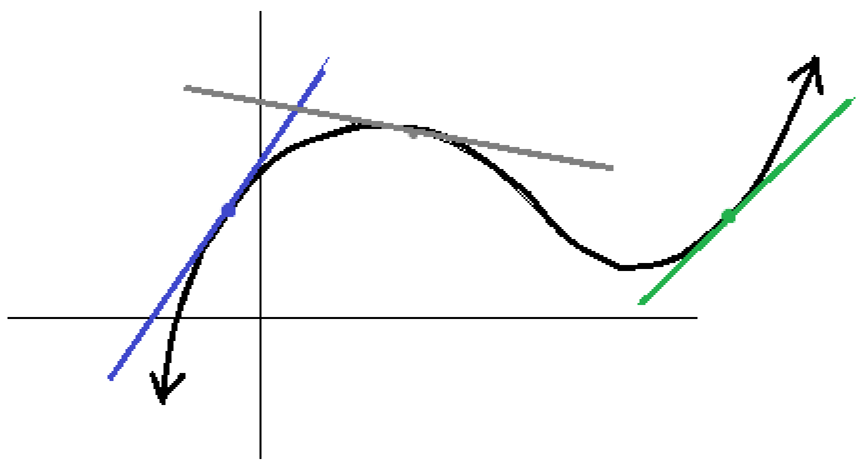
Calculus:

Instantaneous Rate of Change

Notes, Examples, and Quick Quiz (with Solutions)



Topics include Limits, IROC, AROC, tangents, secants, Definition of Derivative, and more.

Using Average Rate of Change (AROC) to estimate Instantaneous Rate of Change (IROC)

Example: For the function $f(x) = \log x$

a) make a table of values that include x = 1, 1.5, 2, 2.5, and 3

b) find the average rate of change between 1 and 3

c) <u>approximate</u> the instantaneous rate of change at x = 2 *using average rates*

$f(x) = \log x$

x	1	1.5	2	2.5	3
$f(x)$	0	.176	.301	.398	.477

Average rate of change is the 'slope': $\frac{.477 - 0}{3 - 1} = .239$ (AROC between x = 1 and x = 3)

To approximate instantaneous rate of change at x = 2, we'll find average rates of change that are near 2.

average rate of change from 1.5 to 2: $\frac{.301 - .176}{2 - 1.5} = .25$

average rate of change from 2 to 2.5: $\frac{.398 - .301}{2.5 - 2} = .194$

The average of the measures is

$(.25 + .194) / 2 = .222$

The actual rate of change? the derivative of logx is $\frac{1}{(\ln 10)x}$

then, at x = 2: $\frac{1}{(2.30)(2)} = .217$

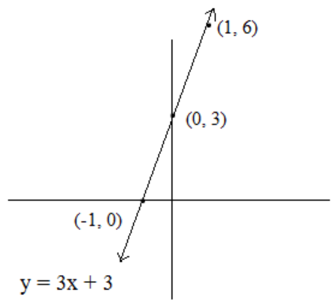
Derivatives: Measure of Slope and Rate of Change

Review: Slope

Slope $m = \frac{\text{"Rise"}}{\text{"Run"}} = \frac{y_1 - y_2}{x_1 - x_2}$

Slope $m =$ "Rate of Change"

$= \frac{\triangle y}{\triangle x}$



$m = \frac{3 - 0}{0 - (-1)} = 3$

$m = \frac{0 - 6}{-1 - 1} = 3$

In fact, the slope at <u>any point</u> on the line is 3...

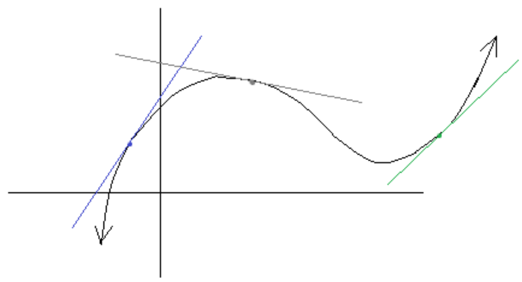
And, the *rate of change* is "for every one unit of x, there are three units of movement in y."

Instantaneous Rate of Change (Slope) on a Curve

The slope of a curve changes:

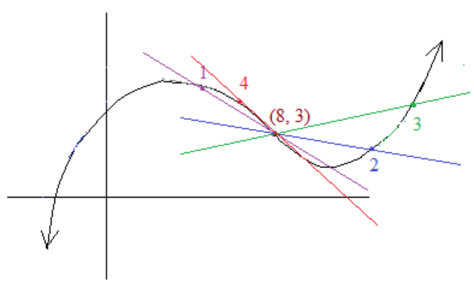
The slope at a given point on the curve is determined by the line tangent to it. How do you measure it?

In the above example, we took two points on the line and found the slope is 3. In a curve, that method doesn't work!



Choosing 2 points on a curved function <u>will not</u> determine slope:

What is the slope of the function at (8, 3)? How do you measure it?



To find the slope at point (8, 3), we can try to find the rate of change between (8, 3) and another point on the curve.. But, notice how the slope varies when measured against different points!!

**The <u>most accurate</u> measure would occur when using the point <u>nearest</u> to (8, 3)...

Is that point x = 8.001?
x = 8.00001?
x = 8.0000001?

The Instantaneous Rate of Change of y at x

$$= \lim_{\triangle x \to 0} \frac{\triangle y}{\triangle x} = \lim_{\triangle x \to 0} \frac{f(x + \triangle x) - f(x)}{\triangle x}$$

The Instantaneous Rate of Change of y at x

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Example: Determine the formula for the slope of the graph of

$$y = x^2 + 3$$

What is the *slope of a line tangent* at (0, 3)? (-2, 7)?

Solutions: 1) Find $f(x + \Delta x)$

$$(x + \Delta x)^2 + 3$$

2) Calculate $f(x + \Delta x) - f(x)$

$$x^2 + 2x(\Delta x) + (\Delta x)^2 + 3 - (x^2 + 3)$$

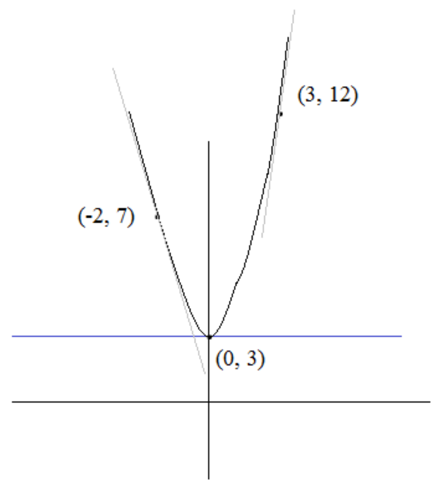
$$= 2x(\Delta x) + (\Delta x)^2$$

3) Divide by Δx

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x + \Delta x$$

4) Apply limit $\Delta x \to 0$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = 2x$$

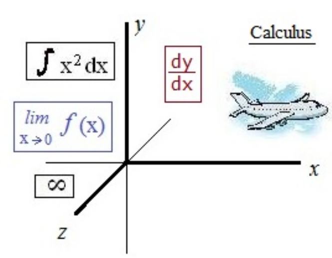


(Instantaneous rate of change of function is 2x)

Slope of line tangent at (0, 3) ----> 2(0) = 0

Slope of line tangent at (-2, 7) ---> 2(-2) = -4

Also, the rate of change at x = 3 is 2(3) = 6
$f(3) = 12$



Word Problem Application: Using AROC and IROC

Example: A ball is thrown out of a window with a path that follows the following projectile function:

$$h(t) = -4.9t^2 + 80$$ where t is time in seconds and h(t) is height in feet

a) What is the Average Rate Of Change during the first 3 seconds?

b) What is the Instantaneous Rate Of Change when t = 2?

c) What is the velocity of the ball when it hits the ground?

a) Average Rate of Change (slope) $\quad \dfrac{f(a) - f(b)}{a - b} \quad \dfrac{y_1 - y_2}{x_1 - x_2}$

$a = 0 \qquad b = 3$

$f(a) = 80 \qquad f(b) = 35.9$

the slope between (0, 80) (3, 35.9)

$$\frac{80 - 35.9}{0 - 3} = -14.7 \text{ feet/second}$$

b) Intantaneous Rate of Change $\quad \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\lim_{h\to 0} \frac{-4.9(x+h)^2 + 80 - [-4.9x^2 + 80]}{h}$$

$$\lim_{h\to 0} \frac{-4.9x^2 - 9.8xh - 4.9h^2 + 80 + 4.9x^2 - 80}{h}$$

$$\lim_{h\to 0} \frac{-9.8xh - 4.9h^2}{h} = \lim_{h\to 0} \frac{h(-9.8x - 4.9h)}{h} =$$

$$\lim_{h\to 0} -9.8x - 4.9h = -9.8x$$

so, the IROC at t = 2 is -9.8(2) = -19.6 feet/second

c) The ball hits the ground when h(t) = 0

$0 = -4.9(t)^2 + 80$

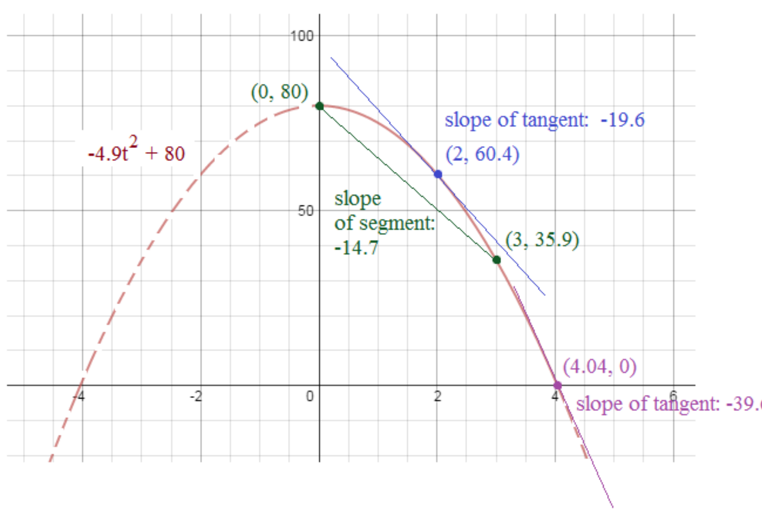
$-80 = -4.9t^2$

$t^2 = 16.3$

$t = 4.04$ or -4.04

since time cannot be negative, the ball hits the ground just after 4 seconds..

$-9.8(4.04) = -39.6$ feet per second



Limits Review

Example: $\lim_{h\to 0} \frac{3(2+h)^2 - 12}{h}$ First, try direct substitution: $\frac{3(2+0)^2 - 12}{0} = \frac{12-12}{0} = \frac{0}{0}$ This is indeterminate

Then, try transforming the equation:

$$\lim_{h\to 0} \frac{3(h^2 + 4h + 4) - 12}{h}$$

$$\lim_{h\to 0} \frac{3h^2 + 12h + 12 - 12}{h}$$

$$\lim_{h\to 0} \frac{3h(h+4)}{h}$$

$$\lim_{h\to 0} \frac{3(h+4)}{1}$$

and, substitute again: $\frac{3(0+4)}{1} = 12$

Example: $\lim_{h\to 0} \frac{(1+h)^3 - 1}{h}$ After direct substitution, we see the equation is indeterminate $\frac{0}{0}$

$$\lim_{h\to 0} \frac{(h+1)(h^2 + 2h + 1) - 1}{h}$$

$$\lim_{h\to 0} \frac{h^3 + 2h^2 + h + h^2 + 2h + 1 - 1}{h}$$

$$\lim_{h\to 0} \frac{h^3 + 3h^2 + 3h}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(h^2 + 3h + 3)}{\cancel{h}}$$

$$\lim_{h\to 0} (h^2 + 3h + 3) = 3$$ (by direct substitution)

Example: $\lim_{h\to 0} \frac{(3+h)^2 - (3-h)^2}{2h}$ After direct substitution, we see the equation is indeterminate

$$\lim_{h\to 0} \frac{h^2 + 6h + 9 - (9 - 6h + h^2)}{2h}$$

$$\lim_{h\to 0} \frac{12h}{2h}$$

$$\lim_{h\to 0} 6 = 6$$

Instantaneous Rate of Change (IROC)
Limits and Slope

Example: Determine the slope of $f(x) = -3x^2 - 4$ at $x = 5$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{-3(x+h)^2 - 4 - (-3x^2 - 4)}{h}$$

$$\lim_{h \to 0} \frac{-3x^2 - 6xh - 3h^2 - 4 + 3x^2 + 4}{h}$$

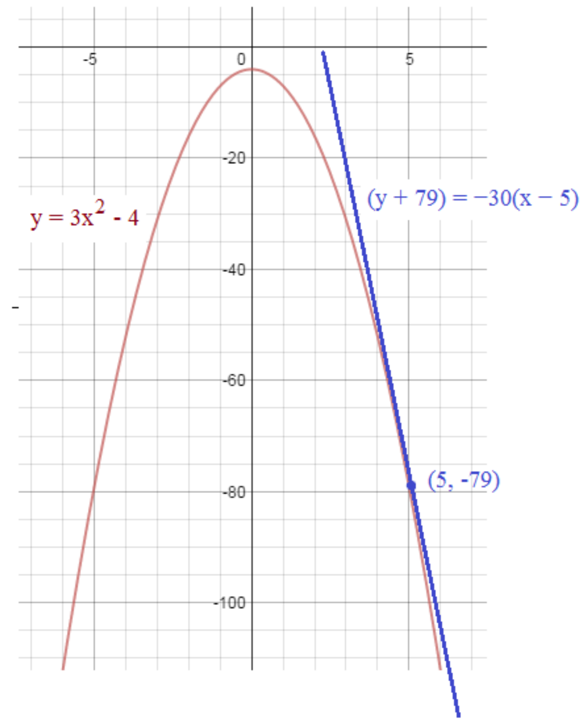
$$\lim_{h \to 0} \frac{-6xh - 3h^2}{h}$$

$$\lim_{h \to 0} \frac{-3h(2x+h)}{h}$$

$$\lim_{h \to 0} \frac{-3(2x+h)}{1}$$

$$= -6x$$

at $x = 5$, -30



also, directly: $\lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$

then, solve...

$$\lim_{h \to 0} \frac{-3(5+h)^2 - 4 - (-3(5)^2 - 4)}{h}$$

$$\lim_{h \to 0} \frac{-75 - 30h - 3h^2 - 4 - -79}{h}$$

$$\lim_{h \to 0} \frac{-30h - 3h^2}{h}$$

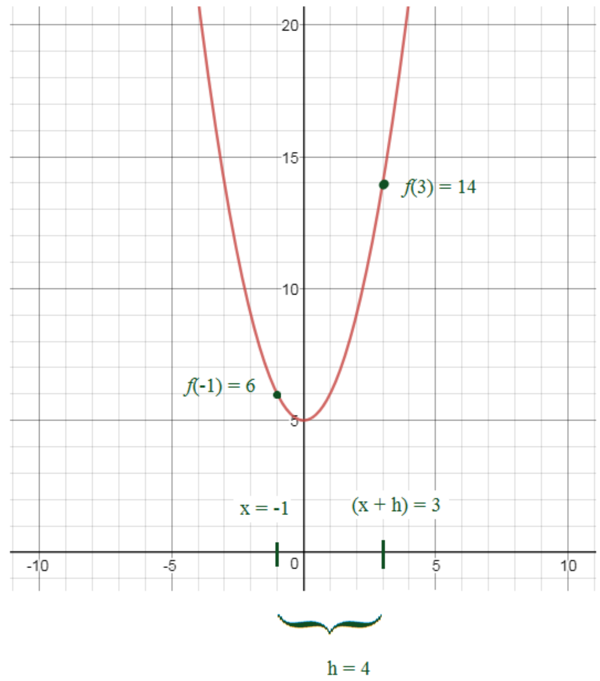
$$\lim_{h \to 0} \frac{h(-30 - 3h)}{h}$$

$$\lim_{h \to 0} -30 - 3h \quad = -30$$

Exploring the Average Rate of Change Formula

Example: Apply the average rate of change formula to this function.
Then, using random inputs, verify the formula!

$f(x) = x^2 + 5$



$$\frac{f(x+h) - f(x)}{h} \Rightarrow \frac{(x+h)^2 + 5 - [x^2 + 5]}{h}$$

$$\frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h}$$

$$\frac{2xh + h^2}{h} = 2x + h$$

Let's find the rate of change when $x = -1$ and $h = 4$

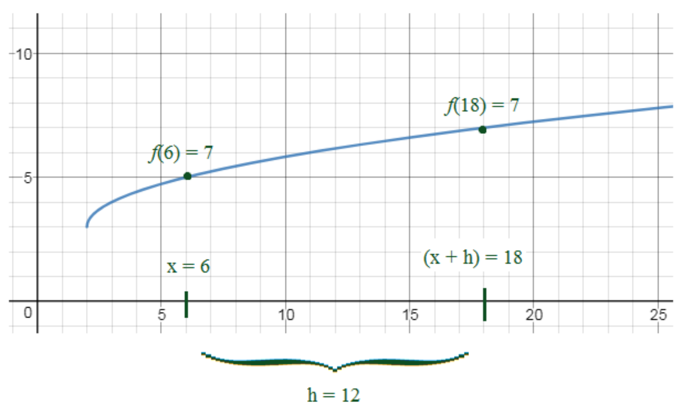
$$2x + h = 2(-1) + (4) = 2$$

Now, let's find the slope between the points $(-1, 6)$ and $(3, 14)$

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{6 - 14}{-1 - 3} = 2$$

Example: Apply the average rate of change formula to this function.
Then, using random inputs, verify the formula!

$f(x) = \sqrt{x - 2} + 3$



$$\frac{f(x+h) - f(x)}{h} \Rightarrow \frac{\sqrt{(x+h) - 2} + 3 - [\sqrt{(x) - 2} + 3]}{h}$$

$$\frac{\sqrt{(x+h) - 2} + 3 - \sqrt{(x) - 2} - 3}{h}$$

(multiply by the conjugate) $$\frac{\left(\sqrt{(x+h) - 2} - \sqrt{(x) - 2}\right)}{h} \cdot \frac{\left(\sqrt{(x+h) - 2} + \sqrt{(x) - 2}\right)}{\left(\sqrt{(x+h) - 2} + \sqrt{(x) - 2}\right)}$$

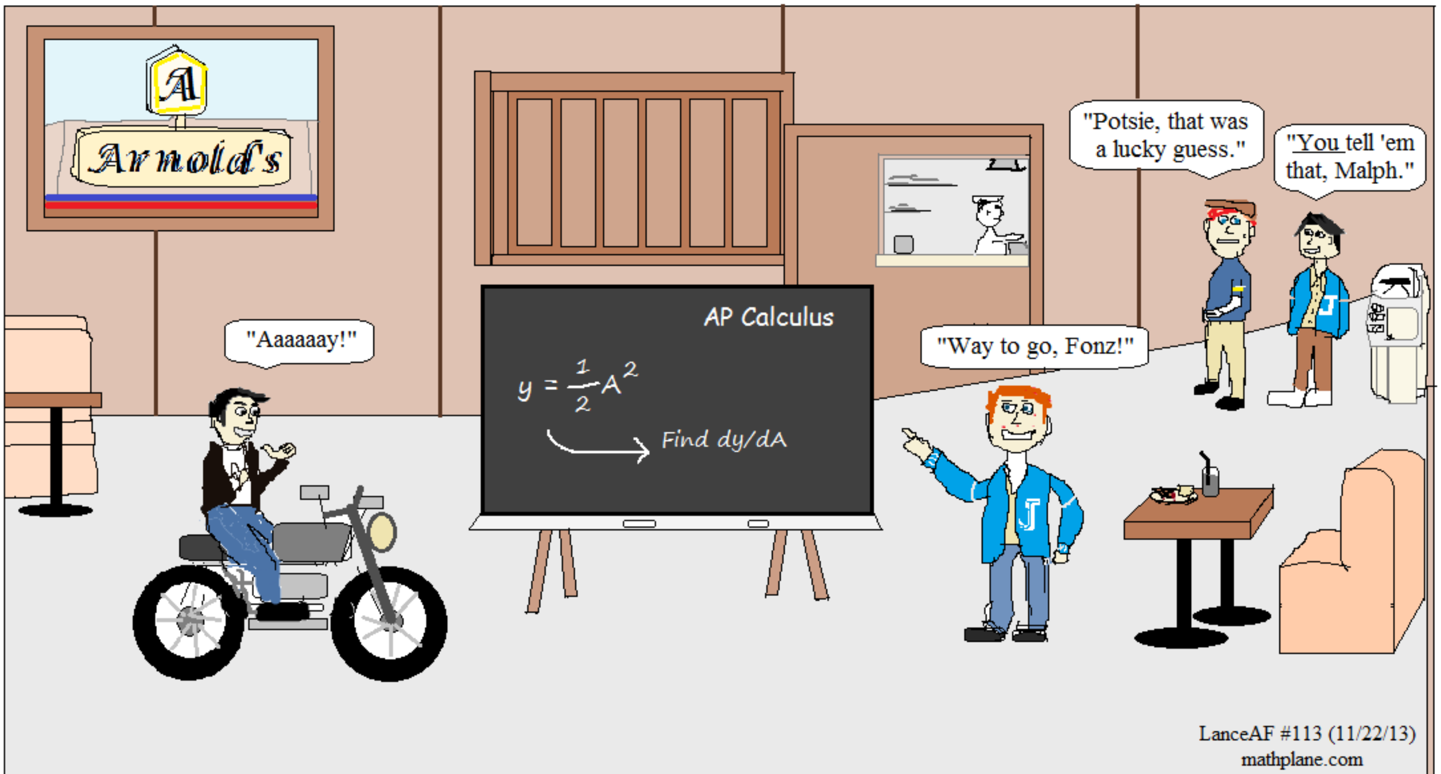
$$\frac{x + h - 2 - [x - 2]}{h\left(\sqrt{(x+h) - 2} + \sqrt{(x) - 2}\right)} = \frac{1}{\left(\sqrt{(x+h) - 2} + \sqrt{(x) - 2}\right)}$$

Let's find the average rate of change $x = 6$ and $h = 12$

$$\frac{1}{\left(\sqrt{(x+h) - 2} + \sqrt{(x) - 2}\right)} = \frac{1}{\sqrt{(6 + 12) - 2} + \sqrt{(6) - 2}} = \frac{1}{6}$$

Now, let's find the slope between the points $(6, 5)$ and $(18, 7)$

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{5 - 7}{6 - 18} = \frac{1}{6}$$



Happy Days

Despite Richie's help, Fonzie dropped out of Calculus.
(... although he did have some success with velocity and acceleration!)

PRACTICE QUIZ -→

Apply the average rate of change formula (with function notation)

$$\frac{f(x+h) - f(x)}{h}$$

Example: $f(x) = 3x + 2$

$f(x + h) = 3(x + h) + 2$

$$\frac{3x + 3h + 2 - (3x + 2)}{h}$$

$$\frac{3h}{h} = \boxed{3}$$

$$\frac{f(x + h) - f(x)}{h} = 3$$

1) $f(x) = 6x - 4$ $\frac{f(x+h) - f(x)}{h} =$	2) $f(x) = 3x^2 + 5$ $\frac{f(x+h) - f(x)}{h} =$	3) $f(x) = x^2 + 4x - 1$ $\frac{f(x+h) - f(x)}{h} =$
4) $f(x) = \frac{2}{x+1}$ $\frac{f(x+h) - f(x)}{h} =$	5) $x^3 - 7$ $\frac{f(x+h) - f(x)}{h} =$	6) $f(x) = \sqrt{x + 1}$ $\frac{f(x+h) - f(x)}{h} =$

Calculus Quiz: Instantaneous Rate of Change

I. Find the instantaneous rate of change for each function (using the limit formula)

a) $f(x) = 3x + 2$

b) $g(x) = 2x^2 + x - 1$

c) $h(x) = \frac{1}{x - 1}$

II. Find the equation of the line tangent to the function at the given point. (Then, sketch the function and tangent line to verify your answer)

a) $f(x) = x^3$; (2, 8)

b) $g(x) = x^2 + 1$; (2, 5)

c) $h(x) = \sqrt{x + 1}$; (3, 2)

III. Find the average rate of change between 2 given points. Then, find the instantaneous rate of change *at each point*. (Graph and compare the results.)

a) $f(t) = \frac{1}{t + 1}$; (0, 1) $(3, \frac{1}{4})$

b) $g(s) = s^2 - 3s - 10$; (−1, −6) (3, −10)

Evaluate the limits.

Definition of Derivative

1) $$\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$$

2) $$\lim_{\Delta x \to 0} \frac{(\Delta x + 2)^3 - 8}{\Delta x}$$

3) $$\lim_{x \to \frac{\pi}{6}} \frac{\sin(x) - \sin(\frac{\pi}{6})}{x - (\frac{\pi}{6})}$$

4) $$\lim_{h \to 0} \frac{\tan(\frac{2\pi}{3} + h) + \sqrt{3}}{h}$$

Apply the average rate of change formula (with function notation)

$$\frac{f(x+h) - f(x)}{h}$$

Example: $f(x) = 3x + 2$

$f(x+h) = 3(x+h) + 2$

$$\frac{3x + 3h + 2 - (3x+2)}{h}$$

$$\frac{3h}{h} = \boxed{3}$$

$$\frac{f(x+h) - f(x)}{h} = 3$$

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1) $f(x) = 6x - 4$

$f(x+h) = 6(x+h) - 4$

then, $\dfrac{6x + 6h - 4 - (6x - 4)}{h}$

$$\frac{6h}{h} = 6$$

$$\frac{f(x+h) - f(x)}{h} = 6$$

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2) $f(x) = 3x^2 + 5$

$f(x+h) = 3(x+h)^2 + 5$

$= 3x^2 + 6xh + 3h^2 + 5$

then, $\dfrac{3x^2 + 6xh + 3h^2 + 5 - (3x^2 + 5)}{h}$

$$\frac{6xh + 3h^2}{h} = \frac{h(6x + 3h)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 6x + 3h$$

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3) $f(x) = x^2 + 4x - 1$

$f(x) = (x+h)^2 + 4(x+h) - 1$

$= x^2 + 2xh + h^2 + 4x + 4h - 1$

then, $\dfrac{x^2 + 2xh + h^2 + 4x + 4h - 1 + (x^2 + 4x - 1)}{h}$

$$\frac{2xh + h^2 + 4h}{h} = \frac{h(2x + h + 4)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 2x + h + 4$$

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4) $f(x) = \dfrac{2}{x+1}$

$f(x+h) = \dfrac{2}{(x+h)+1}$

then, $\dfrac{\dfrac{2}{(x+h)+1} - \dfrac{2}{x+1}}{h}$

$$\frac{\dfrac{2(x+1) - 2(x+h+1)}{(x+h+1)(x+1)}}{h}$$

$$\frac{2x + 2 - 2x - 2h - 2}{(x+h+1)(x+1)} \cdot \frac{1}{h} \; \cdot =$$

$$\frac{-2h}{h(x+h+1)(x+1)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-2}{(x+1)(x+h+1)}$$

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5) $x^3 - 7$

$f(x+h) = (x+h)^3 - 7$

$= (x+h)(x^2 + 2xh + h^2) - 7$

$= x^3 + 2x^2h + xh^2 + hx^2 + 2xh^2 + h^3 - 7$

then, $\dfrac{x^3 + 2x^2h + xh^2 + hx^2 + 2xh^2 + h^3 - 7 - (x^3 - 7)}{h}$

$$\frac{3x^2h + 3xh^2 + h^3}{h}$$

$$\frac{h(3x^2 + 3xh + h^2)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 3x^2 + 3xh + h^2$$

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6) $f(x) = \sqrt{x+1}$

$f(x+h) = \sqrt{(x+h)+1}$

then,

$$\frac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h} \cdot \frac{\left(\sqrt{(x+h)+1} + \sqrt{x+1}\right)}{\left(\sqrt{(x+h)+1} + \sqrt{x+1}\right)}$$

(using conjugate of numerator)

$$\frac{(x+h)+1 - (x+1)}{h\left(\sqrt{(x+h)+1} + \sqrt{x+1}\right)}$$

$$\frac{h}{h\left(\sqrt{(x+h)+1} + \sqrt{x+1}\right)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{\sqrt{(x+h)+1} + \sqrt{x+1}}$$

Calculus Quiz: Instantaneous Rate of Change

SOLUTIONS

(Instantaneous Rate of Change Formula) $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

I. Find the instantaneous rate of change for each function (using the limit formula)

a) $f(x) = 3x + 2$

$\lim_{h\to 0} \frac{3(a+h)+2-(3a+2)}{h}$

$\lim_{h\to 0} \frac{3a+3h+2-3a-2}{h}$

$\lim_{h\to 0} \frac{3h}{h} = \boxed{3}$

b) $g(x) = 2x^2 + x - 1$

$\lim_{\Delta x\to 0} \frac{2(x+\Delta x)^2 + (x+\Delta x) - 1 - (2x^2+x-1)}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{2x^2 + 4x\Delta x + 2\Delta x^2 + x + \Delta x - 2x^2 - x}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{4x\Delta x + 2\Delta x^2 + \Delta x}{\Delta x}$

$\lim_{\Delta x\to 0} 4x + 2\Delta x + 1 = \boxed{4x+1}$

c) $h(x) = \frac{1}{x-1}$

$\lim_{h\to 0} \frac{\frac{1}{(x+h)-1} - \frac{1}{x-1}}{h}$

$\lim_{h\to 0} \frac{\frac{(x-1) - [(x+h)-1]}{[(x+h)-1](x-1)}}{h}$

$\lim_{h\to 0} \frac{\frac{-h}{[(x+h)-1](x-1)}}{h}$

$\lim_{h\to 0} \frac{-1}{(x+h-1)(x-1)} = \boxed{\frac{-1}{(x-1)^2}}$

II. Find the equation of the line tangent to the function at the given point. (Then, sketch the function and tangent line to verify your answer)

a) $f(x) = x^3$; (2, 8)

$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$

$f(2) = 8$

$f(2+h) = (2+h)^3$

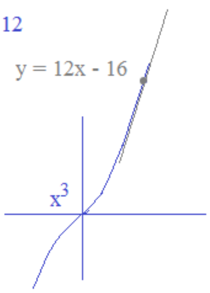
$\lim_{h\to 0} \frac{8 + 12h + 6h^2 + h^3 - 8}{h}$

$\lim_{h\to 0} 12 + 6h + h^2 = 12$

slope: 12

point: (2, 8)

equation of the line is $(y - 8) = 12(x - 2)$



b) $g(x) = x^2 + 1$; (2, 5)

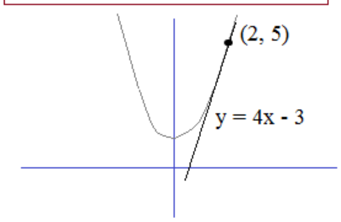
$\lim_{h\to 0} \frac{g(2+h)-g(2)}{h}$

$\lim_{h\to 0} \frac{(2+h)^2 + 1 - [(2)^2+1]}{h}$

$\lim_{h\to 0} \frac{h^2 + 4h + 4 + 1 - 5}{h}$

$\lim_{h\to 0} h + 4 = 4$ (slope at (2, 5))

equation of line: $y - 5 = 4(x - 2)$



c) $h(x) = \sqrt{x+1}$; (3, 2)

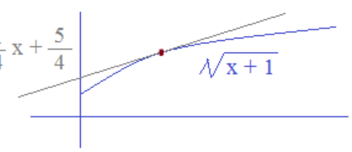
$\lim_{\Delta x\to 0} \frac{h(3+\Delta x) - h(3)}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\sqrt{3+\Delta x+1} - 2}{\Delta x} \cdot \frac{\sqrt{3+\Delta x+1}+2}{\sqrt{3+\Delta x+1}+2}$

$\lim_{\Delta x\to 0} \frac{4+\Delta x - 4}{\Delta x\,[\sqrt{3+\Delta x+1}+2]}$

$\lim_{\Delta x\to 0} \frac{1}{\sqrt{4+\Delta x}+2} = \frac{1}{4}$

equation of tangent line: $y - 2 = \frac{1}{4}(x - 3)$



III. Find the <u>average rate</u> of change between 2 given points. Then, find the <u>instantaneous rate</u> of change *at each point*. (Graph and compare the results.)

(Instantaneous Rate of Change formula) $\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

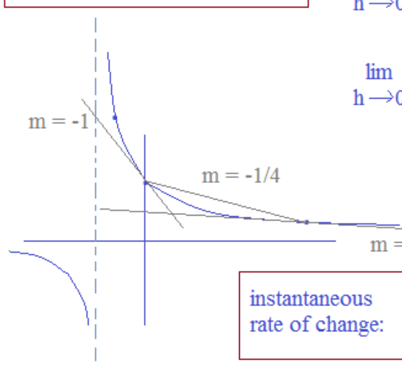
a) $f(t) = \frac{1}{t+1}$; (0, 1) $(3, \frac{1}{4})$ $\lim_{h\to 0} \frac{f(t+h)-f(t)}{h}$

ave. rate: (slope) $\frac{1 - 1/4}{0 - 3} = \frac{-1}{4}$

$\lim_{h\to 0} \frac{\frac{1}{(t+h)+1} - \frac{1}{t+1}}{h}$

$\lim_{h\to 0} \frac{\frac{(t+1) - [(t+h)+1]}{[(t+h)+1](t+1)}}{h}$

$\lim_{h\to 0} \frac{\frac{-h}{[(t+h)+1](t+1)}}{h}$



instantaneous rate of change: $\frac{-1}{[t+1](t+1)}$ (0, 1): -1 (3, 1/4): $\frac{-1}{16}$

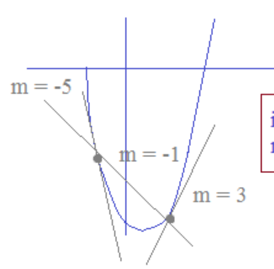
b) $g(s) = s^2 - 3s - 10$; $(-1, -6)$ $(3, -10)$

ave. rate: (slope) $\frac{-10 - (-6)}{3 - (-1)} = -1$

$\lim_{h\to 0} \frac{(s+h)^2 - 3(s+h) - 10 - (s^2 - 3s - 10)}{h}$

$\lim_{h\to 0} \frac{s^2 + 2sh + h^2 - 3s - 3h - 10 - s^2 + 3s + 10}{h}$

$\lim_{h\to 0} \frac{2sh + h^2 - 3h}{h} = \lim_{h\to 0} h + 2s - 3$



instantaneous rate of change: $2s - 3$ (-1, -6): -5 (3, -10): 3

Evaluate the limits.

Recognize the defintion of derivative!! $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Definition of Derivative

SOLUTIONS

1) $\lim_{h \to 0} \frac{\sqrt{4+h} - 2}{h}$

To find the limit, plug in h = 0 ---> the result is $\frac{0}{0}$ (indeterminate)

So, multiply by the conjugate:

$$\frac{\sqrt{4+h} - 2}{h} \cdot \frac{(\sqrt{4+h} + 2)}{(\sqrt{4+h} + 2)} = \frac{4 + h - 4}{h\,(\sqrt{4+h} + 2)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\,(\sqrt{4+h} + 2)} = \boxed{\frac{1}{4}}$$

short-cut: $f(a+h) = \sqrt{4+h}$

$f(a) = 2$ ----> $\sqrt{4}$ $a = 4$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2} x^{-1/2}$ $\boxed{f'(4) = 1/4}$

2) $\lim_{\Delta x \to 0} \frac{(\Delta x + 2)^3 - 8}{\Delta x}$

To find the limit, plug in 0... Result is 0/0 (indeterminate)

So, multiply out the numerator...

$$\lim_{\Delta x \to 0} \frac{\Delta x^3 + 6\Delta x^2 + 12\Delta x + \cancel{8} - \cancel{8}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x^2 + 6\Delta x + 12}{1} = \boxed{12}$$

short-cut: (utilizing the definition of derivative)

$$f'(a) = \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

$f(x) = x^3$. $f(a) = 8$, so $a = 2$

$f'(x) = 3x^2$ $f'(2) = \boxed{12}$

3) $\lim_{x \to \frac{\pi}{6}} \frac{\sin(x) - \sin(\frac{\pi}{6})}{x - (\frac{\pi}{6})}$

Instantaneous rate of change: $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$f(x) = \sin(x)$ $f(a) = \sin(\frac{\pi}{6})$

$a = \frac{\pi}{6}$ $\Longrightarrow$

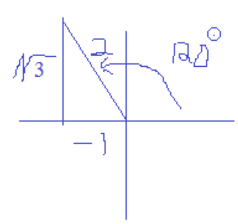
$f'(x) = \cos(x)$

$f'(\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \boxed{\frac{\sqrt{3}}{2}}$

4) $\lim_{h \to 0} \frac{\tan(\frac{2\pi}{3} + h) + \sqrt{3}}{h}$

$\lim_{h \to 0} \frac{\tan(\frac{2\pi}{3} + h) - (-\sqrt{3})}{h}$

$f'(x) = \sec^2(x)$ $\boxed{f'(a) = 4}$



f(a + h)   f(a)

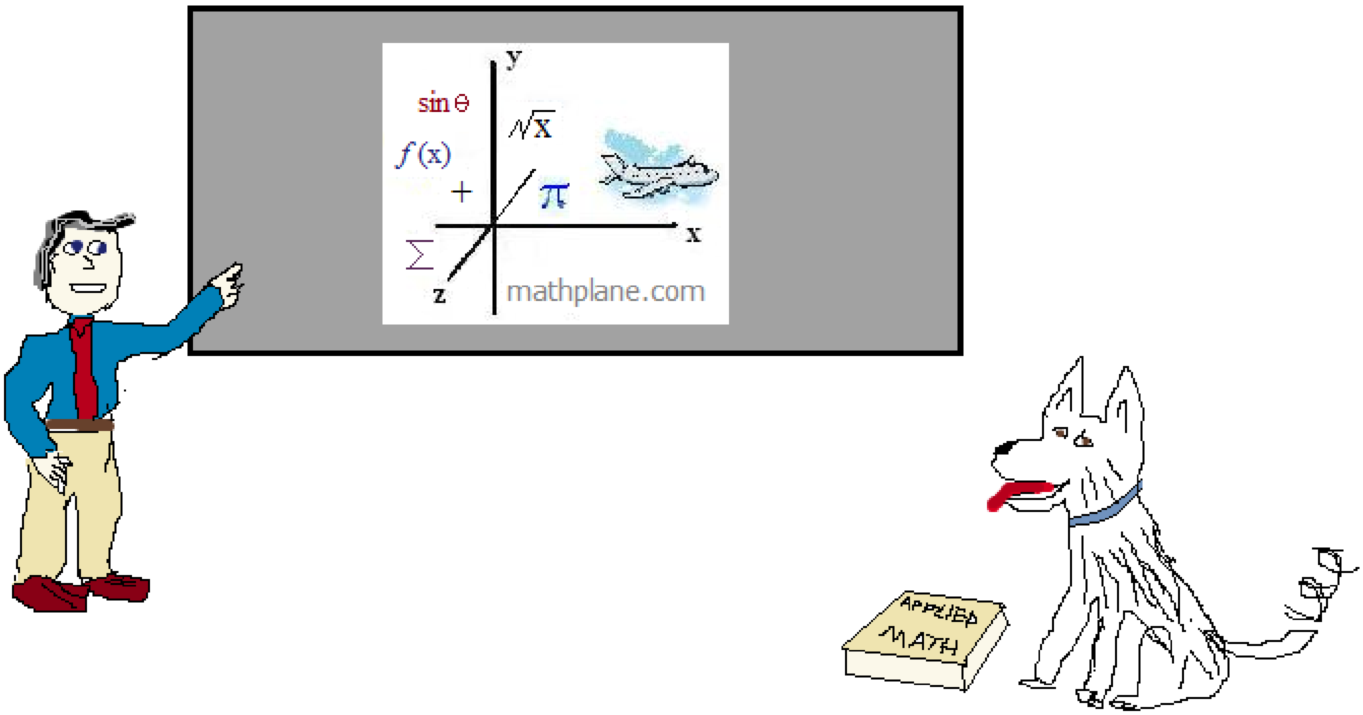
$$\frac{\tan(\frac{2\pi}{3} + h) + \sqrt{3}}{h}$$

$f(x) = \tan(x)$ $a = \frac{2\pi}{3}$ $\tan(\frac{2\pi}{3}) = -\sqrt{3}$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers,



Also, at Pinterest, TES, and TeachersPayTeachers....

And, Mathplane *Express* for mobile at mathplane.ORG

One more question:

At what point is the tangent to $f(x) = x^2 + 4x + 1$ a horizontal line?

ONE MORE QUESTION:

At what point is the tangent to $f(x) = x^2 + 4x + 1$ a horizontal line?

Solution:

A horizontal line has slope = 0. Therefore, if we can find where the instantaneous rate of change is 0, we will have the point of tangency.

Instantaneous rate of change:	$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$$\lim_{h \to 0} \frac{(a+h)^2 + 4(a+h) + 1 - [a^2 + 4a + 1]}{h}$$

$$\lim_{h \to 0} \frac{a^2 + 2ah + h^2 + 4a + 4h + 1 - a^2 - 4a - 1}{h}$$

$$\lim_{h \to 0} \frac{2ah + h^2 + 4h}{h} = \lim_{h \to 0} \frac{h(2a + h + 4)}{h}$$

$$= 2a + 4$$

The instantaneous rate of change at point a is 2a + 4;
Therefore, the instantaneous rate of change is 0 when a = -2

$$f(-2) = (-2)^2 + 4(-2) + 1 = -3$$

The tangent is horizontal at (-2, -3)