Did you know?

A sample of math topics…

Includes the “rule of 72”, pi, venn diagram, roulette wheels, exponents, area and volume, and more..

What are the angle measures of a triangle with sides 3, 4, and 5?

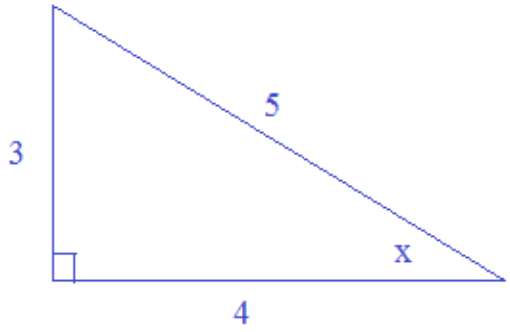
Solution:

As with most problems, it's helpful to draw a picture and write a formula

Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = 5^2$$ confirms that it's a right triangle



Here is the picture of the right triangle --- the largest side must be the *hypotenuse*...
(the other sides are the *legs*)

Then, $\text{Sine } x = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{3}{5} = .60$

Now, find the ArcSine of .60.... 36.87°
(i.e. $\sin(36.87) = 3/5$)

The sum of the interior angles of any triangle is 180 degrees,

therefore, the other angle is

$$180 - (36.87 + 90) = 53.13^\circ$$

The angle measures are 90° 53.13° 36.87°

"The Rule of 72"

What is it? If a sum is compounding at X%, that sum will double in approximately $\frac{72}{X}$ cycles.

Example:

Suppose you deposit $100 in a bank that earns 10% interest per year. How many years will it take to double your money?

Using the "rule of 72"

$$\frac{72}{10} = 7.2$$

So, a little over 7 years...

time	balance
0	100 (initial deposit)
1	110 (100 + 10)
2	121 (110 + 11)
3	133.10 (121 + 12.10)
4	146.41 (133.1 + 13.31)
5	161.05 (146.41 + 10% of 146.41)
6	177.15 (161.05 + interest payment)
7	194.86 (177.15 + interest on 177.15)
8	212.57

Account doubles from 100 to 200 shortly after 7 years...

Formula for compounding interest:

Amount = P (1 + interest rate)^t (where P is the principle and t is the time it compounds)

Using our example:

$200 = 100\,(1 + .10)^t$

$\frac{200}{100} = (1.10)^t$

$2 = (1.10)^t$

(Use a calculator to solve. Or, use logarithms)

$\log 2 = \log (1.10)^t$

$\log 2 = t \cdot \log (1.10)$

$t = \frac{\log 2}{\log (1.10)} = \frac{.3010}{.0414} \cong 7.27$

**If you're earning 10% interest, it will take approximately 7.27 years for you to double your money!

Solve the following:

$9^{2} =$ $\quad$ $9^{-2} =$ $\quad$ $9^{1/2} =$ $\quad$ $9^{-1/2} =$

Review:

$X^{a} \cdot X^{b} = X^{a+b}$ $\quad$ $(X^{a})^{b} = X^{ab}$ $\quad$ $X^{0} = 1$

Solutions and Verification

$9^{2} = 81$ $\quad$ $9 \times 9 = 81$

$9^{-2} = \frac{1}{81}$ $\quad$ $9^{2} \cdot 9^{-2} = 9^{0} \longrightarrow 81 \cdot 9^{-2} = 1$ (reciprocal)

therefore, $9^{-2} = \frac{1}{81}$

$9^{1/2} = 3$ $\quad$ $9^{1/2} \cdot 9^{1/2} = 9^{1} \longrightarrow 9^{1/2} \cdot 9^{1/2} = 9$

therefore, $9^{1/2} = 3$ (square root)

$9^{-1/2} = \frac{1}{3}$ $\quad$ $9^{-1/2} \cdot 9^{1/2} = 9^{0} \longrightarrow 9^{-1/2} \cdot 3 = 1$

therefore, $9^{-1/2} = \frac{1}{3}$

**Bonus Question:

Simplify $(2e^{4})^{3}$

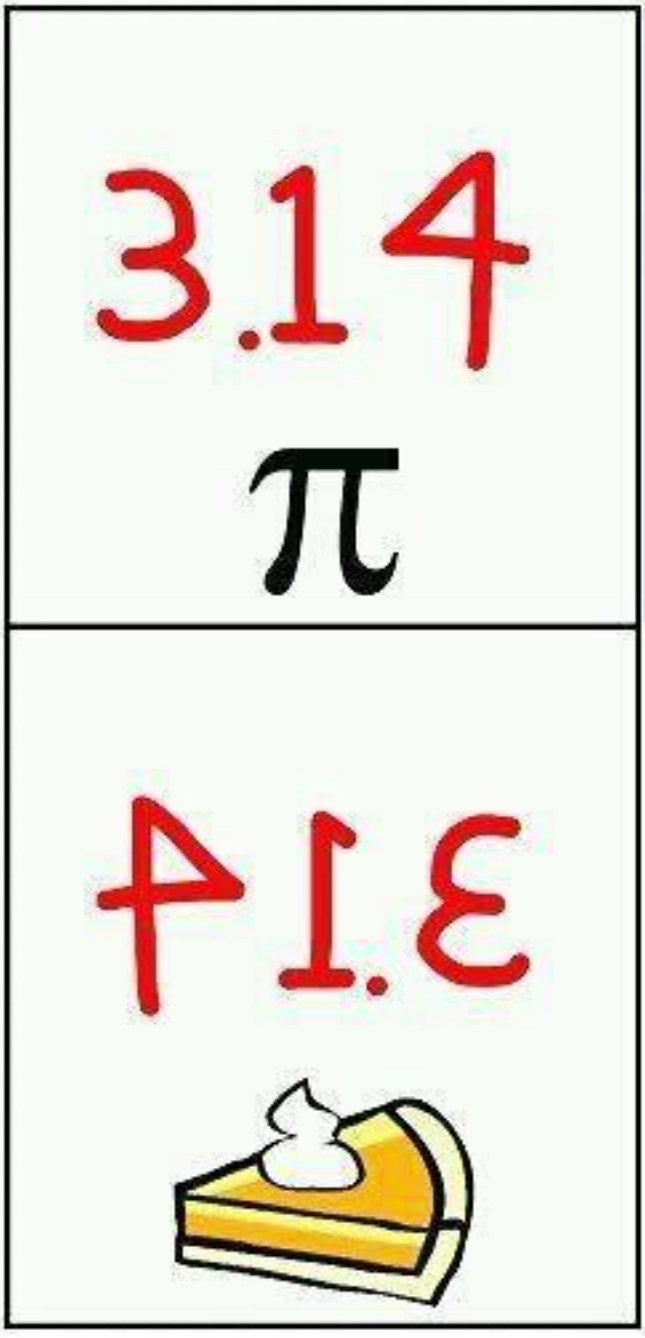
Solution and Verification

$(2e^{4})^{3} = 8e^{12}$ $\quad$ $2e^{4} \cdot 2e^{4} \cdot 2e^{4} = 8e^{12}$

Pi in the mirror….

3.14

π



Prime Numbers

Definition: A natural number (i.e. positive integer) greater than 1 that has no positive divisors other than 1 and itself.

⇨ Its factors are only 1 and itself

* *2 is the only even prime number.* It is divisible by 1 and itself. (Every other even number is divisible by itself, 1, and 2)

* A *non-prime*, positive integer is called a *"composite number"*. It has at least 3 factors: 1, itself, and at least one other number.

* *Zero is neither prime nor composite.*

  Why? Because, zero has an infinite number of factors. (i.e. any number multiplied by 0 is zero!)

* *One is neither prime nor composite.*

  Why? Because, one has only 1 divisor: itself. So, it does not fit either definition.

* *Negative numbers*, such as -7, *are not prime*.

  Why are negative numbers not included in the definition of prime?

  Allowing negatives would double the number of divisors/factors.

  Example: 7 would have factors of -1, 1, 7, -7

  -7 would have factors of -1, 1, 7, -7

Other Comments:

300 BC	Euclid demonstrated that there are infinitely number of primes.
3rd Century BC	Greek mathematician Eratosthenes figured out a way to generate a list of primes. ('sieve of Eratosthenes')
7th Century	Rules for negative numbers were stated

-- The concept of primes preceded the idea of negative numbers. So, primes excluded non-positive integers. The definition of prime numbers was never modified to include negatives.

* The *Fundamental Theorem of Arithmetic* -- Any integer greater than one can be expressed uniquely as a product of primes. To maintain unique factorization, 1's and negative numbers must be omitted.

* A *Marsenne Number* is a positive integer that is 1 less than a power of 2

$$M_p = 2^P - 1$$

So, a *Marsenne Prime* is any Marsenne number that is prime.

Find more information at these sites:

http://en.wikipedia.org/wiki/Prime_number
http://wiki.answers.com/Q/Is_1_a_composite_number
http://nrich.maths.org/5961
http://primes.utm.edu/notes/faq/one.html
http://mathworld.wolfram.com/PrimeNumber.html
http://mathforum.org/library/drmath/view/57036.html
http://numberphile.com/videos/31.html

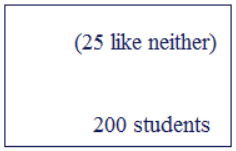
Venn Diagram Application

1) In a survey of 200 students, 115 like math, 80 like english, 25 like neither.

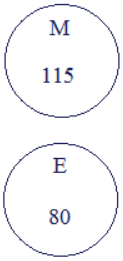
A) What is the probability that a selected student likes *both* english and math?

B) What is the probability that a selected student likes *either* math or engish?

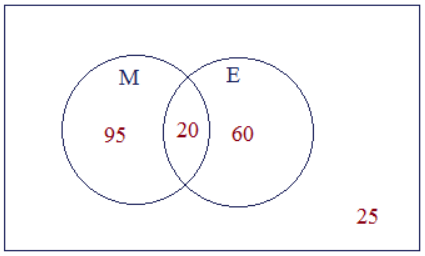
An effective method of solving is to use a Venn Diagram:



Since 25 like neither,
175 must like either
math or english.
200 − 25 = 175



Since 115 like math
and 80 like english,
there is an overlap of 20
(195 − 175 = 20)



Math only = 115
English only = 80
Math AND English 20
Neither = 25

A) P(both M and E) = $\frac{20}{200}$ = 10%

B) P(either M or E) = $\frac{175}{200} = \frac{7}{8}$ = 87.5%

or, $1 - \frac{25}{200}$

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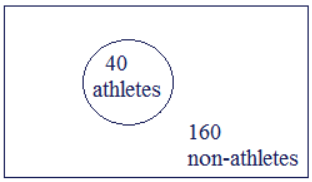
2) At the local high school, 20% of the students are athletes that play a sport.
Of the athletes, 25% play football, 10% play ONLY basketball, and 5% play football and basketball.
(The rest of the athletes play other sports.)

A) What percent of athletes play sports other than football or basketball?

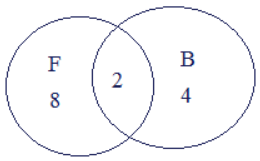
B) If I pick a random student, what is the probability that he plays basketball?

To simplify, let's assume the high school has 200 students



20% athletes
80% non-athletes

Then, let's break up the athletes

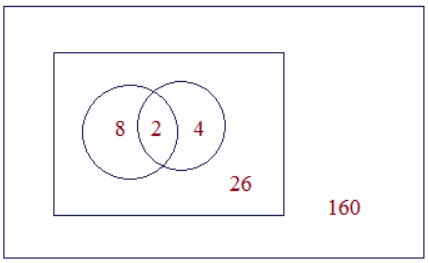


8 football only
4 basketball only
2 football/basketball

A) Therefore, 40 athletes − 14 basketball/football = 26
26 out of 40 play a different sport!

$\frac{26}{40}$ = 65%

Assuming 200 students:

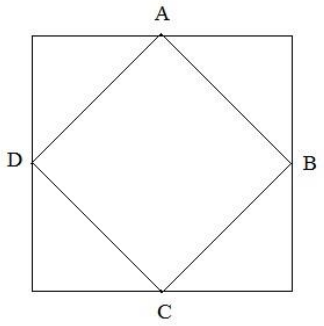


In the diagram, there are 200 students.
And, 6 play basketball
(4 play only basketball; 2 play
basketball and football)

B) P(student plays basketball) = $\frac{6}{200}$ = 3%

"Area of a square inscribed in a square"

Square ABCD is inscribed in the larger square.
If the area of the larger square is 100 sq. feet, what is the area of ABCD?

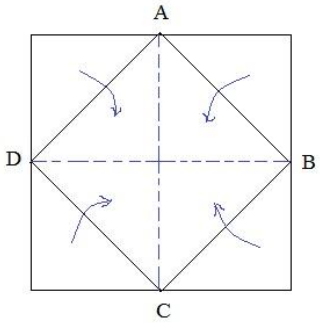


Note: There are a few approaches to solving.

Hint: Since it is a square inscribed in a square, A, B, C, and D are midpoints of the sides of the larger square!

Solution using different approaches:

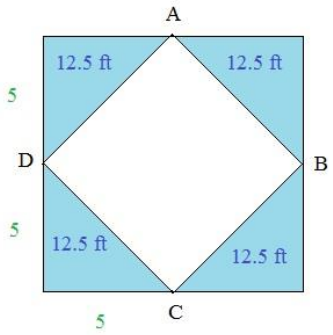
1) Observations and recognizing triangles



The large triangle contains 8 congruent triangles
and
the small triangle contains 4 of those triangles..

Therefore, the area is 4/8 (100) = 50 square feet!

2) large square − extra triangles = inside square



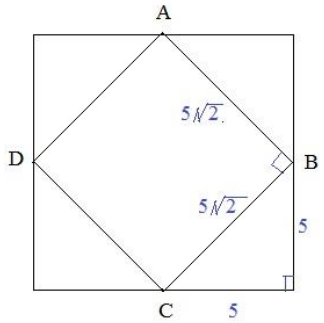
Area of large square = 100 sq. feet
Therefore, each side is 10 feet

Since A, B, C, and D are midpoints, each triangle has a base of 5 and height of 5..
Therefore, the area of each triangle is 1/2bh = 25/2

Since there are 4 triangles, the extra area is 50 sq. feet.

So, the inside square is 100 feet - 50 feet = 50 feet...

3) Calculate the area of the inside square



Consider the right triangle (45-45-90)
From this (or the pythagorean theorem),
we know the length of each side of the inside square is $5\sqrt{2}$

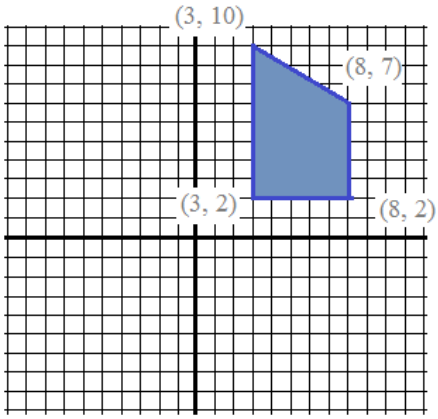
Area of a square is S^2

So, area of inscribed square is $\left(5\sqrt{2}\right)^2 =$ 50 square feet!

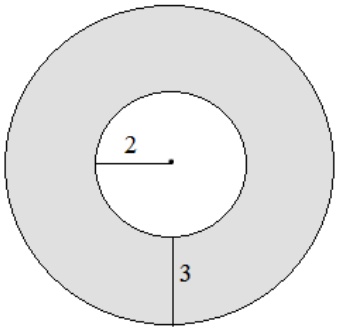
Three Area Problems....

What is the area of each shaded region?

1) Quadrilateral

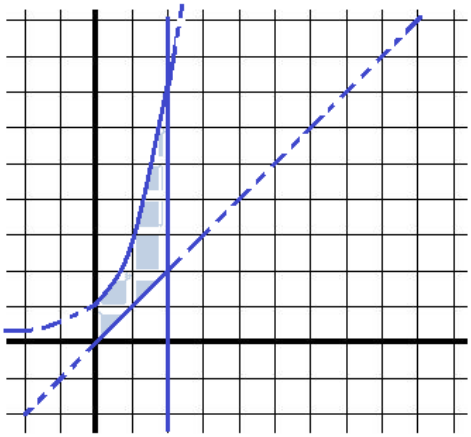


2) Concentric Circles



3) The area bordered by:

$y = e^x$
$y = x$
$x = 2$
the y-axis

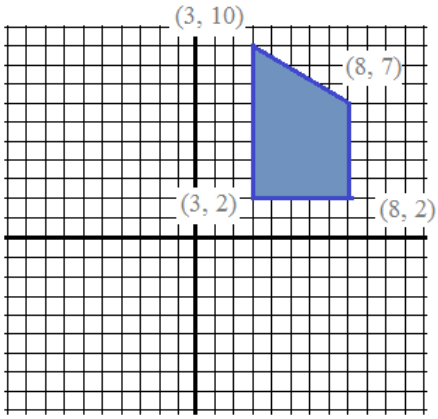


SOLUTIONS-→

Three Area Problems....

What is the area of each shaded region?

1) Quadrilateral



method 1:
square + triangle

(side x side) $\frac{1}{2}$ (base)(height)

$(5 \times 5) + \frac{1}{2}(5)(3)$

$25 + 15/2 = 32.5$

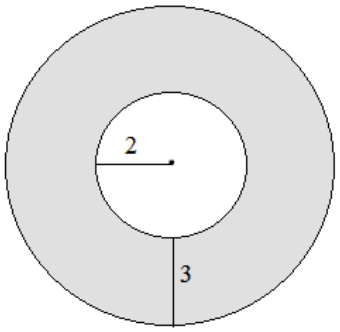
method 2:
(sideways) trapezoid

$\frac{1}{2}$(base1 + base2)(height)

$\frac{1}{2}(8 + 5)(5)$

$\frac{1}{2}\ 65 = 32.5$

2) Concentric Circles



radius of big circle: $2 + 3 = 5$
area of big circle: $\pi r^2 = 25\pi$

radius of small circle: 2
area of small circle: $\pi r^2 = 4\pi$

area of shaded region = $\text{area}_{big} + \text{area}_{small}$

$25\pi - 4\pi = 21\pi$

3) $y = e^x$
$y = x$
$x = 2$
the y-axis

$y = e^x$

x	y
-1	.37
0	1
1	2.72
2	7.39

(approx.)

The shaded area consists of the area under the log function MINUS the right triangle (i.e. the area under the line y = x)

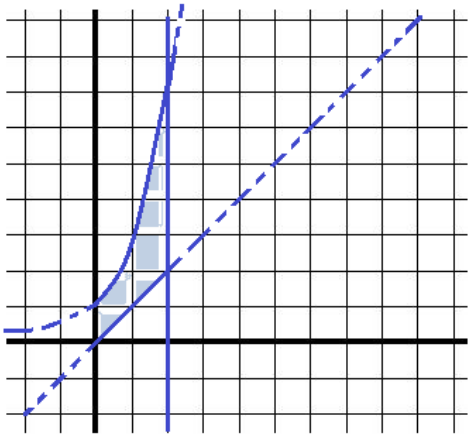
(use definite integral to find area)

$$A_{log} = \int_0^2 e^x\,dx = e^2 - e^0$$

$$= 7.39 - 1$$

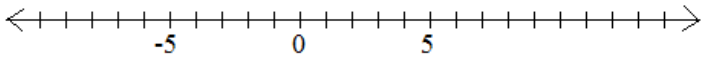
$$A_{tri} = \frac{1}{2}(2)(2) = 2$$

Total Area is approx. 4.39 sq. units

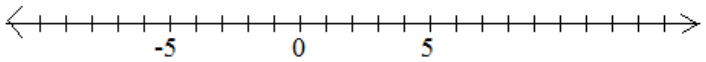


Sketch each of the following on a number line:

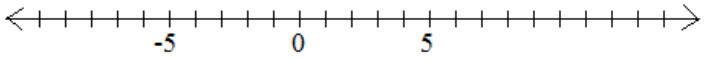
1) $|x - 4| \leq 0$



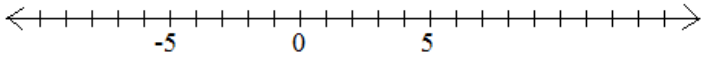
2) $|x + 5| > 0$



3) $x < 6$ OR $x \leq -2$

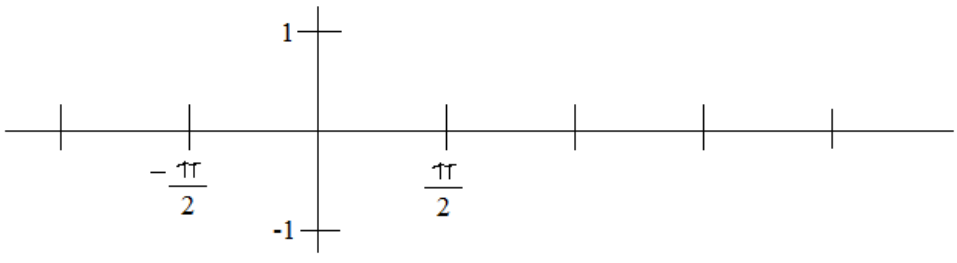


4) $|x - 6| + 8 < 4$



5) Extra:

Sketch the function $|\sin x|$

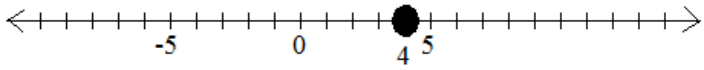


SOLUTIONS-→

Sketch each of the following on a number line:

1) $|x - 4| \leq 0$

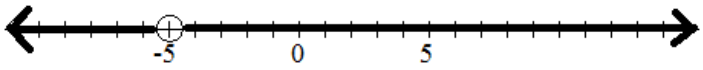
At $x = 4$, $|x - 4| = 0$... At all other x, the absolute value output will be > 0



2) $|x + 5| > 0$

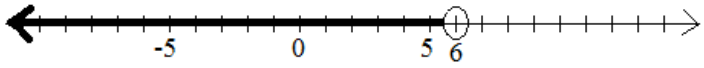
All real numbers EXCEPT -5

At $x = -5$, $|x + 5| = 0$...
At all other x, the output is > 0



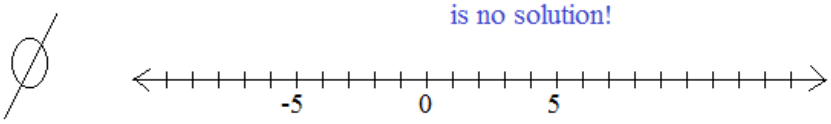
3) $x < 6$ OR $x \leq -2$

If $x \leq -2$, then it must be less than 6..
So, the solution will be $x < 6$



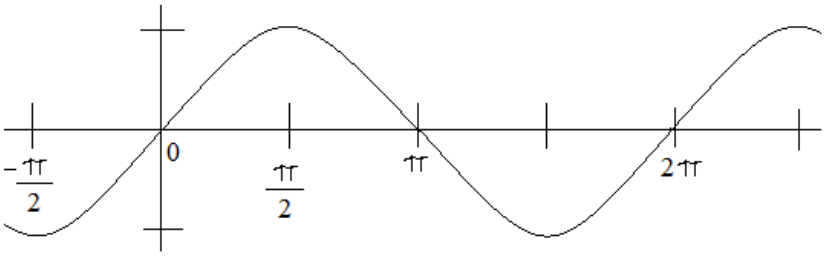
4) $|x - 6| + 8 < 4$ ----> $|x - 6| < -4$

Since the absolute value of any term is non-negative, there is no solution!

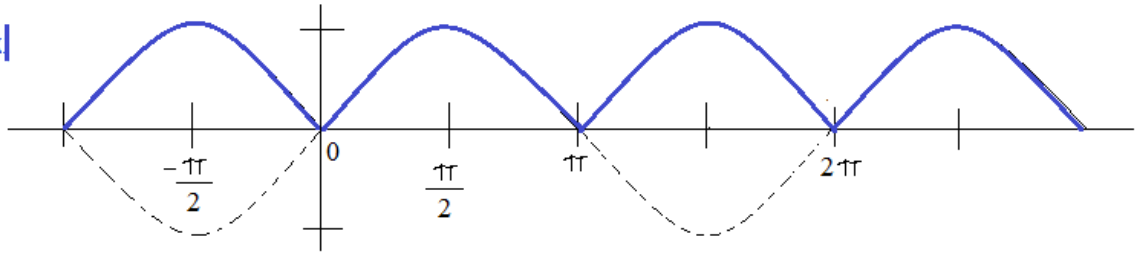


5) Extra: Sketch the function $|\sin x|$

Sin x

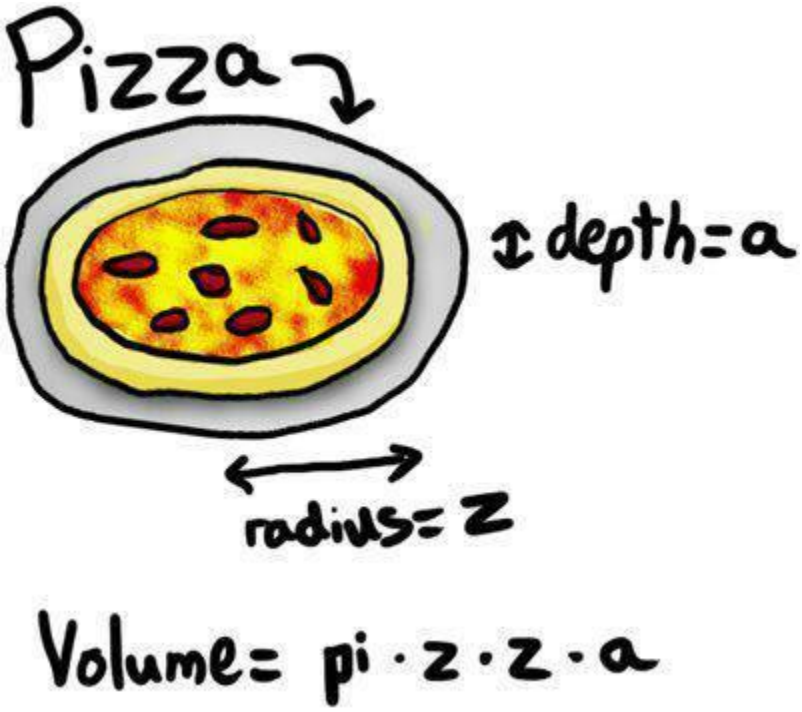


$|\sin x|$



SOLUTIONS

Pizza Pi(e)

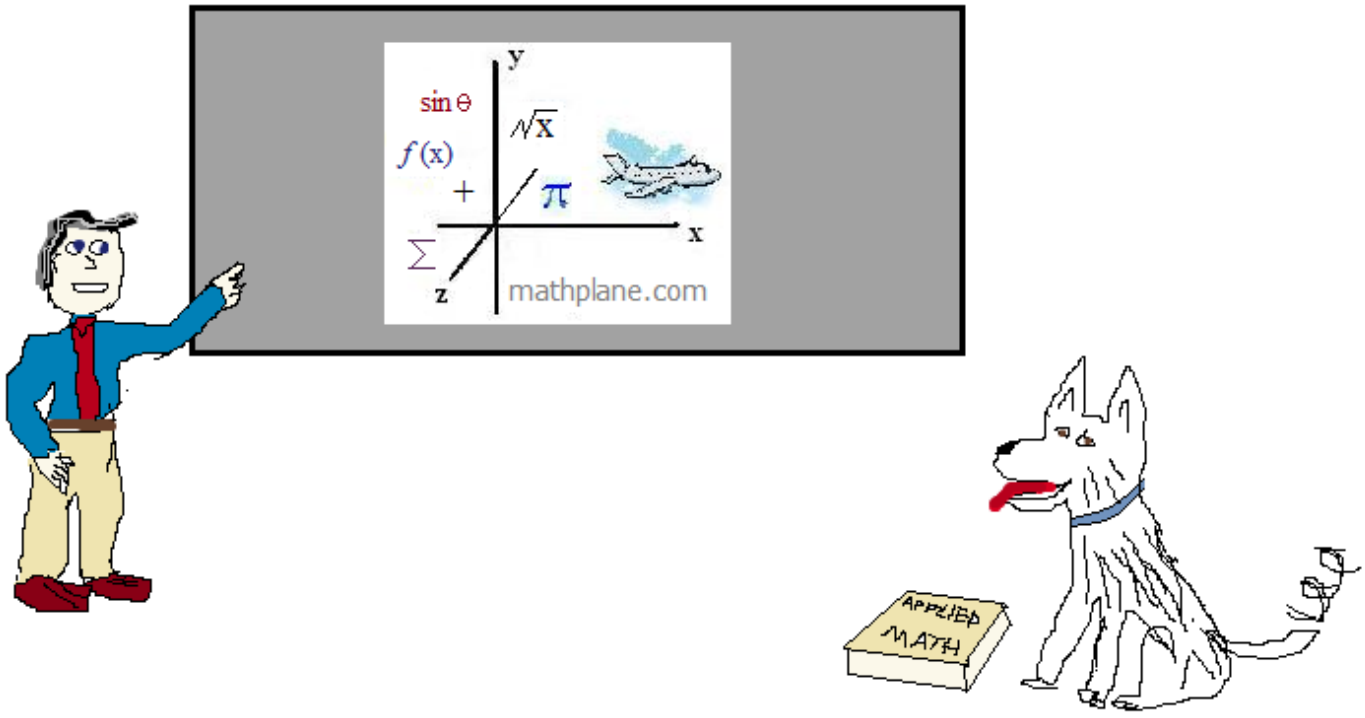


What is the <u>sum</u> of all the numbers on a roulette wheel? 666

Thanks for visiting!

If you have questions, suggestions, or requests, let us know..

Cheers



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