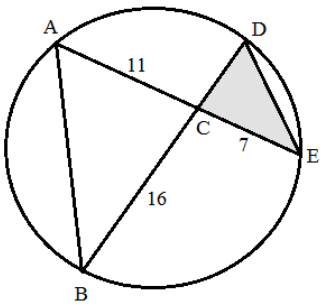
If length of material is constraint, then round area is greater...
But, if cost of material is constraint, then square area is greater!

Example: If the area of triangle ABC is 86,
what is the area of triangle CDE?

$\overline{AC} = 11$

$\overline{BC} = 16$

$\overline{CE} = 7$



$\overline{AE}$ and $\overline{BD}$ are intersecting chords...
therefore, (power theorem)

$(AC)(CE) = (BC)(CD)$

$(11)(7) = (16)(CD)$

$\overline{CD} = 4.8125$

Area of triangle $= \frac{1}{2}ab(\sin C)$

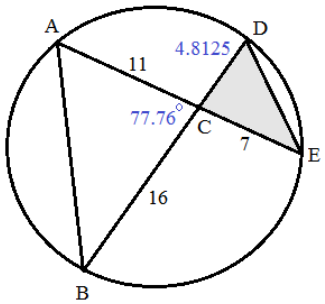
$86 = \frac{1}{2}(11)(16)\sin C$

$.977 = \sin C$

$\angle ACB = 77.76^\circ$

$\angle ACB = \angle DCE$ (vertical angles)

77.76 degrees



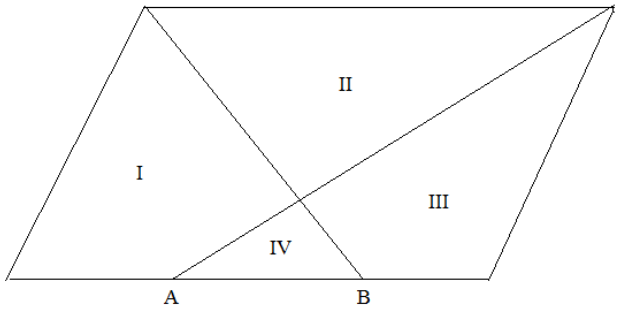
Area $\triangle CDE = \frac{1}{2}(4.8125)(7)\sin(77.76)$

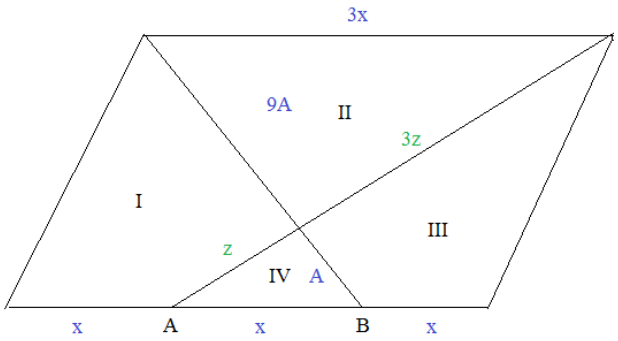
= 16.46

Example: Given: Parallelogram and

trisection points A and B

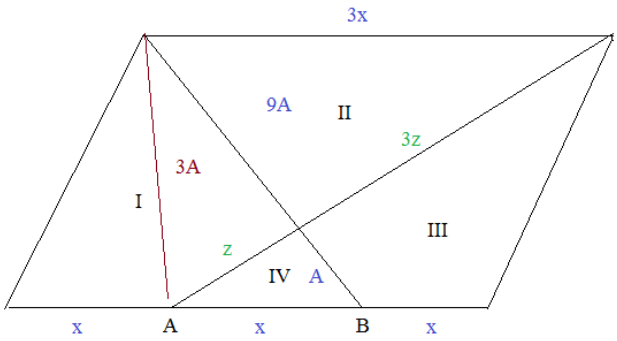
Find the ratio of the areas of the sections I : II : III : IV





Since triangles II and IV are similar (angle-angle) and their sides are proportional with a ratio of x:3x, the ratio of the areas is 1 : 9

Draw an auxilary line bisecting triangle I/IV



Since triangles II and IV are proportional 3 : 1, we know the sides z and 3z....

since triangles have same height and bases of ratio z : 3z,

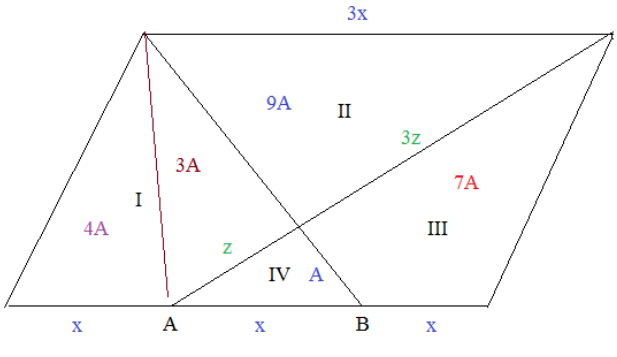
the triangles are 3A : 9A

Answer: I 7A

II 9A

III 7A

IV A



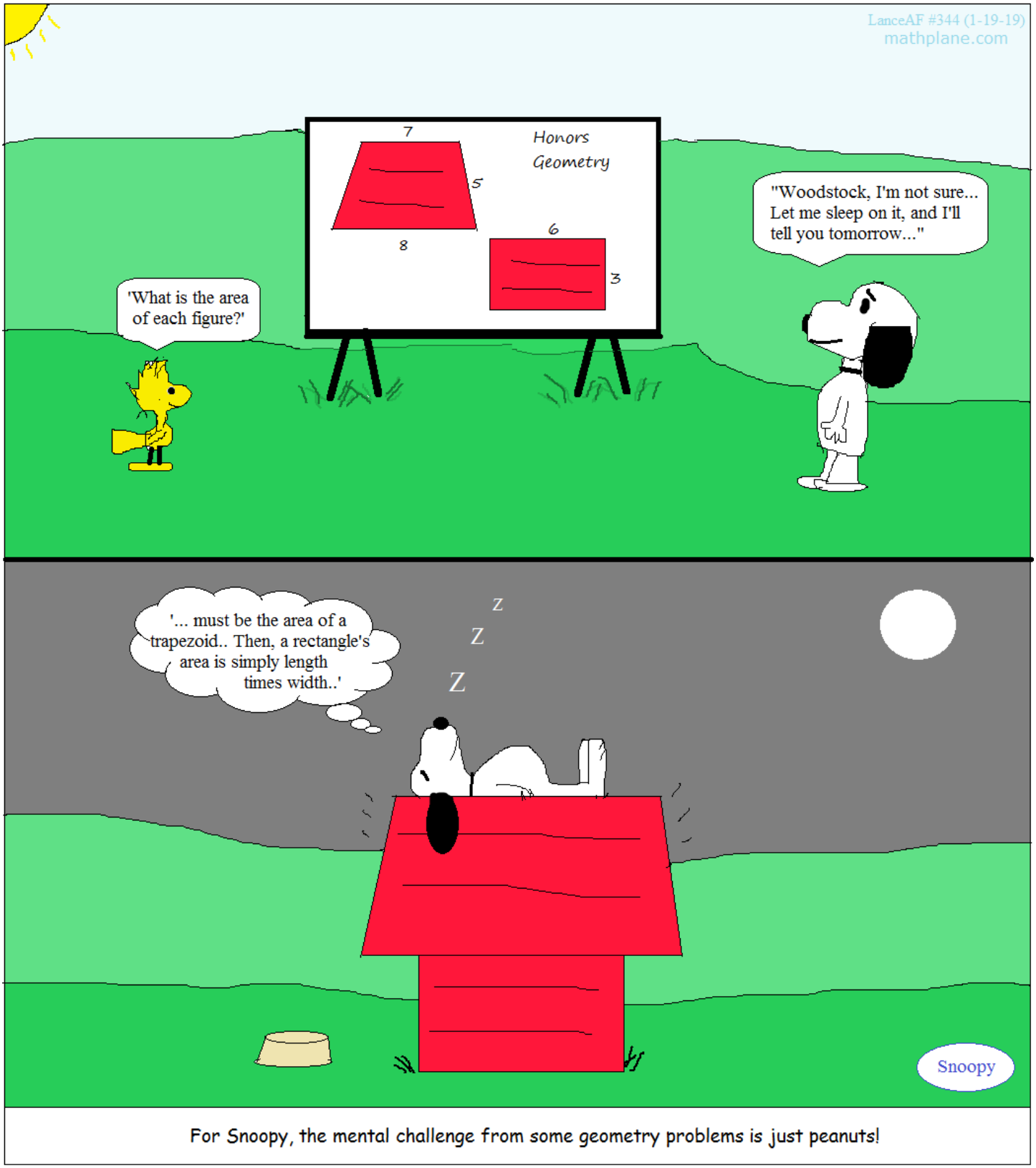
3A + A = 4A

because the triangles have the same base x and x and the same height...

Section I has an area of 4A (with a base of x)

Sections IV and III has an area of 8A because the base is 2x..

Both triangles have the same height...

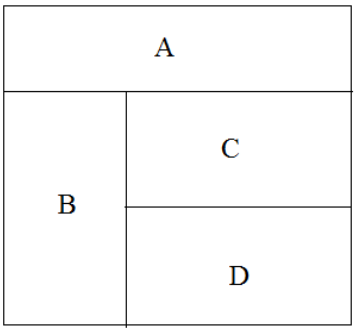


For Snoopy, the mental challenge from some geometry problems is just peanuts!

Practice Questions -→

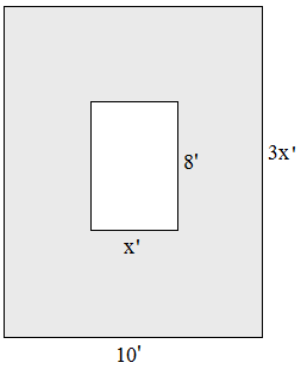
1) The area of the square is 81 units ^{2}.

If rectangles A, B, C, and D have equal areas, then what is the perimeter of rectangle C?



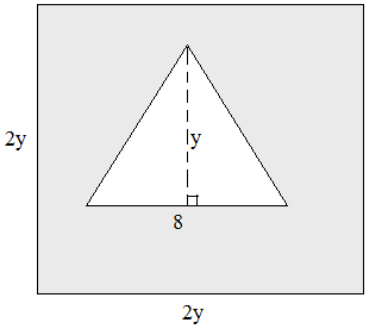
2a) Shaded area = 132 square feet

Find x.



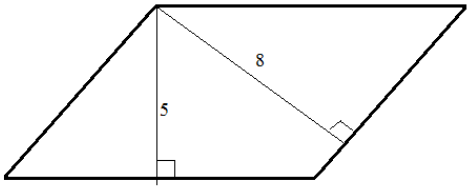
2b) Area of shaded area = 168 square units

Find y.

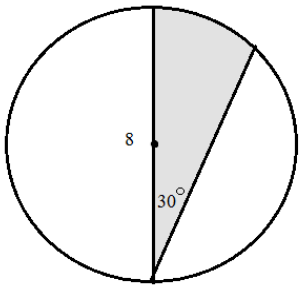


3) If perimeter is 104, then

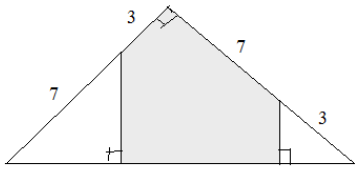
What is the area of the parallelogram?



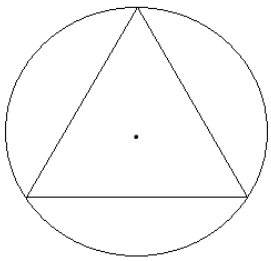
4) Find the shaded area...



5) What is the shaded area?



6) An equilateral triangle is inscribed in a circle with radius 12.
What is the area of the triangle?



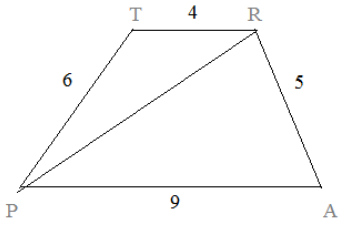
7) Find the area of parallogram with sides 10 and 18,
and an angle of 60 degrees....

8) Assuming TRAP is a trapezoid with bases $\overline{PA}$ and $\overline{TR}$, what are the following ratios?

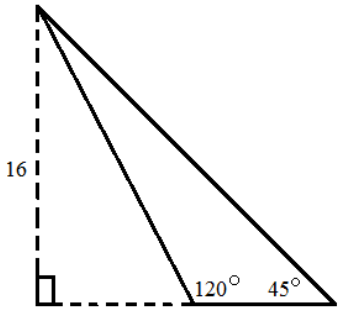
a) $\triangle TRP : \triangle PAR$

(ratio of areas)

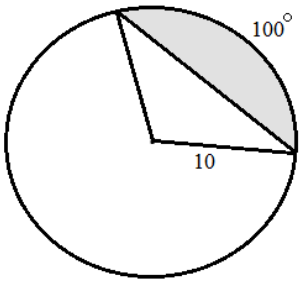
b) $\triangle TRP : \text{trapezoid } TRAP$



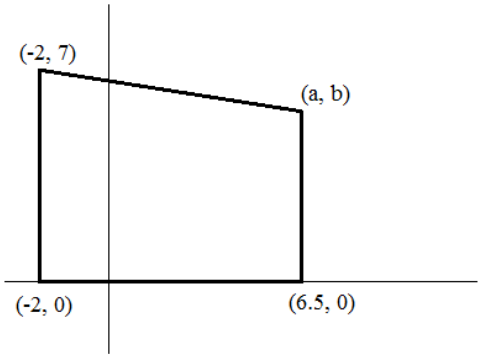
9) What is the area of the obtuse triangle?

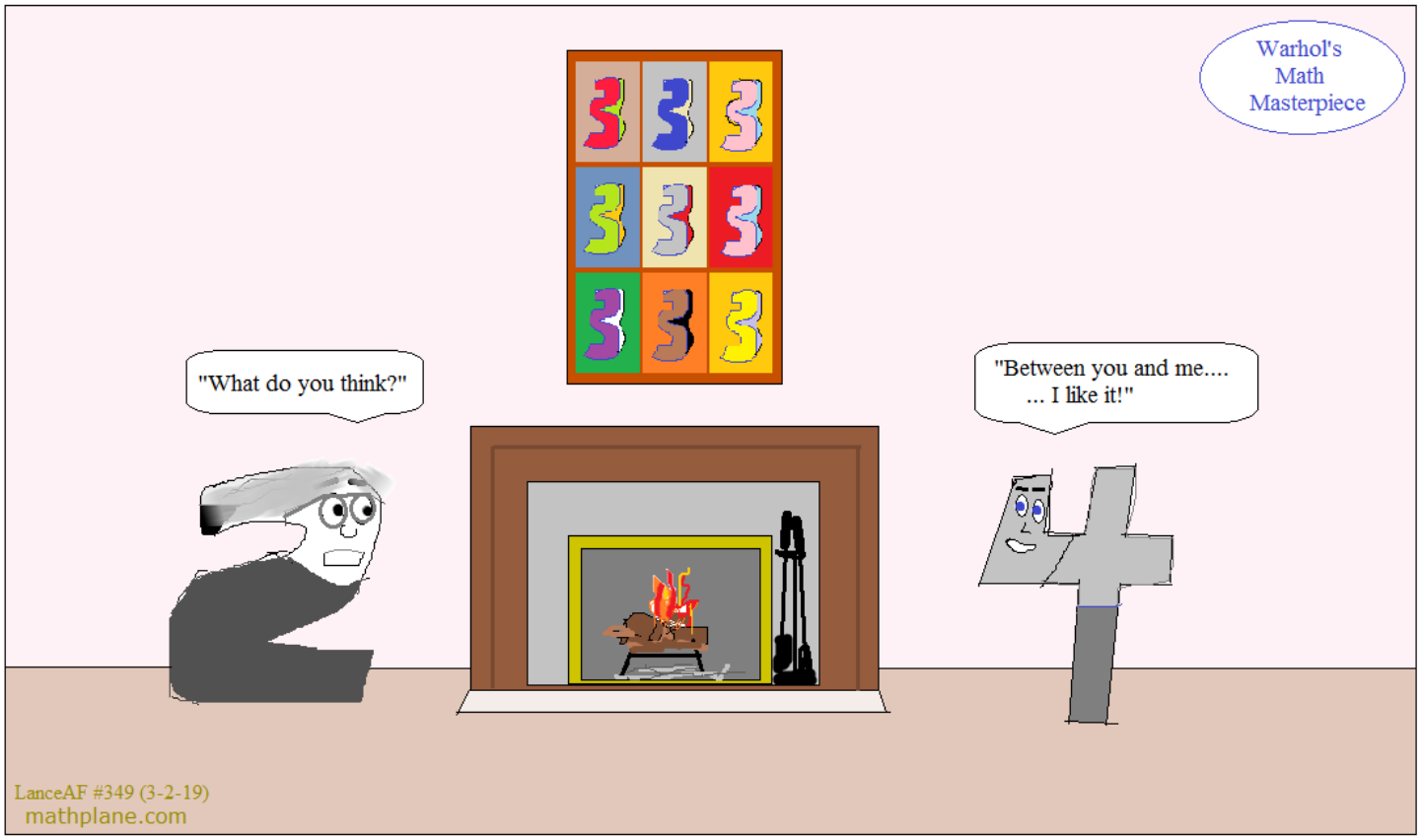


10) What is the area of the shaded segment?



11) If the area of the trapezoid is 53, what is the coordinate of (a, b)?

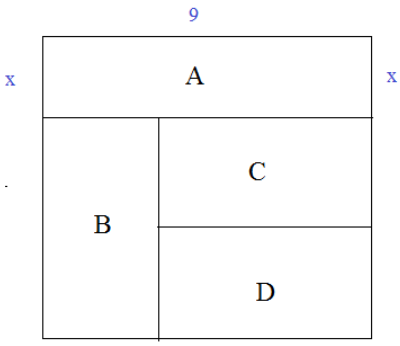



Warhol's
Math
Masterpiece
"What do you think?"
"Between you and me....
... I like it!"
LanceAF #349 (3-2-19)
mathplane.com

SOLUTIONS-→

1) The area of the square is 81 units ^{2}.

If rectangles A, B, C, and D have equal areas, then what is the perimeter of rectangle C?



SOLUTIONS

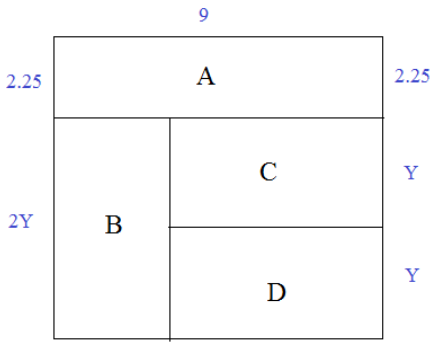
Step 1: Since the area of the square is 81, each side is 9...

And, the areas of A, B, C, and D = 81/4 = 20.25

area of A = 20.25 = 9x

x = 2.25

Step 2:

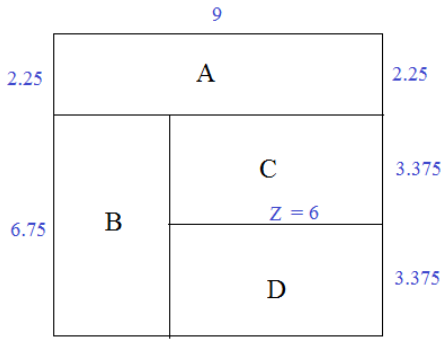


2Y + 2.25 = 9

2Y = 6.75

Y = 3.375

Step 3:



3.375 x Z = 20.25 Z = 6

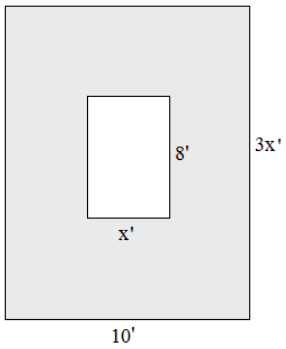
Step 4

Perimeter is

3.375 + 6 + 3.375 + 6 = 18.75

2a) Shaded area = 132 square feet

Find x.



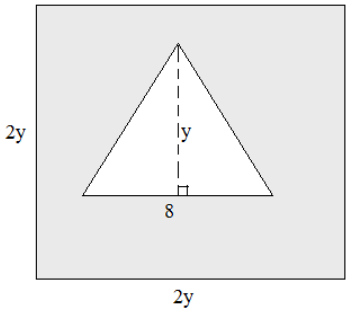
10(3x) − 8(x) = 132

22x = 132

x = 6

2b) Area of shaded area = 168 square units

Find y.



$(2y)^2 - \frac{1}{2}(8)(y) = 168$

$4y^2 - 4y = 168$

$4\left(y^2 - y - 42 \right) = 0$

4(y + 6)(y − 7) = 0

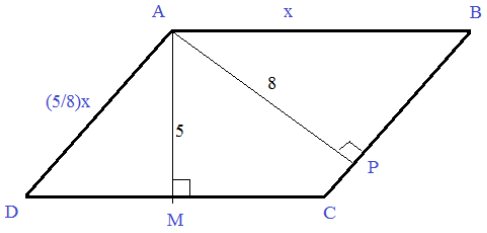
y = -6 or 7...

(but, it cannot be negative...)

y = 7

3) If perimeter is 104, then

What is the area of the parallelogram?



SOLUTIONS

Since it's a parallelogram,
angles B and D are congruent

Therefore, $\triangle ADM \sim \triangle ABP$

The ratio of sides is 5:8

$2 \cdot x + 2 \cdot (5/8)x = 104$

$2x + \frac{10}{8}x = 104$

$\frac{26}{8}x = 104$

$x = 32$

$(5/8)x = 20$

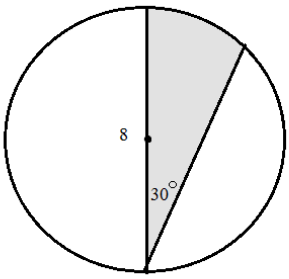
area = 32 x 5 = 160

OR

area = 20 x 8 = 160

4) Find the shaded area...

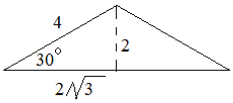
Method 1: Find area of semicircle and subtract the white segment...



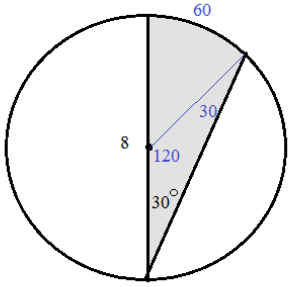
semicircle is 8π

white segment = sector − triangle

$= \frac{16\pi}{3} - 4\sqrt{3}$



$8\pi - \left(\frac{16\pi}{3} - 4\sqrt{3}\right)$



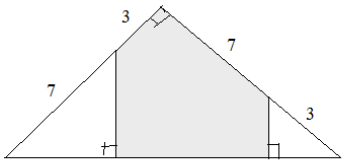
Method 2: Find area of sector plus triangle

area of triangle $4\sqrt{3}$

area of sector $\frac{1}{6}16\pi$

shaded area: $4\sqrt{3} + \frac{8}{3}\pi$

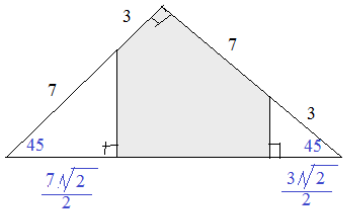
5) What is the shaded area?



This is a rotated right triangle...
The sides are 10 and 10, so it's a 45-45-90 triangle...

The area of entire triangle is (10 x 10)/2 = 50

Then, the two white right triangles:

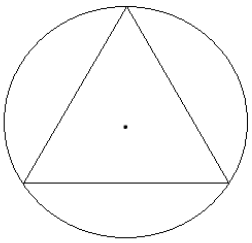


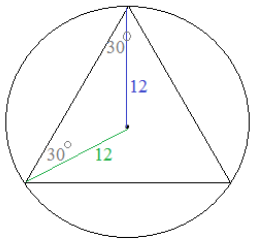
areas are 49/2 and 9/2

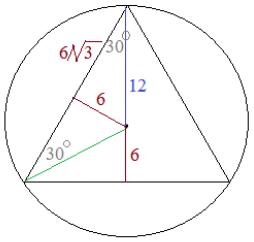
so, shaded area = 50 − 49/2 − 9/2 = 21

6) An equilateral triangle is inscribed in a circle with radius 12.
What is the area of the triangle?

SOLUTIONS







sides of triangle are $12\sqrt{3}$

height is 18

Each little right triangle is

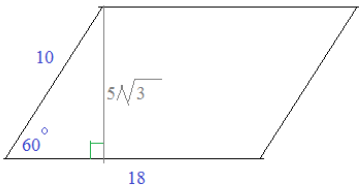
$\frac{1}{2}(6\sqrt{3})(6) = 18\sqrt{3}$

Therefore, area of entire equilateral triangle is

$6 \times 18\sqrt{3} = \boxed{108\sqrt{3}}$

7) Find the area of parallogram with sides 10 and 18,
and an angle of 60 degrees....

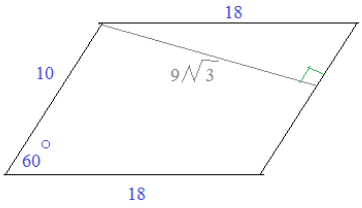
approach 1:



area = (base)(height)

$\boxed{90\sqrt{3}}$

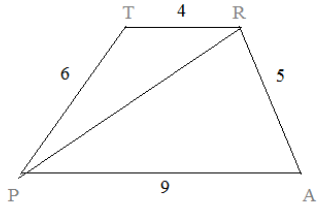
approach 2:



8) Assuming TRAP is a trapezoid with bases $\overline{PA}$ and $\overline{TR}$, what are the following ratios?

a) $\triangle TRP : \triangle PAR$ (ratio of areas)

b) $\triangle TRP : \square TRAP$



Since bases $\overline{TR}$ and $\overline{PA}$ are parallel,
the heights of each triangle will be the same....

TRP = (1/2)(4)(height)

PAR = (1/2)(9)(height)

$\boxed{\text{TRP:PAR} \quad 4:9}$

TRP = (1/2)(4)(height)

TRAP = (1/2)(4 + 9)(height)

$\boxed{\text{TRP:TRAP} \quad 4:13}$

SOLUTIONS

9) What is the area of the obtuse triangle?

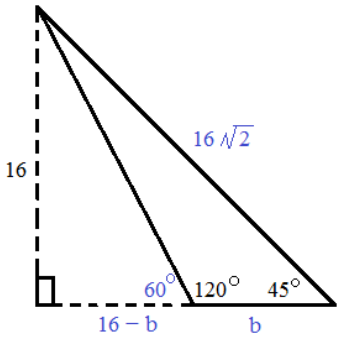
$$(16 - b) = \frac{16}{\sqrt{3}}$$

$$(16 - b) = 9.24$$

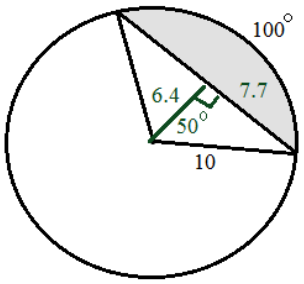
$$b = 6.76$$

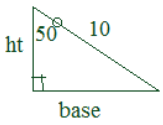
$$\text{Area} = \frac{1}{2}\,bh$$

$$\frac{1}{2}(6.76)(16) = 54.1$$



10) What is the area of the shaded segment?

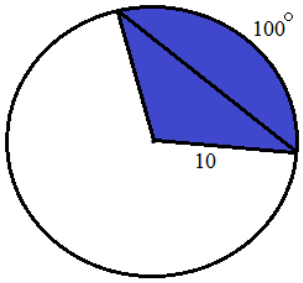




$$\frac{1}{2}(15.4)(6.4) = 49 \quad \text{TRIANGLE}$$

$$\sin(50) = \frac{\text{base}}{10} \quad \text{base} = 7.7$$

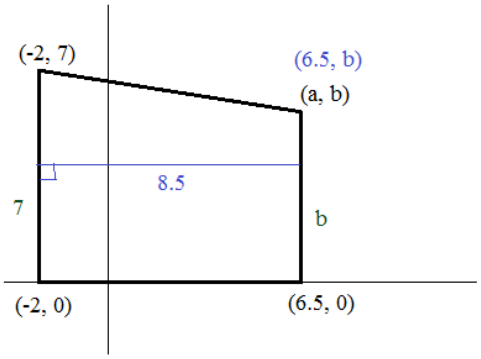
$$\cos(50) = \frac{\text{height}}{10} \quad \text{height} = 6.4$$



$$\left(\frac{100^0}{360^0}\right)\pi(10)^2 = 87.27 \quad \text{SECTOR AREA}$$

SECTOR − TRIANGLE = 38.3 SEGMENT

11) If the area of the trapezoid is 53, what is the coordinate of (a, b)?



Since it's a trapezoid, we know a = 6.5

and, the height is 8.5

$$\text{Area of trapezoid} = \frac{1}{2}(\text{base1} + \text{base2})(\text{height})$$

$$53 = \frac{1}{2}(7 + b)(8.5)$$

$$106 = (7 + b)(8.5)$$

$$12.47 = 7 + b$$

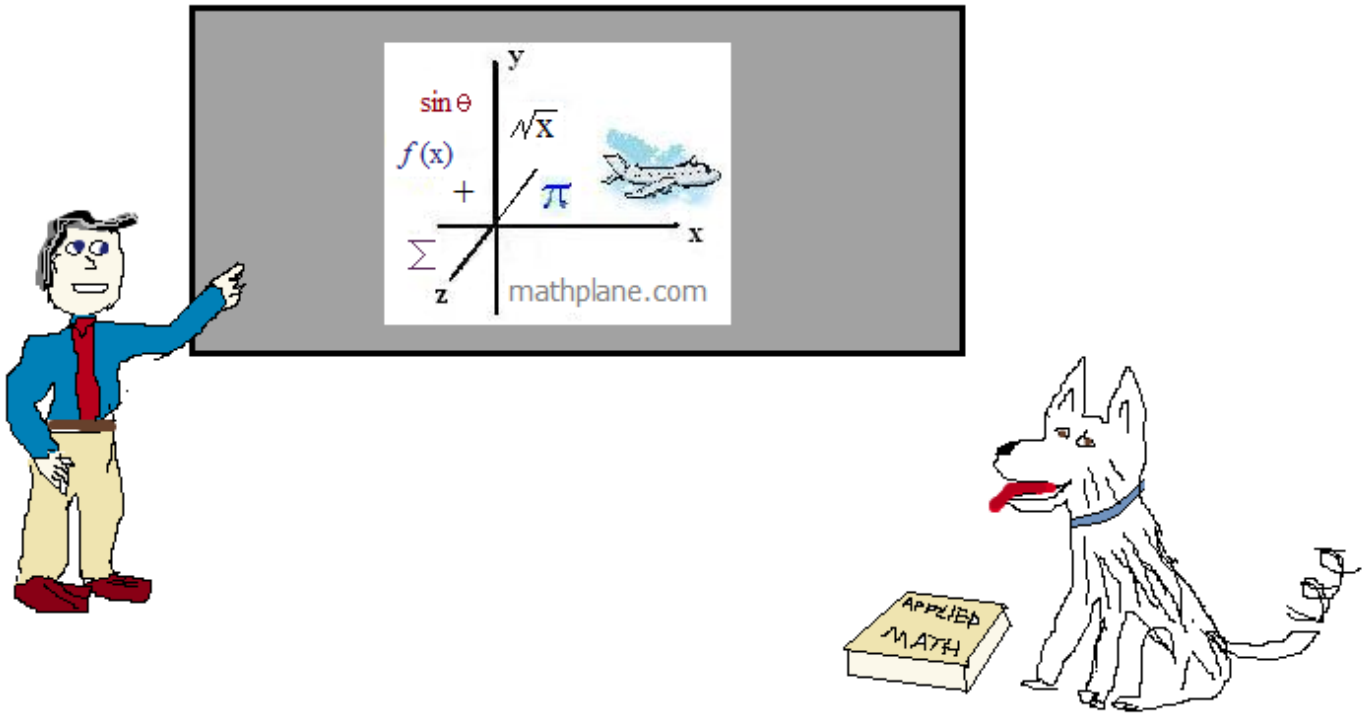
$$b = 5.47$$

(6.5, 5.47)

Thanks for visiting!

Hope the review helped.

If you have questions, suggestions, or requests, let us know.



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