Algebra 2 Review 001

Examples and practice worksheets (with solutions)

Topics include conics, arc length and sector area, factoring, venn diagram, domain, radicals, and more.

Find the equation of a circle where the endpoints of the diameter are $(-2, -5)$ and $(2, 1)$

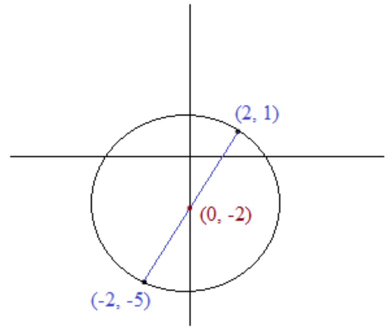
To express the equation of a circle, we need the radius and center of the circle.

Center of the Circle: Since $(-2, -5)$ and $(2, 1)$ are endpoints of a diameter, the midpoint is the center of the circle.

Midpoint formula: $\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2} \right)$

$$\left(\frac{(-2+2)}{2}, \frac{(-5+1)}{2} \right) = (0, -2)$$

The center (h, k) is $(0, -2)$



Radius of the Circle: Using the distance formula, we can find the length of the diameter. (The radius is 1/2 the diameter)

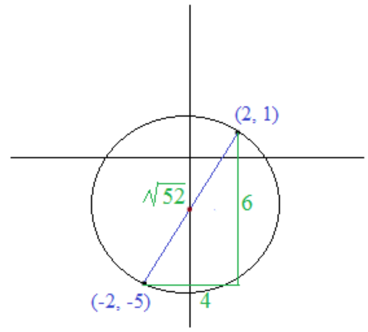
Distance Formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(-2-2)^2 + (-5-1)^2}$$

$$d = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$$

The diameter is $2\sqrt{13}$, so the radius is $\sqrt{13}$



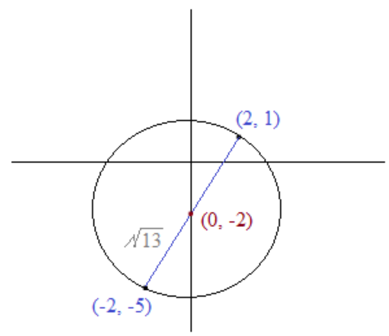
Equation of the Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

where r is the radius and (h, k) is the center of the circle.

$$(x-0)^2 + (y-(-2))^2 = (\sqrt{13})^2$$

$$\boxed{x^2 + (y+2)^2 = 13}$$



Quick Check: Do endpoints fit into the equation?

(2, 1) $\quad (2)^2 + ((1)+2)^2 = 13$

$4 + 9 = 13$ ✓

(-2, -5) $\quad (-2)^2 + ((-5)+2)^2 = 13$

$4 + 9 = 13$ ✓

Solve and Graph the following System:

$$9x^2 + y^2 - 2y = 80$$
$$x^2 + y^2 - 10y = 0$$

$9x^2 + y^2 - 2y = 80$ (complete the square)

$9x^2 + y^2 - 2y + 1 = 80 + 1$

$9x^2 + (y-1)^2 = 81$

$$\frac{x^2}{9} + \frac{(y-1)^2}{81} = 1$$

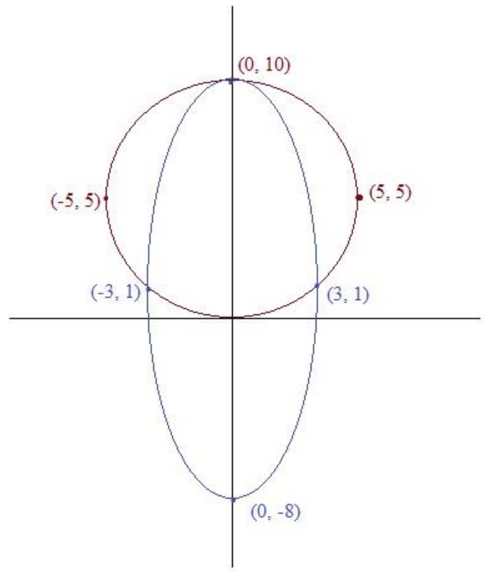
Ellipse: minor axis x, a = 3
major axis y, b = 9
Center: (0, 1)

$x^2 + y^2 - 10y = 0$

$x^2 + y^2 - 10y + 25 = 0 + 25$

$x^2 + (y-5)^2 = 25$

Circle: Radius = 5
Center: (1, 5)



Algebraic Solution:

$9x^2 + y^2 - 2y = 80$ $\quad x^2 + y^2 - 10y = 0$ (isolate x^2)

$x^2 = -y^2 + 10y$

(Use substitution) $9(-y^2 + 10y) + y^2 - 2y = 80$

$-9y^2 + 90y + y^2 - 2y = 80$

$-8y^2 + 88y - 80 = 0$

(factor) $8y^2 - 88y + 80 = 0$

$y^2 - 11y + 10 = 0$

(solve) $(y-10)(y-1) = 0$

$y = 1, 10$

(using the y solutions, find x)

$x^2 + y^2 - 10y = 0$

$x^2 + (1)^2 - 10(1) = 0$

$x^2 - 9 = 0$

$x = 3, -3$

$x^2 + (10)^2 - 10(10) = 0$

$x^2 + 100 - 100 = 0$

$x = 0$

Solutions are	(0, 10)
	(3, 1)
	(-3, 1)

To check, plug solutions into other equation:

$9x^2 + y^2 - 2y = 80$

(0, 10) $9(0)^2 + (10)^2 - 2(10) = 80$

$0 + 100 - 20 = 80$ ✓

(3, 1) $9(3)^2 + (1)^2 - 2(1) = 80$

$81 + 1 - 2 = 80$ ✓

(-3, 1) $9(-3)^2 + (1)^2 - 2(1) = 80$

$81 + 1 - 2 = 80$ ✓

Solve the following. (Use interval notation) Then, sketch the equations.

1) $X^3 - 3X^2 - 13X + 15 < 0$

2) $\frac{2X - 5}{X + 2} \le 1$

---

1) $X^3 - 3X^2 - 13X + 15 < 0$

Factor the polynomial. In this case, find the X-intercepts (zeros).

(Using Rational Root Theorem), possible intercepts are:

$\pm 1, \pm 3, \pm 5, \pm 15$

(Using the Remainder/Factor Theorem), try 1:

$(1)^3 - 3(1)^2 - 13(1) + 15 = 0$

Therefore, 1 is a root!

(Synthetic Division)

1	1	−3	−13	15
		1	−2	−15
	1	−2	−15	0

$(X - 1)(X^2 - 2X - 15)$

$(X - 1)(X - 5)(X + 3) < 0$

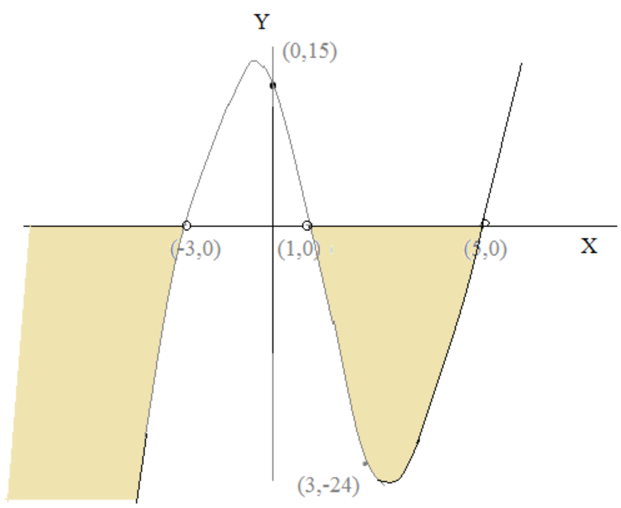
Now, test the intervals:

−3 1 5

(-4, -45) (0, 15) (3, -24) (6, 45)

− + − +

$(-\infty, -3) \cup (1, 5)$



2) $\frac{2X - 5}{X + 2} \le 1$

First, set equation = 1.
and, cross multiply...

$\frac{2X - 5}{X + 2} = \frac{1}{1}$

$(2X - 5) \cdot 1 = (X + 2) \cdot 1$

$2X - 5 = X + 2$

$X = 7$

Now, test points on each side of 7.

plug 5 into original equation:

$\frac{2(5) - 5}{(5) + 2} \le 1$? YES

plug 9 into original equation:

$\frac{2(9) + 5}{(9) + 2} \le 1$? NO

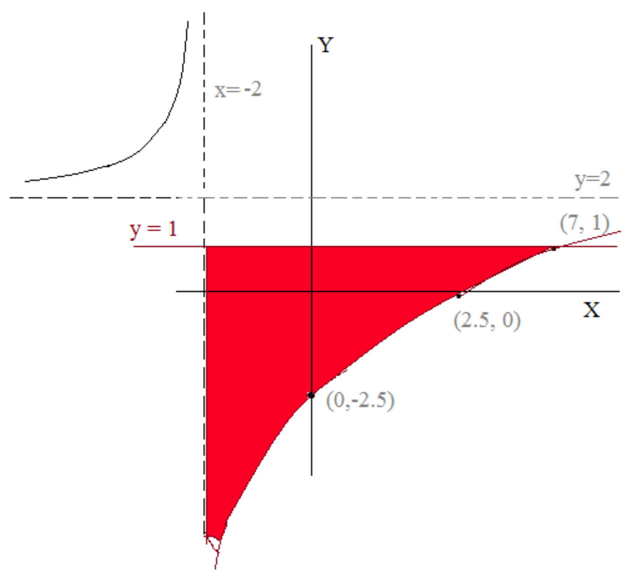
Therefore, $X \le 7$

**Identify the behavior of the function at the vertical asymptote!

At X = - 2, the output is undefined.
At X > - 2, the output Y < 2
At X < -2, the output Y > 2

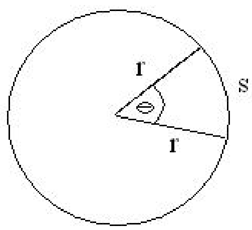
Therefore, $X > -2$

$(-2, 7]$



The sector area of a circle is $3cm^2$. And, the perimeter of the sector is 7cm. What is the (possible) length(s) of the radius?

Step 1: Draw the figure; label the parts



Step 2: List measurements and formulas.

$$\text{Sector Area} = \frac{\theta}{360}\pi r^2 = 3cm^2$$

$$\text{Arc Length} = \frac{\theta}{360} 2\pi r = s$$

$$\text{Perimeter of sector} = r + r + s$$
$$= 2r + s = 7$$

Step 3: Combine formulas and use algebra to find missing variables.

$$\frac{\theta}{360} = \frac{3cm^2}{\pi r^2} \quad \text{(from sector area)}$$

$$\frac{\theta}{360} = \frac{s}{2\pi r} \quad \text{(from arc length)}$$

$$\frac{3cm^2}{\pi r^2} = \frac{s}{2\pi r} \quad \text{(substitution)}$$

$$\frac{3cm^2}{r} = \frac{s}{2} \quad \text{(multiply both by } \pi r\text{)}$$

(multiply both by 2)

$$\frac{6cm^2}{r} = s$$

Step 4: Place s into perimeter formula to find r

$$2r + s = 7$$

$$2r + \frac{6cm^2}{r} = 7 \quad \text{(substitution)}$$

$$2r^2 + 6cm^2 = 7r \quad \text{(multiply entire equation by r)}$$

$$2r^2 - 7r + 6cm^2 = 0 \quad \text{(Factor and solve)}$$

$$(2r - 3cm)(r - 2cm) = 0$$

radius = 1.5 cm or 2 cm

Step 5: Check your answer

If r = 2 cm

Area of circle = 4π

Circumference = 4π

s = 3 (because we were given 2r + s = 7)

$$\text{Arc Length} = \frac{\theta}{360} 2\pi r = s$$

$$\frac{\theta}{360} 2\pi(2cm) = 3$$

$$\frac{\theta}{360} 4cm = \frac{3}{\pi}$$

$$\theta = \frac{270}{\pi}$$

$$\text{Sector Area} = \frac{\theta}{360}\pi r^2 = 3cm^2$$

$$\frac{\frac{270}{\pi}}{360}\pi(2)^2 = 3cm^2$$

$$\frac{270}{360}(4) = 3cm$$

$$\frac{270}{360} = \frac{3}{4} \checkmark$$

(**Then, check r = 1.5)

Solving radical equations

Can you solve?

1) $\sqrt{2x+5} = x - 5$

2) $\sqrt{7x+4} = x + 2$

3) $\sqrt{x} - \sqrt{x-5} = 1$

4) $\sqrt{x+3} + 2\sqrt{x} > 10$

Answers below the line...

---

1) $\sqrt{2x+5} = x - 5$

square both sides $2x + 5 = (x-5)^2$

collect "like" terms $0 = x^2 - 10x + 25 - 2x - 5$

$0 = x^2 - 12x + 20$

factor and solve $(x-2)(x-10) = 0$

$x = 2$ or 10

check (in original equation) $\sqrt{2(2)+5} = (2) - 5$

$3 \neq -3$ ✗ 2 is extraneous

$\sqrt{2(10)+5} = (10) - 5$

$5 = 5$ ✓ 10 is the solution

2) $\sqrt{7x+4} = x + 2$

square both sides $7x + 4 = (x+2)^2$

collect "like" terms $0 = x^2 + 4x + 4 - 7x - 4$

$x^2 - 3x = 0$

factor and solve $x(x+3) = 0$

$x = 0$ or 3

check answers $\sqrt{7(0)+4} = (0) + 2$

$2 = 2$ ✓

$\sqrt{7(3)+4} = (3) + 2$

$5 = 5$ ✓

Both 0 and 3 are solutions

3) $\sqrt{x} - \sqrt{x-5} = 1$

Separate the radicals. Then, square both sides! $\sqrt{x} = 1 + \sqrt{x-5}$

$x = (1 + \sqrt{x-5})(1 + \sqrt{x-5})$

$x = 1 + \sqrt{x-5} + \sqrt{x-5} + x - 5$

Collect "like" terms and isolate the radical $4 = 2\sqrt{x-5}$

Square both sides again $16 = 4(x-5)$

$4 = x - 5$

$x = 9$

Check answer $\sqrt{(9)} - \sqrt{(9)-5} = 1$ 9 is the solution

$3 - 2 = 1$ ✓

4) $\sqrt{x+3} + 2\sqrt{x} > 10$

First, ignore the inequality and solve

$\sqrt{x+3} = 10 - 2\sqrt{x}$ Square both sides

$x + 3 = 100 - 40\sqrt{x} + 4x$ Collect "like" terms

$40\sqrt{x} = 97 + 3x$ Square both side

$1600x = 9x^2 + 582x + 9409$

$9x^2 - 1018x + 9409 = 0$ Quadratic formula

$x = 102.96$ and 10.15 (approx.)

plug 10.15 into the original equation, and we can see the solution is $x > 10.15$

$x > 10.15$ (approx)

Example: Find the domain of $y = \sqrt{x+2} + \sqrt{3-x}$

Separate the equation into two functions:

$\sqrt{x+2}$ $\quad\quad$ $\sqrt{3-x}$

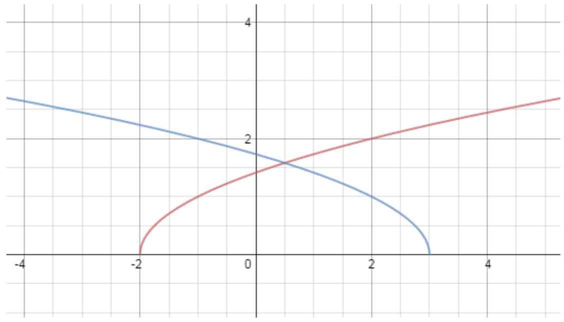
Find the domain of each function:

$x \geq -2$ $\quad\quad$ $x \leq 3$

(cannot have a negative under radical)

Then, identify the <u>intersection</u> of the domains!

The intersection is $-2 \leq x \leq 3$

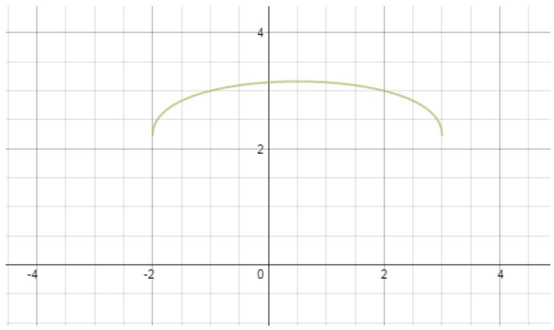


domain of $\sqrt{3-x}$ is $x \leq 3$

domain of $\sqrt{x+2}$ is $x \geq -2$

$y = \sqrt{x+2} + \sqrt{3-x}$

domain: $\{x \mid -2 \leq x \leq 3\}$



Example: $f(x) = \sqrt{x+1}$

$g(x) = \dfrac{2}{x-1}$ $\quad\quad$ Find the domain of $f(g(x))$

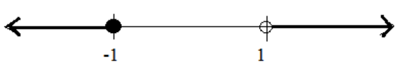
First, the domain of $g(x)$ is all real numbers <u>except</u> 1 (cannot have 0 in the denominator)

Then, the domain of the composite:

$$f(g(x)) = \sqrt{\frac{2}{x-1} + 1}$$

$\dfrac{2}{x-1} + 1 \geq 0$ $\quad\quad$ $\dfrac{2}{x-1} + 1 = 0$ when $x = -1$

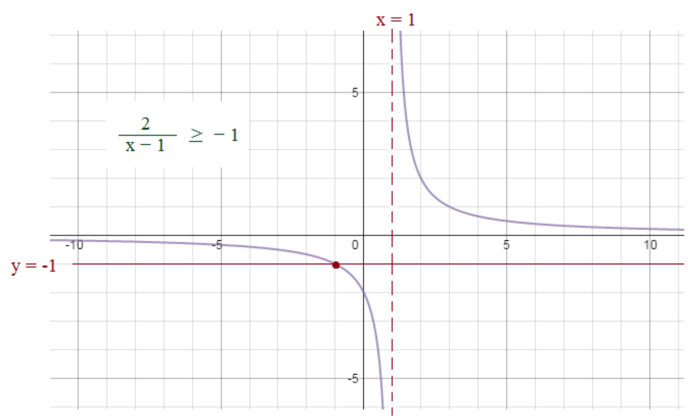
$\dfrac{2}{x-1} + 1$ is undefined at $x = 1$



If $x = -2$, $\dfrac{2}{-2-1} + 1 \geq 0$

if $x = 0$, $\dfrac{2}{0-1} + 1 \ngeq 0$

if $x = 2$, $\dfrac{2}{2-1} + 1 \geq 0$



The composite domain is the intersections:

D: $g(x)$ ∩ D: $f(g(x))$ $= \{x \mid x \leq -1 \text{ or } x > 1\}$

Domain of rational expression

Find the domain: $f(x) = \sqrt{\dfrac{x+3}{2x^2 - 12x - 14}}$

To determine the domain, recognize that the denominator cannot equal zero AND the expression cannot be negative (because of the radical)

$$\frac{x+3}{2x^2 - 12x - 14} \geq 0$$

$$x + 3 = 0$$

The equation equals zero when $x = -3$

$$2x^2 - 12x - 14 = 0$$

$$2(x^2 - 6x - 7) = 0$$

$$2(x - 7)(x + 1) = 0$$

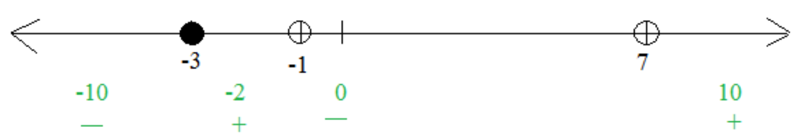
The denominator equals zero when $x = 7$ or $x = -1$

(undefined)

The 'critical values' are at -3, -1, and 7

The domain includes -3, but excludes -1 and 7

Test regions:



$$\frac{x+3}{2(x-7)(x+1)} \geq 0$$

at $x = -10$

$\dfrac{-7}{2(-17)(-9)}$ is negative

at $x = -2$

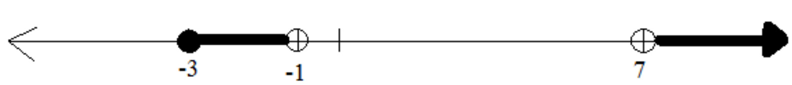
$\dfrac{1}{2(-9)(-1)}$ is positive

at $x = 0$

$\dfrac{3}{2(-7)(1)}$ is negative

at $x = 10$

$\dfrac{13}{2(3)(11)}$ is positive



domain: $[-3, -1) \cup (7, \infty)$

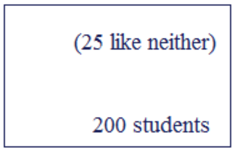
Venn Diagram Application

1) In a survey of 200 students, 115 like math, 80 like english, 25 like neither.

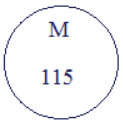
A) What is the probability that a selected student likes *both* english and math?

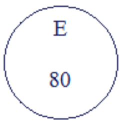
B) What is the probability that a selected student likes *either* math or engish?

An effective method of solving is to use a Venn Diagram:

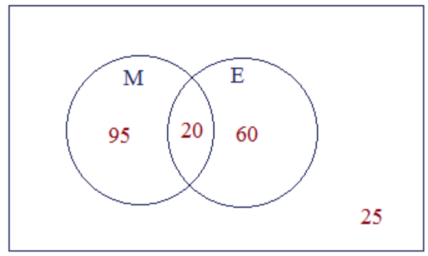


Since 25 like neither,
175 must like either
math or english.
200 − 25 = 175





Since 115 like math
and 80 like english,
there is an overlap of 20
(195 − 175 = 20)



Math only = 115
English only = 80
Math AND English 20
Neither = 25

A) $P(\text{both M and E}) = \frac{20}{200} = 10\%$

B) $P(\text{either M or E}) = \frac{175}{200} = \frac{7}{8} = 87.5\%$

or, $1 - \frac{25}{200}$

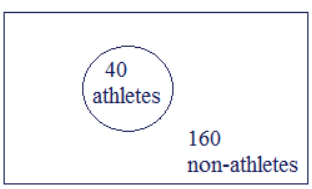
---

2) At the local high school, 20% of the students are athletes that play a sport.
Of the athletes, 25% play football, 10% play ONLY basketball, and 5% play football and basketball.
(The rest of the athletes play other sports.)

A) What percent of athletes play sports other than football or basketball?

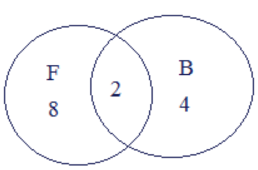
B) If I pick a random student, what is the probability that he plays basketball?

To simplify, let's assume the high school has 200 students



20% athletes
80% non-athletes

Then, let's break up the athletes

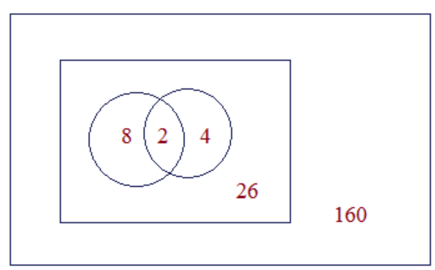


8 football only
4 basketball only
2 football/basketball

A) Therefore, 40 athletes − 14 basketball/football = 26
26 out of 40 play a different sport!

$\frac{26}{40} = 65\%$

Assuming 200 students:



In the diagram, there are 200 students.
And, 6 play basketball
(4 play only basketball; 2 play basketball and football)

B) $P(\text{student plays basketball}) = \frac{6}{200} = 3\%$

Algebra II Factoring Polynomials

Factor the following polynomials:

(Hints and approaches are at the bottom of the page)

1) $8a^3 - 27b^3$

2) $12x^2 + 3xy - 9y^2$

3) $64m^2 - n^2 - 4 + 4n$

4) $x^4 - (y^2 - 25)^2$

5) $x^4 - 19x^2 + 9$

6) $2x(1 - x) + 3(x - 1)$

7) $x^3 - y^2 + 2xy - y^3 - x^2$

8) $3d(x + y) + 2c(y^2 - x^2)$

1) Difference of cubes
2) Greatest Common Factor & Trinomial Quadratic
3) Re-group
4) Difference of Squares
5) Re-write the middle term (and form a perfect square)
6) Re-write (x - 1) and re-group
7) Difference of cubes and Difference of squares
8) Difference of squares Re-group

Algebra II - Simplifying Rational Expressions

Reduce/Simplify the following:

1) $2 + \dfrac{x}{x+5}$

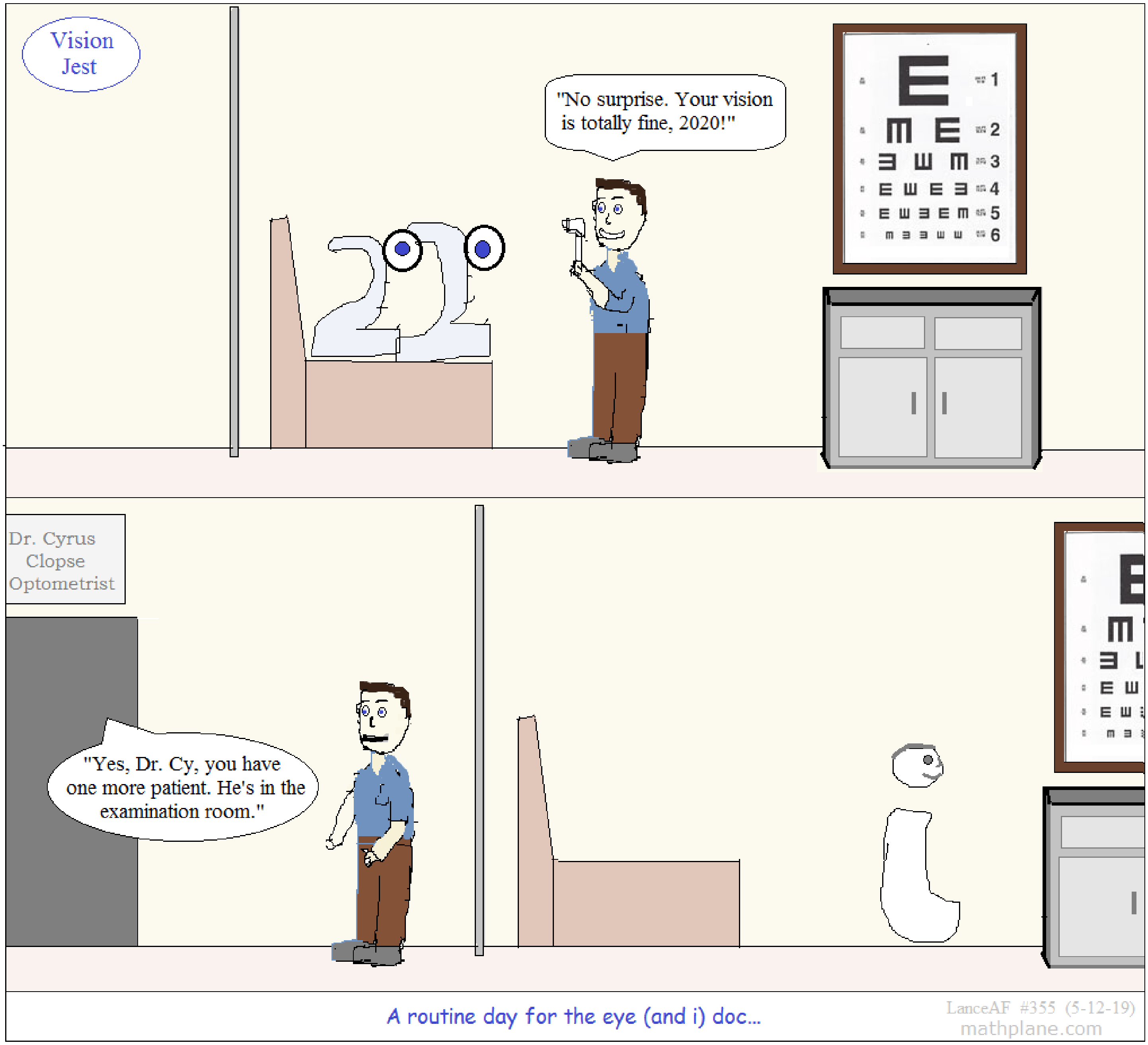
2) $\dfrac{2b^3 - b^2 - 6b}{2b^2 - 7b + 6}$

3) $\dfrac{2x+1}{2x^2 + x - 15} \div \dfrac{6x^2 - x - 2}{x+3}$

4) $\dfrac{x+3}{4x^2 - 9} \div \dfrac{x^2 + 7x + 12}{2x^2 + 7x - 15}$

5) $\dfrac{x + \dfrac{1}{x+2}}{x - \dfrac{1}{x+2}}$

6) $\dfrac{3(1+x)^{1/3} - x(1+x)^{-2/3}}{(1+x)^{2/3}}$


Vision Jest
"No surprise. Your vision is totally fine, 2020!"
Dr. Cyrus Clopse Optometrist
"Yes, Dr. Cy, you have one more patient. He's in the examination room."
A routine day for the eye (and i) doc...
LanceAF #355 (5-12-19)
mathplane.com

ANSWERS-→

Algebra II Factoring Polynomials

Solutions

Factor the following polynomials:

(Hints and approaches are at the bottom of the page)

1) $8a^3 - 27b^3$

cube root of 1st term: 2a
cube root of 2nd term: 3b

$(2a - 3b)(4a^2 + 6ab + 9b^2)$

2) $12x^2 + 3xy - 9y^2$

GCF is 3

$3(4x^2 + xy - 3y^2)$

Factor the quadratic trinomial

$(\quad 3y)(\quad y)$

$(4x \quad 3y)(x \quad y)$

$3(4x - 3y)(x + y)$

3) $64m^2 - n^2 - 4 + 4n$

$64m^2 - (n^2 + 4 - 4n)$

$64m^2 - (n - 2)(n - 2)$

$64m^2 - (n - 2)^2$

difference of squares

$(8m + (n - 2))(8m - (n - 2))$

$(8m + n - 2)(8m - n + 2)$

4) $x^4 - (y^2 - 25)^2$

$[x^2 + (y^2 - 25)][x^2 - (y^2 - 25)]$

$[x^2 + (y + 5)(y - 5)][x^2 - (y + 5)(y - 5)]$

5) $x^4 - 19x^2 + 9$

$x^4 + 6x^2 + 9 - 25x^2$

$(x^2 + 3)(x^2 + 3) - 25x^2$

$(x^2 + 3)^2 - 25x^2$

difference of squares

$(x^2 + 3 + 5x)(x^2 + 3 - 5x)$

6) $2x(1 - x) + 3(x - 1)$

$2x(1 - x) + 3(-1)(1 - x)$

$(2x - 3)(1 - x)$

7) $x^3 - y^2 + 2xy - y^3 - x^2$

$x^3 - y^3 - x^2 + 2xy - y^2$

$x^3 - y^3 - 1(x^2 - 2xy + y^2)$

$x^3 - y^3 - (x - y)(x - y)$

$(x - y)(x^2 + xy + y^2) - (x - y)^2$

$(x - y)[(x^2 + xy + y^2) - (x - y)]$

$(x - y)[x^2 + y^2 + xy - x + y]$

8) $3d(x + y) + 2c(y^2 - x^2)$

$3d(x + y) + 2c(y + x)(y - x)$

$(x + y)[3d + 2c(y - x)]$

1) Difference of cubes
2) Greatest Common Factor & Trinomial Quadratic
3) Separate
4) Difference of Squares
5) Re-write the middle term (and form a perfect square)
6) Re-write (x - 1) and re-group
7) Difference of cubes and Difference of squares
8) Difference of squares Re-group

Algebra II - Simplifying Rational Expressions

SOLUTIONS

Reduce/Simplify the following:

1) $2 + \frac{x}{x+5}$

$\frac{2(x+5)}{(x+5)} + \frac{x}{x+5}$ (common denominator)

$\frac{2x+10+x}{x+5}$ (distribute and add fractions)

$\frac{3x+10}{x+5}$ (collect "like" terms)

2) $\frac{2b^3 - b^2 - 6b}{2b^2 - 7b + 6}$

$\frac{b(2b^2 - b - 6)}{(2b-3)(b-2)}$ (factor out the 'b')

$\frac{b(2b+3)(b-2)}{(2b-3)(b-2)}$ (factor the quadratics)

$\frac{b(2b+3)}{(2b-3)}$

3) $\frac{2x+1}{2x^2+x-15} \div \frac{6x^2-x-2}{x+3}$ (Factor)

$\frac{2x+1}{(2x-5)(x+3)} \div \frac{(3x-2)(2x+1)}{x+3}$ ("invert and multiply")

$\frac{2x+1}{(2x-5)(x+3)} \cdot \frac{x+3}{(3x-2)(2x+1)}$ (cancel and combine terms)

$\frac{1}{(2x-5)(3x-2)}$

4) $\frac{x+3}{4x^2-9} \div \frac{x^2+7x+12}{2x^2+7x-15}$

$\frac{x+3}{(2x+3)(2x-3)} \div \frac{(x+3)(x+4)}{(2x-3)(x+5)}$ (factor)

$\frac{x+3}{(2x+3)(2x-3)} \cdot \frac{(2x-3)(x+5)}{(x+3)(x+4)}$ (invert and multiply)

(reduce)

$\frac{x+5}{(2x+3)(x+4)}$

5) $\frac{x + \frac{1}{x+2}}{x - \frac{1}{x+2}}$

$\frac{\frac{x(x+2)}{(x+2)} + \frac{1}{x+2}}{\frac{x(x+2)}{(x+2)} - \frac{1}{x+2}}$ (combine fractions in the numerator... and, combine fractions in the denominator...)

$\frac{\frac{x^2+2x+1}{(x+2)}}{\frac{x^2+2x-1}{(x+2)}}$ (Divide -- "invert and multiply" the rational expressions)

$\frac{x^2+2x+1}{(x+2)} \cdot \frac{(x+2)}{x^2+2x-1}$ (simplify)

$\frac{(x+1)^2}{x^2+2x-1}$

6) $\frac{3(1+x)^{1/3} - x(1+x)^{-2/3}}{(1+x)^{2/3}}$

(factor numerator)
(use the SMALLER rational exponent)

$\frac{(1+x)^{-2/3}[3(x+1) - x]}{(1+x)^{2/3}}$

(collect terms)

$\frac{(1+x)^{-2/3}[2x+3]}{(1+x)^{2/3}}$

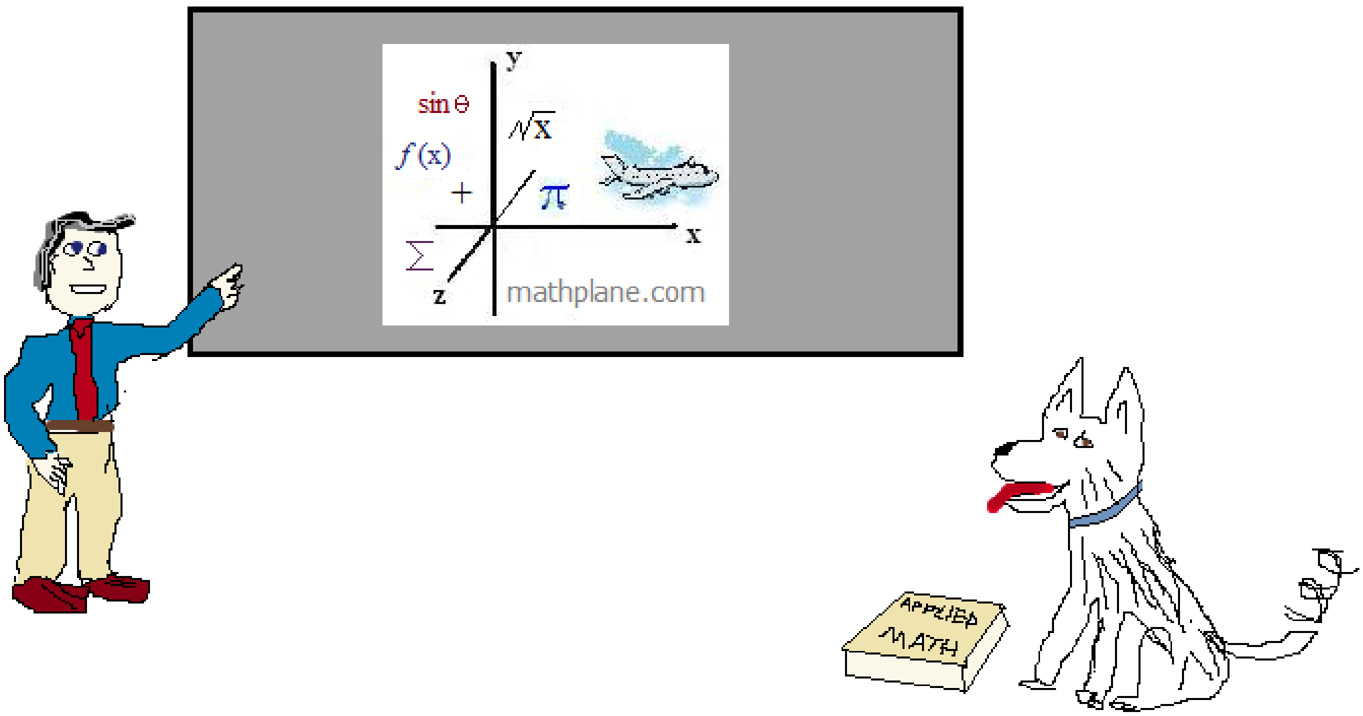
(simplify)

$\frac{2x+3}{(1+x)^{4/3}}$

Thanks for visiting! (Hope it helped.)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, find us at mathplane.ORG for mobile and tablets.

And, TeachersPayTeachers.com