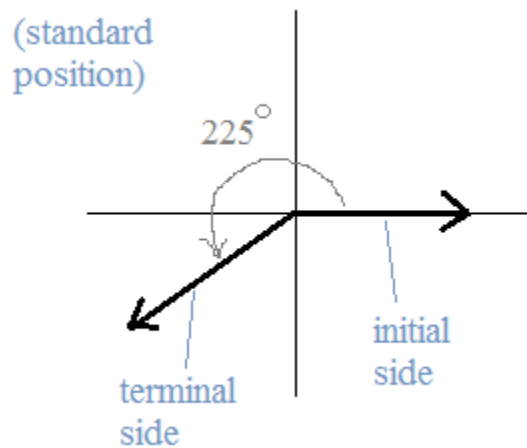# Trigonometry:

# Angle Measurement

Notes, Examples, Practice Quiz, and Puzzle (with Solutions)



Includes coterminal and reference angles, Degrees/Minutes/Seconds, angle vs. linear speed, radians, and more.

Degrees/Minutes/Seconds Measurement of Angles

Angle measurements are not always whole numbers.

For example: when you bisect a $45^\circ$ angle, what is the measure of the resulting angles?

$$\frac{45^\circ}{2} = ?$$

The solution contains a 'fractional degree'..

It can be expressed in 'decimal form'     $22.5^\circ$
Or,
It can be expressed in 'DMS form'     $22^\circ 30'$

Circle: $360^\circ$    (360 degrees)

1 degree = 60' (60 minutes)

1 minute = 60" (60 seconds)

## Degrees/Minutes/Seconds (DMS) Form

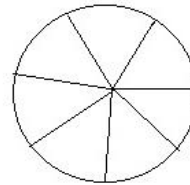Angle measurements can be expressed using standard divisions of a degree.

| 1 degree = 60 minutes | $1^\circ = 60'$ |
|---|---|
| 1 minute = 60 seconds | $1' = 60''$ |
| Then, 1 degree = 3600 seconds | $1^\circ = 3600''$ |

Example:   Suppose you divide a circle into 7 equal parts.
What is the measure of each angle?

$$\frac{360^\circ}{7} = \boxed{51.429^\circ}$$    Decimal Form (rounded to 3 decimal places)

$$51.429^\circ = 51^\circ + .429^\circ \times \frac{60'}{1^\circ}$$

$$= 51^\circ + 25.74'$$

$$= 51^\circ + 25' + .74' \times \frac{60''}{1'}$$

$$= \boxed{51^\circ + 25' + 44.4''}$$    DMS Form

Example:   Write $72^\circ 15' 23''$ in Decimal Form (rounded to 3 places)

$$72^\circ 15' 23''$$

$$72^\circ + \frac{15^\circ}{60} + \frac{23^\circ}{3600}$$

$$72^\circ + .25^\circ + .006^\circ$$

approximately $72.256^\circ$

$$72^\circ = 72^\circ$$

$$15' \cdot \frac{1^\circ}{60'} = .25^\circ$$

$$23'' \cdot \frac{1^\circ}{3600} = .006^\circ$$

$$72.256^\circ \text{ total}$$
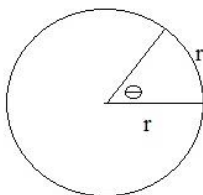
Radian Measurement of Angles

Formal Definition: Measure on an angle with vertex at the center of a circle that subtends an arc equal to the radius of the circle.
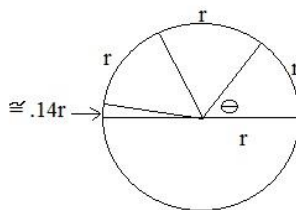
What does that mean?

If you take the radius of the circle and lay it on the circle, the angle formed by that arc is ONE RADIAN.

$180° = \pi$ radians

(In other words, if you laid the radius around the circle, it would take approx. 3.14 radii to cover half of the circle.. )

$\cong .14r \rightarrow$

It follows that the degree measure of an angle that is 1 radian would be

$$\frac{180°}{\pi} \cong 57.3°$$

Converting Radians/Degrees

$$180° = \pi \text{ radians} \implies \frac{180°}{\pi \text{ radians}} = 1 = \frac{\pi \text{ radians}}{180°}$$

*Examples:*

Degrees ---> Radians

$$60° \cdot \frac{\pi \text{ radians}}{180°} = \frac{\pi}{3} \text{ radians (rad)}$$

$$360° \cdot \frac{\pi \text{ radians}}{180°} = 2\pi \text{ radians}$$

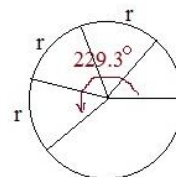$$147° \cdot \frac{\pi \text{ radians}}{180°} \cong 0.817\,\pi \text{ radians}$$

$$\cong 0.817 \cdot (3.14) \text{ radians}$$

approximately 2.56 radians

Radians ---> Degrees

$$\frac{\pi}{2} \text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = 90°$$

$$\frac{7\pi}{3} \text{ rad} \cdot \frac{180°}{\pi \text{ radians}} = 420°$$

$$4 \text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = \frac{720°}{\pi} = \frac{720°}{3.14}$$

$$\cong 229.3°$$

229.3°

(r = length of radius)

Angular vs. Linear Speed

*Example:* A bicycle wheel spins at a rate of 400 rotations/minute.
If the diameter of the wheel is 26",

    a) what is the *angular* speed?

    b) what is the *linear* speed?

a) Angular speed describes the amount of distance covered in terms of *angles* and time.

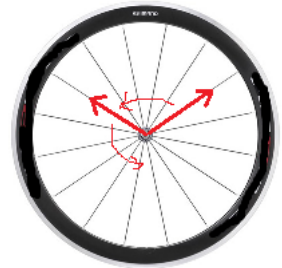If a bicycle wheel (or any circle) goes around once, the angular distance is $360°$ or $2\pi$ radians

So, if the bicycle wheel rotates 400 times, the angular distance is $400 \cdot 360°$

$$= 144{,}000 \text{ degrees/minute}$$
$$\text{or}$$
$$= 800\pi \text{ radians/minute}$$
$$\text{approx. } 2513 \text{ radians/minute}$$

b) Linear speed describes the distance covered by a point on the circumference path of the rotating item.

Suppose a little person went around the bicycle wheel one time. He would travel the circumference of the wheel:

$$\text{circumference} = 2\pi \text{ radius} \quad \text{or} \quad \pi \text{ diameter}$$

Since the wheel's circumference is $\pi$ x 26 inches = 81.68 inches,

the linear distance of 400 trips around would be $400(\pi \text{ x } 26 \text{ inches}) = 32{,}672$ inches

Therefore, the linear speed of the wheel is approximately $32{,}672$ inches/minute
or 2723 feet/minute

Now, suppose we measure the angular and linear speed of the bicycle rim.
Again, the wheel spins at a rate of 400 rotations/minute.
If the radius of the rim is 11 inches (diameter is 22 inches), then

    a) what is the angular speed?

    b) what is the linear speed?

a) Since the number of rotations/minute is the same, the angular speed is the same!

$$360 \frac{\text{degrees}}{\text{rotation}} \text{ x } 400 \frac{\text{rotations}}{\text{minute}} = 144{,}000 \frac{\text{degrees}}{\text{minute}}$$

b)
$$\text{linear speed} = \frac{\text{distance traveled}}{\text{time}}$$

$$= \frac{400 \text{ rotations} \cdot 22\pi \text{ inches/rotation}}{1 \text{ minute}} = 27{,}645 \text{ inches/minute} \quad \text{or } 2304 \text{ feet/minute}$$
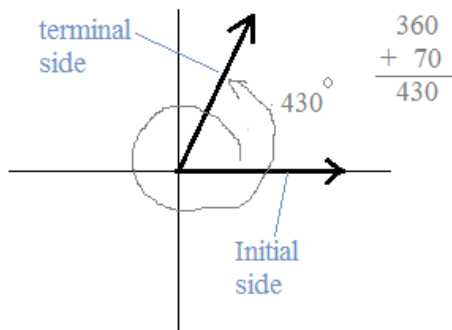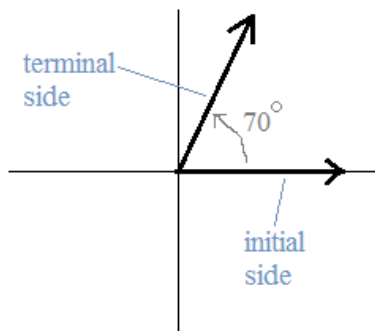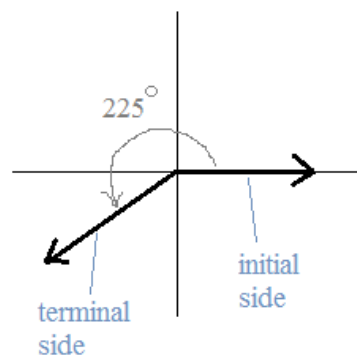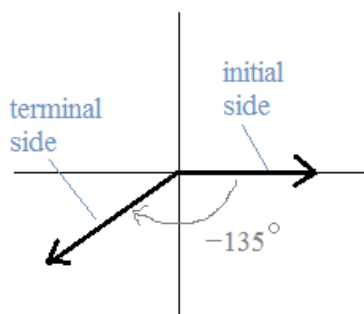$$\text{(approximately)}$$

Coterminal vs. Reference Angles

Coterminal Angles:   Angles that share the same terminal side  (when drawn in standard position)

*Examples:*

70 degree and 430 degree
angles are coterminal

terminal
side

$70^\circ$

initial
side

terminal
side

$430^\circ$

Initial
side

$$\begin{array}{r} 360 \\ + \phantom{0}70 \\ \hline 430 \end{array}$$

-135 degree and 225 degree
angles are coterminal

note:  $-135^\circ + 360^\circ = 225^\circ$

terminal
side

initial
side

$-135^\circ$

$225^\circ$

initial
side

terminal
side

If the angle is
measured in radians,
adding or subtracting
$2\pi$ will reveal
coterminal angles

$\dfrac{2\pi}{3}$

$\dfrac{2\pi}{3} + 2\pi$
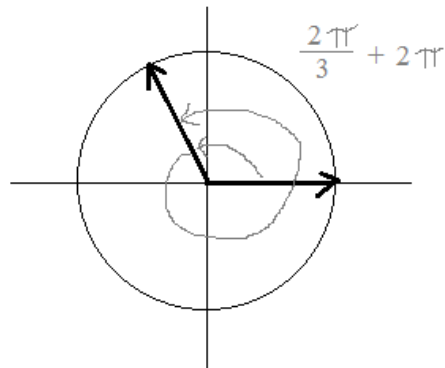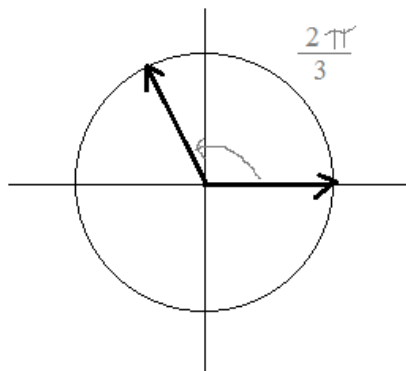
Question:  Can you identify one *positive* and one *negative* coterminal angle to 140 degrees?

Here is a $140^\circ$ angle in *standard position:*

$140^\circ$
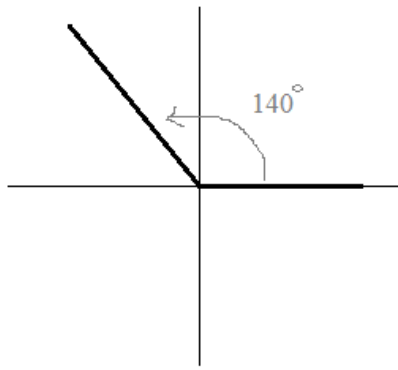
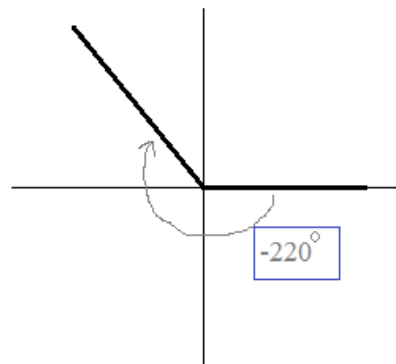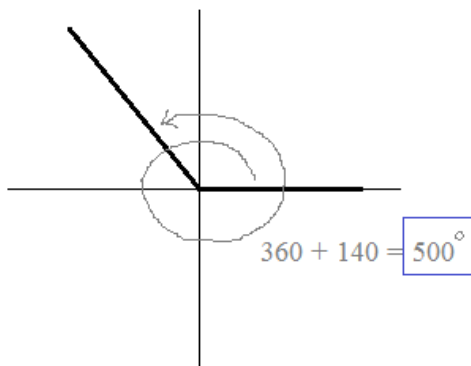**To find coterminal angles, add/subtract $360^\circ$

$140^\circ, 500^\circ, 860^\circ, ...$

360      360

... $-580^\circ, -220^\circ, 140^\circ$

360      360

So, one positive coterminal angle to $140^\circ$ is $500^\circ$     and   one negative coterminal angle to $140^\circ$ is $-220^\circ$

$360 + 140 = \boxed{500^\circ}$

$\boxed{-220^\circ}$

Question:  Are  $217^\circ$ and  $-143^\circ$ coterminal angles?

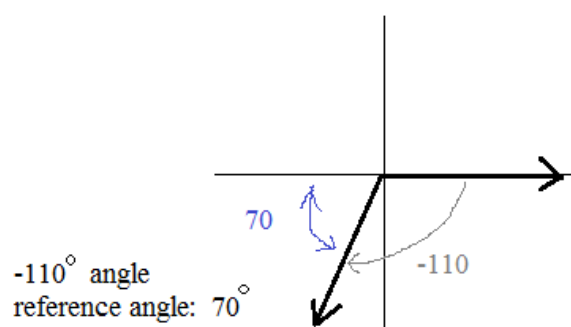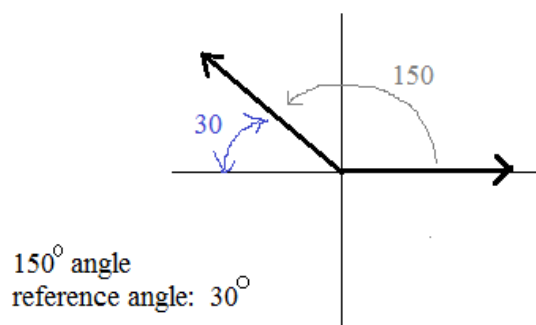Since $-143 + 360 = 217$, these are coterminal angles..

Question:  What are _all_ the coterminal angles to $-20^\circ$ ?

One way to express the answer:  $340^\circ + 360^\circ n$ , where n is any integer...

Reference angle: the *acute* angle between the x-axis and the terminal side of an angle (in standard position)
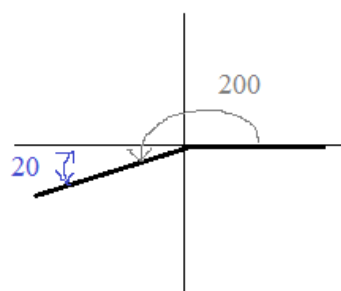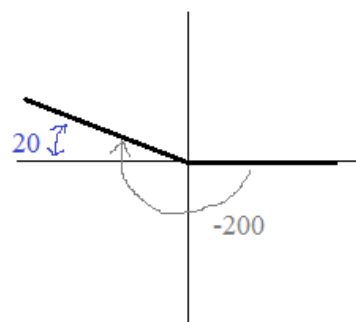
("an acute angle version of an angle")

*Examples:*

150° angle
reference angle: 30°

-110° angle
reference angle: 70°

Observations: 1) since reference angles are measures, they have a positive value
2) any acute angle in quadrant I has an identical reference angle

38° angle
reference angle: 38°

angle ⊖: -x°
reference angle: x°
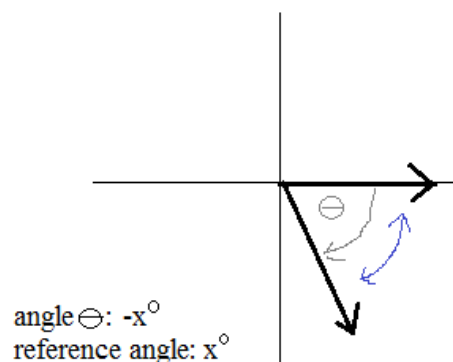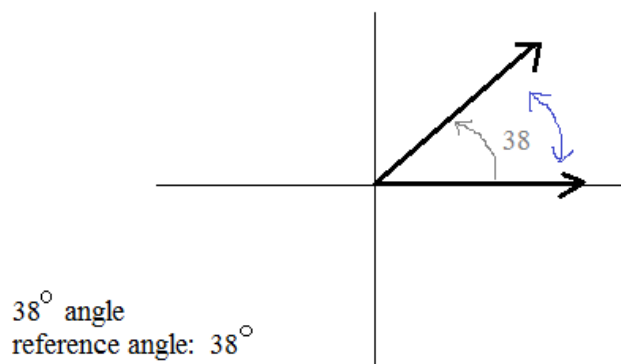
Questions: Find the reference angles for 200° and -200°

The reference angles for 200 and -200 are the same!

It's 20°

Trig

Newtons

*Math snacks made with real symbols!*

Ingredients: sine, cosine, degrees, angles

3.14 oz.
baked at 360 degrees

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (*t*), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

PRACTICE TEST (w/SOLUTIONS)-→

Measuring Angles Quiz
*(Decimal Form, DMS, Radians, Degrees)*

1) Write  $21^\circ 18' 49''$  using decimal degrees.

2) Write  $88.297^\circ$  using DMS  (Degrees, Minutes, & Seconds)

3)  $\angle A$  and  $\angle B$  are complementary angles.

$\angle A = 62^\circ 15' 23''$      What is the measure of  $\angle B$ ?

4)  At  2:30pm, what is the angle measure between the hour and minute hands?
Express your answer in decimal degree form  <u>and</u>  DMS form.

5)  What is the radian measure of  $120^\circ$?  $75^\circ$ ?

6)  Convert  a) $\dfrac{\pi}{6}$  radians to degrees.   b) 3.6 radians to degrees

I. Coterminal Angles

   1)  Determine if the following pairs are coterminal:

       a) $57^\circ$   $357^\circ$                        c) $-80^\circ$   $280^\circ$

       b) $-20^\circ$   $160^\circ$                        d) $-40^\circ 30'$   $319^\circ 30'$


   2) Identify one *positive* and one *negative* coterminal angle to -30 degrees



   3) Identify one *positive* and one *negative* coterminal angle to $\dfrac{\pi}{3}$     (use radian measures)



   4)  Write an expression that describes *all* coterminal angles to 100 degrees



II. Reference Angles

   Identify the reference angle for each angle:

   1)  $110^\circ$                    2) $42^\circ$                    3) $-72^\circ$
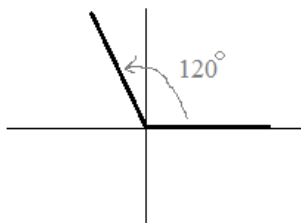


   4)  $210^\circ$                    5)  $-150^\circ$                    **6) $180^\circ$



III. Coterminal vs Reference Angles

   For each of the following,
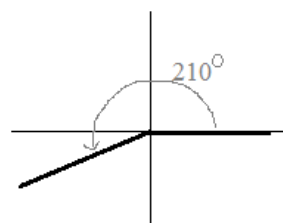   identify *the reference* angle
   and *any coterminal* angle:

   A)

   

   Reference Angle:

   Coterminal Angle:

   B)

   

   Reference Angle:

   Coterminal Angle:
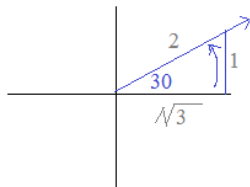
Trigonometry angle measurements and terms

1)  30°

reference angle?   30°

coterminal angle?   ex: 390, -330

sine?   $\frac{1}{2}$

cosine?   $\frac{\sqrt{3}}{2}$

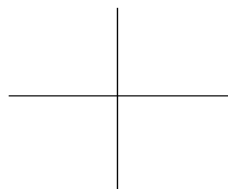convert to radians:  $30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$ radians



2)  −60°

reference angle?

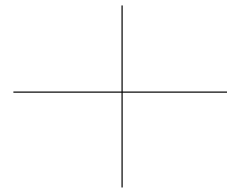coterminal angle?

sine?

cosine?

convert to radians:

3)  −120°

reference angle?

2 coterminal angles?

tangent?

cosine?

convert to radians:
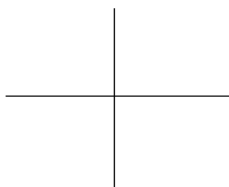
4)  $\frac{3\pi}{4}$  radians

reference angle?

coterminal angle?

sine?

tangent?

convert to degrees:

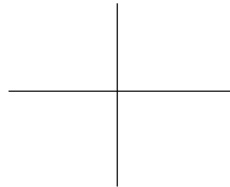5)  $\frac{-\pi}{6}$  radians

reference angle?

2 coterminal angles?

sine?

cosine?

convert to degrees:
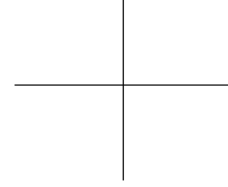
6)  $\frac{9\pi}{4}$  radians

reference angle?

coterminal angle?

sine?

tangent?
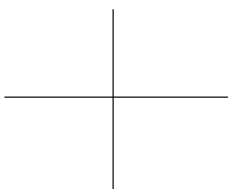
convert to degrees:

7)  90°

coterminal angle?

tangent?

sine?

cosine?

convert to radians:

8)  $\pi$ radians

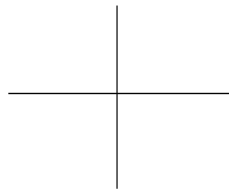negative coterminal angle?

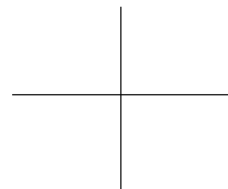positive coterminal angle?

sine?

cosine?

convert to degrees:

9)  480°

reference angle?

coterminal angle?

sine?

cosine?

convert to radians:

1) Given:    $\sin\ominus = \dfrac{3}{5}$    $\tan\ominus < 0$

    Find  a)  $\sin(-\ominus)$

          b)  $\cos(\ominus + \pi)$

          c)  $\tan(\pi - \ominus)$

          d)  $\sin(\ominus + \dfrac{\pi}{2})$

2) Given:   $\tan\ominus = \sqrt{3}$    in Quad I

    Find  a)  $\sec\ominus$
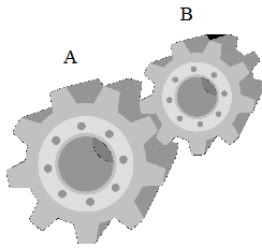
          b)  $\sin(\ominus + 180^\circ)$

          c)  $\tan(\ominus - 90^\circ)$

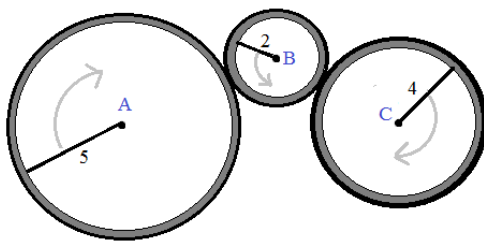          d)  $\cos(180^\circ - \ominus)$

1) Gear A has a radius of 12 inches.
Gear B has a radius of 7 inches.

If A makes 20 revolutions per minute, how many revolutions per minute does B make?



2) If wheel A makes 10 revolutions, how many revolutions will wheel C make?



3) A computerized spin balance machine rotates a 25" diameter tire at 480 revolutions per minute.

a) find the road speed (in miles per hour) at which the tire is being balanced.

b) At what rate should the spin balance machine be set so that the tire is being tested for 70 miles per hour?

1) Write $21°18'49''$ using decimal degrees.

$$21° + 18'\frac{1°}{60'} + 49''\frac{1°}{3600''}$$

$$21° + .3° + .014° = \boxed{21.314°}$$

2) Write $88.297°$ using DMS (Degrees, Minutes, & Seconds)

$$88° + .297°\frac{60'}{1°}$$

$\boxed{88°\ 17'\ 49.2''}$

$$17.82' \longrightarrow 17' + .82'\frac{60''}{1'}$$

$$49.2''$$

3) $\angle A$ and $\angle B$ are complementary angles.

$\angle A = 62°15'23''$          What is the measure of $\angle B$?

Complementary -- therefore,
A + B = 90

$90° - 62°15'23'' = \boxed{27°44'37''}$

$90° - 62.26° = 27.74°$

4) At 2:30pm, what is the angle measure between the hour and minute hands?
Express your answer in decimal degree form **and** DMS form.

3:00 would obviously be $90°$

2:30 takes a little bit of thought...
1) There are 360 degrees in a circle.. Since there are 12 hours -- $30°$ between each hour
2) At 2:30, the hour hand is HALFWAY between the 2 and the 3..

Degrees from 3 to 6 =
$90°$

Degrees from 3 to the middle of 2 and 3 =
$15°$

Angle is 90 + 15 = $\boxed{105°}$

5) What is the radian measure of $120°$? $75°$?

$$120° \cdot \frac{\pi \text{ radians}}{180°} = \frac{2}{3}\pi \text{ radians}$$

$$75° \cdot \frac{\pi \text{ radians}}{180°} = \frac{5}{12}\pi \text{ radians}$$

$$\cong .417\,(3.14)\text{ rad}$$

$$\boxed{\text{approximately } 1.308 \text{ rad}}$$

6) Convert  a) $\frac{\pi}{6}$ radians to degrees.   b) 3.6 radians to degrees

$$\frac{\pi}{6}\text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = 30°$$

$$3.6 \text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = \frac{3.6\,(180°)}{\pi}$$

$$\cong \frac{3.6\,(180°)}{3.14}$$

$$\boxed{\text{approximately } 206.369°}$$

**Coterminal and Reference Angles**　　　　　　　　SOLUTIONS

## I. Coterminal Angles

1) Determine if the following pairs are coterminal:

　　a) $57°$　$357°$　| NO |　$357 - 57 = 300$　　　c) $-80°$　$280°$　| YES |　$280 - (-80) = 360°$

　　b) $-20°$　$160°$　| NO |　$160 - (-20) = 180$　　d) $-40°30'$　$319°30'$　| YES |　$-40°30' - (319°30') = -360°$

2) Identify one *positive* and one *negative* coterminal angle to -30 degrees

　　　　$-30 + 360 =$ | 330 degrees |

　　　　$-30 - 360 =$ | $-390$ degrees |



3) Identify one *positive* and one *negative* coterminal angle to $\dfrac{\pi}{3}$　　(use radian measures)

　　　add $2\pi$　| $\dfrac{7\pi}{3}$ |　　subtract $2\pi$　| $\dfrac{-5\pi}{3}$ | .

4) Write an expression that describes *all* coterminal angles to 100 degrees

　　　　$100° + 360° n$　　where n is any integer....

## II. Reference Angles

Identify the reference angle for each angle:　　(reference angle is measured from the *terminal side to the x-axis*)
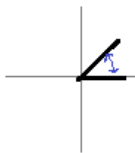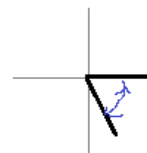
1) $110°$

　70 degrees

2) $42°$

　42 degrees

3) $-72°$

　72 degrees

4) $210°$

　30 degrees
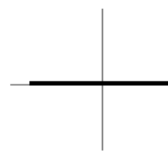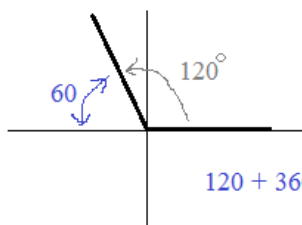
5) $-150°$

　30 degrees

**6) $180°$

　NONE

　(zero degrees)

## III. Coterminal vs Reference Angles

For each of the following, identify *the reference* angle and *any coterminal* angle:
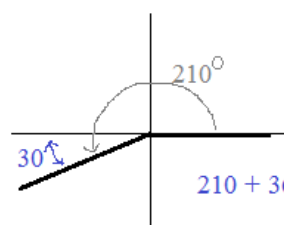
A)

$120°$　$60$

$120 + 360 = 480$

Reference Angle:　60 degrees

Coterminal Angle:　480 degrees

B)

$210°$　$30$

$210 + 360 = 570$

Reference Angle:　30 degrees

Coterminal Angle:　570 degrees
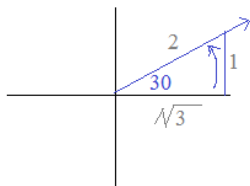
**SOLUTIONS**

**1)** $30^{\circ}$

reference angle? $30^{\circ}$

coterminal angle? ex: 390, -330

sine? $\dfrac{1}{2}$

cosine? $\dfrac{\sqrt{3}}{2}$

convert to radians: $30 \cdot \dfrac{\pi}{180} = \dfrac{\pi}{6}$ radians
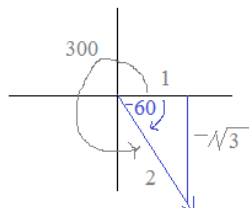


**2)** $-60^{\circ}$

reference angle? $60^{\circ}$

coterminal angle? ex: 300, 660

sine? $\dfrac{-\sqrt{3}}{2}$

cosine? $\dfrac{1}{2}$

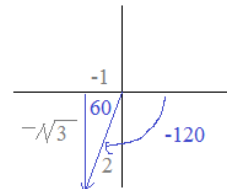convert to radians: $-60^{\circ} \cdot \dfrac{\pi\,\text{rad}}{180^{\circ}} = \dfrac{-\pi}{3}$ radians



**3)** $-120^{\circ}$

reference angle? $60^{\circ}$

examples:

2 coterminal angles? $-120 + 360 = 240^{\circ}$

$-120 - 360 = -480^{\circ}$

tangent? $\sqrt{3}$

cosine? $-\dfrac{1}{2}$

convert to radians: $-120 \cdot \dfrac{\pi\,\text{rad}}{180^{\circ}} = \dfrac{-2\pi}{3}$ rad
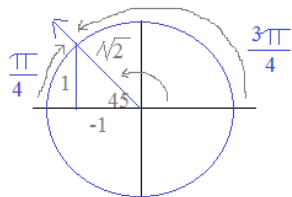


**4)** $\dfrac{3\pi}{4}$ radians

reference angle? $\dfrac{\pi}{4}$ or 45 degrees

coterminal angle? ex: $\dfrac{11\pi}{4}$, $\dfrac{-5\pi}{4}$

sine? $\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

tangent? -1

convert to degrees: $\dfrac{3\pi}{4}$  $135^{\circ}$
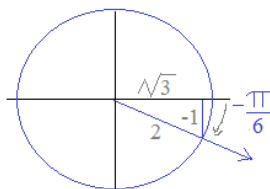


**5)** $\dfrac{-\pi}{6}$ radians

reference angle? $\dfrac{\pi}{6}$ or 30 degrees

2 coterminal angles? $\dfrac{-\pi}{6} + 2\pi = \dfrac{11\pi}{6}$

sine? -1/2    $\dfrac{-\pi}{6} - 2\pi = \dfrac{-13\pi}{6}$

cosine? $\sqrt{3}/2$

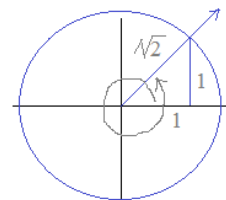convert to degrees: $-\dfrac{\pi}{6} \cdot \dfrac{180}{\pi} = $ -30 degrees



**6)** $\dfrac{9\pi}{4}$ radians

reference angle? $\dfrac{\pi}{4}$ or 45 degrees

coterminal angle? ex: $\dfrac{\pi}{4}$

sine? $\dfrac{\sqrt{2}}{2}$

tangent? 1

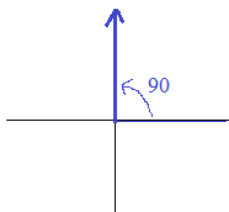convert to degrees: $\dfrac{9\pi}{4} \cdot \dfrac{180^{\circ}}{\pi} = 405^{\circ}$



**7)** $90^{\circ}$

ex: $90 + 360 = 450$

coterminal angle? $90 - 360 = -270$

tangent? undefined

sine? 1

cosine? 0

convert to radians: $90 \cdot \dfrac{\pi}{180} = \dfrac{\pi}{2}$ radians
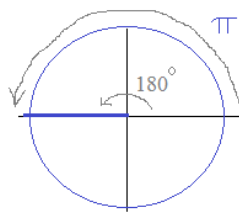


**8)** $\pi$ radians

negative coterminal angle? ex: $-\pi$ or $-3\pi$

positive coterminal angle? ex: $3\pi$ or $5\pi$

sine? 0

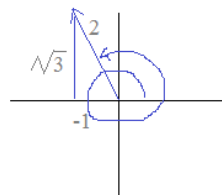cosine? $-1$

convert to degrees: 180 degrees



**9)** $480^{\circ}$

reference angle? $60^{\circ}$

coterminal angle? $480 - 360 = 120^{\circ}$

sine? $\dfrac{\sqrt{3}}{2}$

cosine? $-\dfrac{1}{2}$

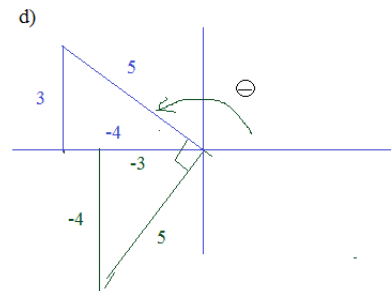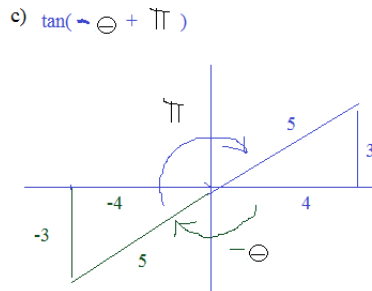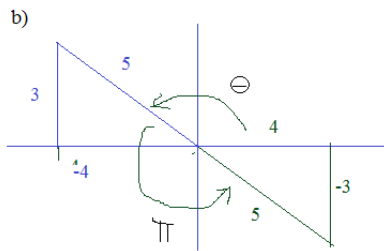convert to radians: $480 \cdot \dfrac{\pi}{180} = \dfrac{8\pi}{3}$ radians
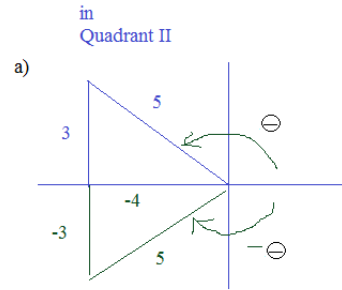
Applying reference angles

1) Given: $\sin\ominus = \dfrac{3}{5}$    $\tan\ominus < 0$
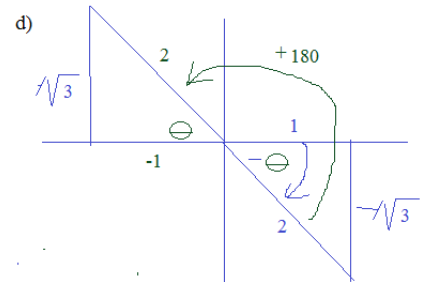
Step 1: Using the restrictions, find the angle....

in
Quadrant II

Find   a) $\sin(-\ominus)$  $= \boxed{\dfrac{-3}{5}}$

   b) $\cos(\ominus + \pi)$  $= \boxed{\dfrac{4}{5}}$

   c) $\tan(\pi - \ominus)$  $= \boxed{\dfrac{3}{4}}$

   d) $\sin(\ominus + \dfrac{\pi}{2})$  $= \boxed{\dfrac{-4}{5}}$

tan < 0
sin > 0        sin > 0

tan < 0

a)

5
3
-4
-3
5

b)

5
3
-4
4
-3
5

c) $\tan(-\ominus + \pi)$

5
3
-4
4
-3
5

d)

5
3
-4
-3
-4
5
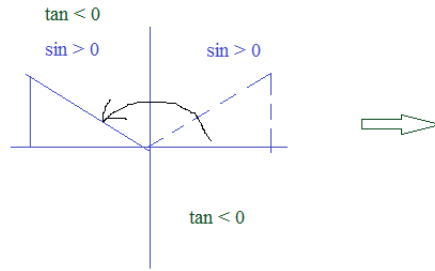
2) Given: $\tan\ominus = \sqrt{3}$    in Quad I

First, find the reference angle we're going to use...

Find   a) $\sec\ominus = \boxed{2}$

   b) $\sin(\ominus + 180°) = \boxed{\dfrac{-\sqrt{3}}{2}}$

   c) $\tan(\ominus - 90°) = \boxed{\dfrac{-1}{\sqrt{3}}}$

   d) $\cos(180° - \ominus)$  $= \boxed{\dfrac{-1}{2}}$
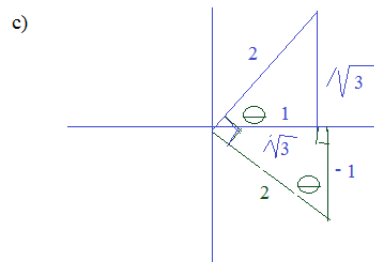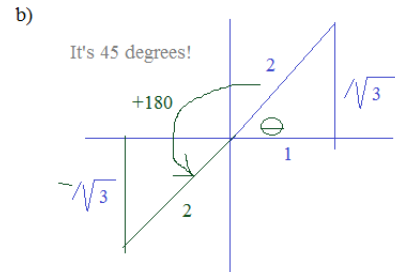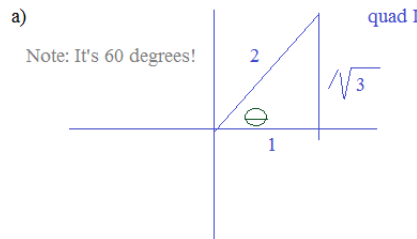
a)                                   quad I
Note: It's 60 degrees!

2        $\sqrt{3}$
1

b)
It's 45 degrees!

+180

2        $\sqrt{3}$
1
$-\sqrt{3}$        2

c)

2        $\sqrt{3}$
1
$\sqrt{3}$
-1
2

d)

+180
2        $\sqrt{3}$
$\sqrt{3}$        1
-1
2
$-\sqrt{3}$

1) Gear A has a radius of 12 inches.
   Gear B has a radius of 7 inches.

If A makes 20 revolutions per minute, how many revolutions per minute does B make?
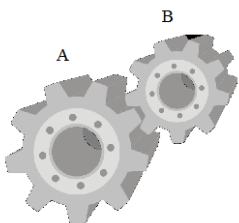
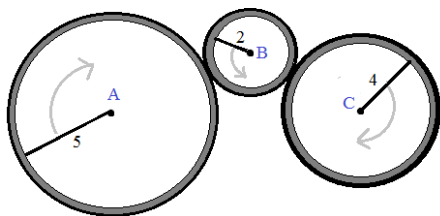Gear A: $\dfrac{20 \text{ revolutions}}{1 \text{ minute}}$ x $\dfrac{24 \text{ }\pi\text{ inches}}{1 \text{ revolution}}$ = $\dfrac{480 \text{ }\pi\text{ inches}}{1 \text{ minute}}$ 

Gear A's linear speed
----> this will move
Gear B

Gear B: $\dfrac{480 \text{ }\pi\text{ inches}}{1 \text{ minute}}$ ÷ $\dfrac{14 \text{ }\pi\text{ inches}}{1 \text{ revolution}}$ = $34\dfrac{2}{7}$ revolutions/minute

2) If wheel A makes 10 revolutions, how many revolutions will wheel C make?

A   10 revolutions:  100 $\pi$   (linear distance)

B   100 $\pi$   (25 revolutions)

C   100 $\pi$ ⟶ $\dfrac{100 \text{ }\pi}{8 \text{ }\pi}$ = 12.5 revolutions

3) A computerized spin balance machine rotates a 25" diameter tire at 480 revolutions per minute.

a) find the road speed (in miles per hour) at which the tire is being balanced.

b) At what rate should the spin balance machine be set so that the tire is being tested for 70 miles per hour?

a) since we're looking for the "road speed", we need to find 'traveling distance as it relates to time' i.e. miles/hour

since diameter is 25", every rotation is the circumference:  25 $\pi$       $\dfrac{25 \text{ }\pi \text{ inches}}{1 \text{ rotation}}$

since the machine rotates the tire 480 revolutions per minute,       $\dfrac{28,800 \text{ rotation}}{1 \text{ hour}}$
it rotates the tire  480 x 60  =  28,800 revolutions per hour

Now, convert to the units  miles/hour

$\dfrac{25 \text{ }\pi \text{ inches}}{1 \text{ rotation}}$ X $\dfrac{28,800 \text{ rotation}}{1 \text{ hour}}$ X $\dfrac{1 \text{ mile}}{63,360 \text{ inches}}$ = 35.7 miles/hour

NOTE: all the units cancel, leaving miles/hour

travels $\dfrac{720,000 \text{ }\pi \text{ inches}}{\text{hour}}$

b) First, let's find out how many rotations are required to travel 70 miles...

$\dfrac{25 \text{ }\pi \text{ inches}}{1 \text{ rotation}}$ X $\dfrac{1 \text{ mile}}{63,360 \text{ inches}}$ X (Rotations)  = 70 miles

# of rotations = 56,470.7
(revolutions)

$\dfrac{56,470.7 \text{ revolutions}}{70 \text{ miles}}$ ⟹

since we're seeking
70 miles/hour,

we know 56,470.7 revolutions/hour
are required....

Divide that by 60 minutes:

941.18  revolutions/minute

Express method/check:
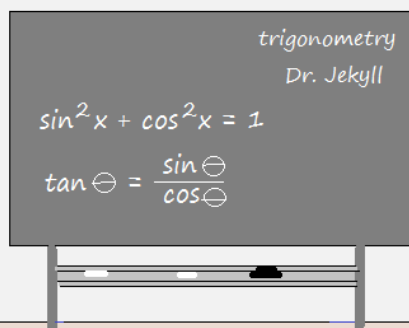
$\dfrac{941.18 \text{ rev/minute}}{70 \text{ miles/hour}}$ = $\dfrac{480 \text{ rev/minute}}{35.7 \text{ miles/hour}}$ ✓

Yes, the short-cut from a) to b)

$\dfrac{S}{70 \text{ miles/hour}}$ = $\dfrac{480 \text{ rev/minute}}{35.7 \text{ miles/hour}}$
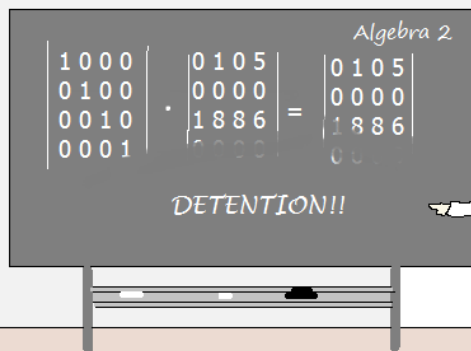
S = 941.18 rev/minute

Hidden Message Puzzle-→

## Hidden Message

Hint: The ultimate clue is in the puzzle

Answer the 12 questions below. Then, convert the numbers into letters to reveal the hidden term!

Number Key:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | C | D | E | G | H | K | N | R | T |

1) Seconds between $24°\ 12'$ and $24°\ 11'\ 41"$

     1 [ ] →

2) $\dfrac{3\pi}{4}$ Radians = x degrees. What is x?

     13 [ ] →

3) Minutes in 1/2 degree

     [ ] 0 →

4) $63°\ 41'\ 45"$ : convert to decimal form
                (round to nearest tenth)

     63. [ ] →

5) $71.15° = 71°\ x'\ y"$ What is x?

     [ ] →

6) $36° = \dfrac{\pi}{z}$ Radians What is z?

     [ ] →
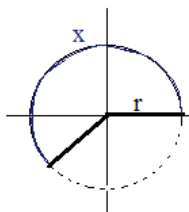
7) Angles A and B are complementary.
     $m\angle A = 43°\ 32'\ 30"$     $m\angle B = 46°\ x'\ 30"$
               What is x?

     [ ] 7 →

8) At 1:00, what is the degree measure between the hour hand and minute hand?

     [ ] 0 →

9) Approximate arc length of x (measured in radians - rounded to nearest integer)



     [ ] →

10) Degrees in 5 Radians

     2 [ ] $6.5°$ →

11) Seconds in 1 degree

     [ ] 600 →

12) Convert to degrees (decimal form): $144°\ 18'$

     144. [ ] →

Answer the 12 questions below. Then, convert the numbers into letters to reveal the hidden term!

Number Key:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | C | D | E | G | H | K | N | R | T |

SOLUTIONS

1) Seconds between $24°\ 12'$ and $24°\ 11'\ 41''$

$$24\ 11'\ 60''$$
$$-\ 24\ 11'\ 41''$$
$$19''$$

2) $\dfrac{3\pi}{4}$ Radians = x degrees. What is x?

$$\dfrac{3\ pi\ radians}{4} \cdot \dfrac{180\ degrees}{pi\ radians} = 135\ degrees$$

3) Minutes in 1/2 degree     60 minutes = 1 degree... therefore, 30 minutes = 1/2 degree

4) $63°\ 41'\ 45''$ : convert to decimal form (round to nearest tenth)

$$63° + 41'\dfrac{1°}{60'} + 45''\dfrac{1°}{3600''} \cong 63 + .6833 + .0125$$
$$\cong 63.6958$$

5) $71.15° = 71°\ x'\ y''$   What is x?

$$.15\ degrees\ x\ \dfrac{60\ minutes}{1\ degree} = 9\ minutes$$
$$x = 9\quad y = 0$$

6) $36° = \dfrac{\pi}{z}$ Radians   What is z?

$$\dfrac{\pi}{z}\ Rad\left(\dfrac{180°}{\pi Rad}\right) = \dfrac{180°}{z} = 36\ degrees \qquad z = 5$$

7) Angles A and B are complementary.
$m\angle A = 43°\ 32'\ 30''\qquad m\angle B = 46°x'\ 30''$
What is x?

$$43°\ 32'\ 30'' + 46°\ x'\ 30'' = 90°$$
$$89°\ (32+x)' + 60'' = 90°$$
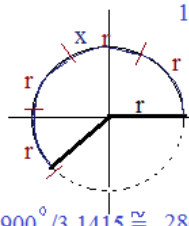$$x = 27$$

8) At 1:00, what is the degree measure between the hour hand and minute hand?

clock is 360 degrees
12 hours; 30 degrees between each hour!

9) Approximate arc length of x (measured in radians - rounded to nearest integer)

approx: 4 radians

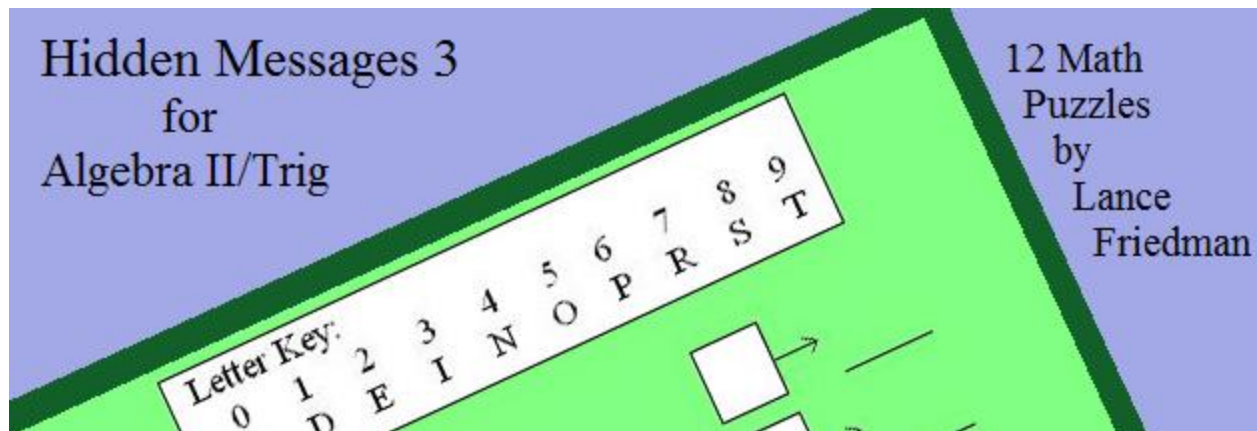10) Degrees in 5 Radians   $5\ Radians\left(\dfrac{180°}{\pi\ radians}\right)$   $900°/3.1415 \cong 286.5°$

11) Seconds in 1 degree   $\left(\dfrac{60\ seconds}{1\ minute}\right)\cdot\left(\dfrac{60\ minutes}{1\ degree}\right) = 3600\ secondes/degree$

12) Convert to degrees (decimal form): $144°18'$

$$144\ degrees\ +\ 18\ minutes\ x\ \dfrac{1\ degree}{60\ minutes}\ =\ 144.3\ degrees$$

| | | | |
|---|---|---|---|
| 1 | 9 | → | T |
| 13 | 5 | → | H |
| 3 | 0 | → | E |
| 63. | 7 | → | N |
| | 9 | → | T |
| | 5 | → | H |
| 2 | 7 | → | D |
| 3 | 0 | → | E |
| | 4 | → | G |
| 2 8 | 6.5° | → | R |
| 3 | 600 | → | E |
| 144. | 3 | → | E |

Hidden Messages 3
for
Algebra II/Trig

Letter Key:
0   1   2   3   4   5   6   7   8   9
D   E   I   N   O   P   R   S   T

12 Math
Puzzles
by
Lance
Friedman

More puzzles available in the "travel log collection" at mathplane.com. Proceeds go to site maintenance and improvement. (And, treats for Oscar the Dog!)

## Clock Question:
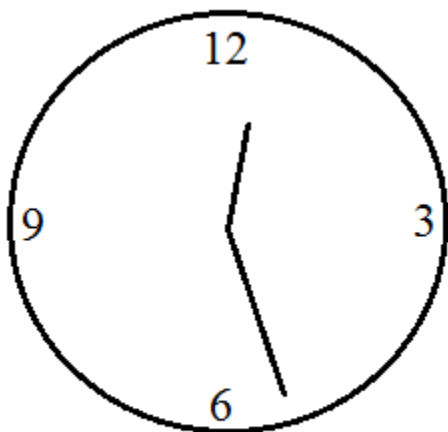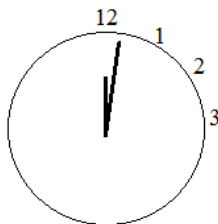


12:26:35

When will the minute hand cross the hour hand?

Express the answer to the nearest second.

(Answer and Explanation on the next page)

A clock sits at exactly 12:02...

    What time will the minute hand cross the hour hand?
Express the answer to the nearest second...

Measurements to remember:

60 minutes = 1 hour

60 seconds = 1 minute

3600 seconds = 1 hour

Solution:

    The hour hand will move from one number to the next every 60 minutes  (or 3600 seconds)

    The minute hand will move from one number to the next every 5 minutes (or 300 seconds)

The minute hand goes around the clock and returns to the top. (1:00)...   The minute hand is on the 12, and the hour hand is on the 1....

Then, the minute hand reaches the 1 five minutes later (1:05)...

***But, during those 5 minutes, the hour hand moved!!

    How far?

The hour hand moves from 1 to 2 in 60 minutes...
So, during the 5 minutes, the hour hand moved  5/60  or  1/12 of the way to 2...
So, how long would it take for the minute hand to travel 1/12 of the way to 2?
Well, it takes 300 seconds for the minute hand to travel from 1 to 2...
Therefore, it takes  25 seconds to travel  1/12 of the way!!

But, wait....  During those 25 seconds, the hour hand is still moving...
So, during the 25 seconds, how far did the hour hand move?

Well, it takes 3600 seconds for the hour hand to move from 1 to 2...
Therefore, in 25 seconds, the hour hand moved  25/3600 of the distance..   This is approx.  .007

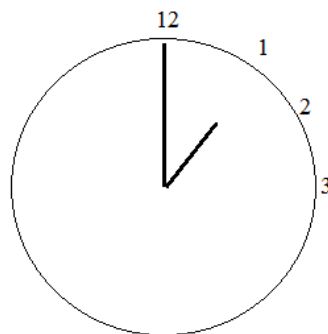Again, it takes 300 seconds for the minute hand to travel from 1 to 2.
Therefore, it would take approx. 2 seconds to travel .007 of the way between 1 and 2.
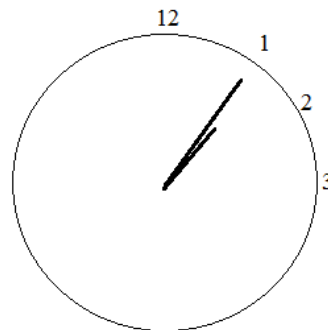
    So, the minute hand has moved
63 minutes to get to 1:05...
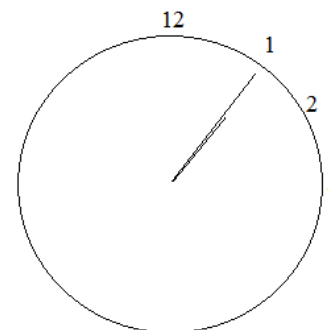Then, 25 seconds to close in on the hour hand...  1:05:25...
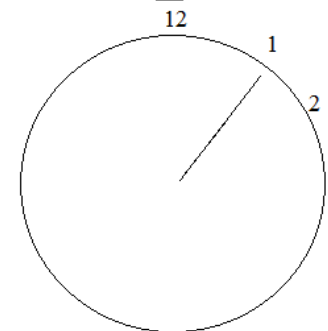And, finally, 2 more seconds to reach the hour hand...  1:05:27
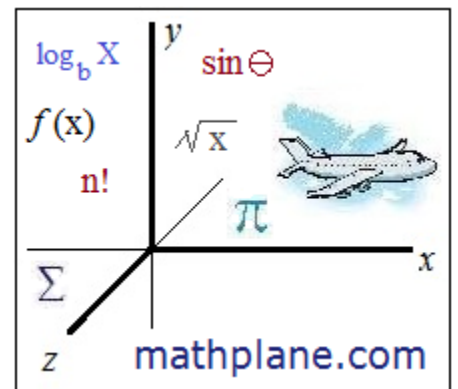
1:00

1:05

1:05:25

1:05:27

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers

Also, at TeachersPayTeachers

And, mathplane.ORG for mobile and tablets