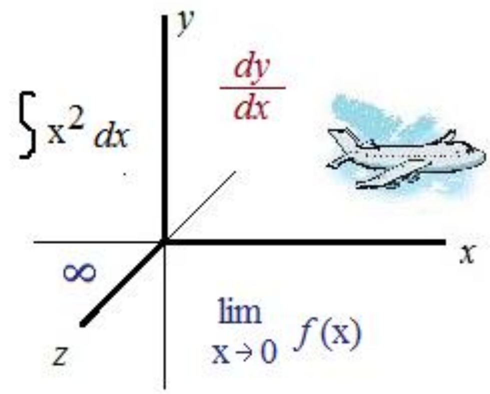
Calculus, Natural Log, and *e*

Practice Test and Solutions



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Topics include logarithms, area, tangent lines, implicit differentiation, graphing, inverses, partial fractions, and more.

Calculus: Logarithms, ln, and *e*

I. Logarithm Review

1) Answer: $\ln 1 =$ $\ln e =$ $\ln 0 =$

(no calculator)

2) $\log 4 = .602$ $\log 3 = .477$

Find: $\log 12 =$ $\log 400 =$ $\log(.75) =$

(no calculator)

3) Solve for x:

A) $\log_5 x + \log_5 (x-4) = 1$

B) $3^x = 8$

C) $2^{6-x} = 4^{2+x}$

Challenge: $4^x - 2^{x+1} = 3$

II. Calculus: *e* and ln

Calculus: Logarithms, ln, and *e*

1) Find $\frac{dy}{dx}$

$y = e^{2x}$ $y = -e^{-x}$ $y = e^{2}$ $y = \ln(2x + 4)$

$y = \ln(3)$ $y = \ln(x + 3)^2$ $y = \ln((x + 3)^2)$ $y = \frac{2}{e^{3x}}$

2) What is the equation of the line tangent to $y = e^{2x-3}$ at the point $(\frac{3}{2}, 1)$?

Optional: graph your result

3) What is the equation of the normal to $y = \ln(x - 2) + 4$ at the point $(3, 4)$?

Optional: graph your result

4) $\int e^{2x}\, dx$ $\qquad$ $\int \frac{3x}{3x^2 + 2}\, dx$ $\qquad$ $\int \frac{2}{3x + 3}\, dx$

5) What is the area of the region above the x-axis that is bounded by the y-axis, $x = 3$, and e^x ?

6) What is the area of the region bounded by $y = \ln(x) + 2$, $y = 2$, and $x = 5$?
(Use Calculator)

7) Find the equation of the line that is tangent to $f(x) = 3x^2 - \ln x$ at $(1, 3)$

(Optional: Use a graphing calculator to confirm your answer)

8) $y = \dfrac{x^2}{2} - \ln x$

What are the extrema?

Points of inflection?

(Optional: Use a graphing calculator to check your answers)

III. Inverses and derivatives

1) $f(x)$ and $g(x)$ are one to one inverses.
If the slope of $f(x)$ at (3, 8) is 2, where is the slope of $g(x)$ equal to 1/2?

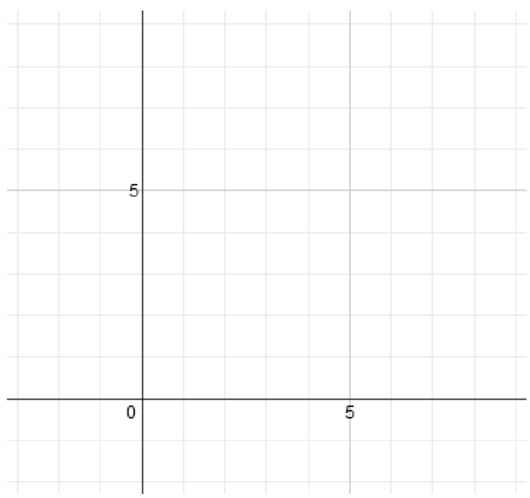
2) $f(x) = x^3 - x - 6$

What is $f^{-1\prime}(0)$?

3) $h(x) = \ln(x) + 4$

a) What is the inverse of $h(x)$?

b) $h(3) = \ln(3) + 4$ ($\approx$ 5.1) What is the slope at $h(3)$?



c) Graph $h(x)$ and $h^{-1}(x)$

Sketch the tangent lines at (3, 5.1) and (5.1, 3)

What are the equations of the tangent lines?

IV. Exponential Functions

Find the first derivatives:

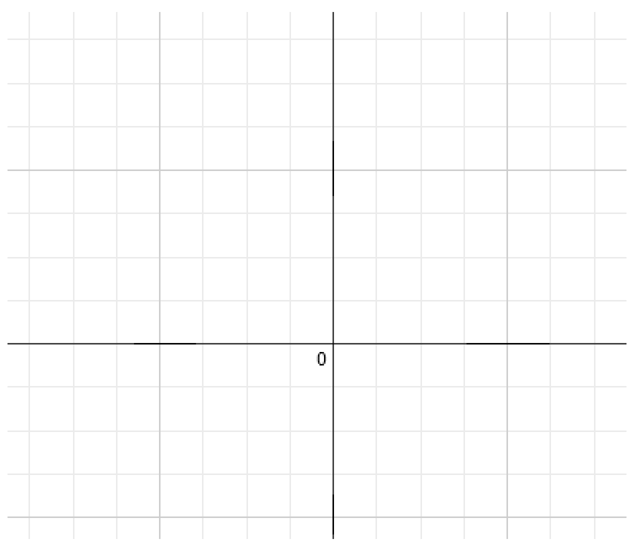
1) $g(x) = 2^{x+3}$

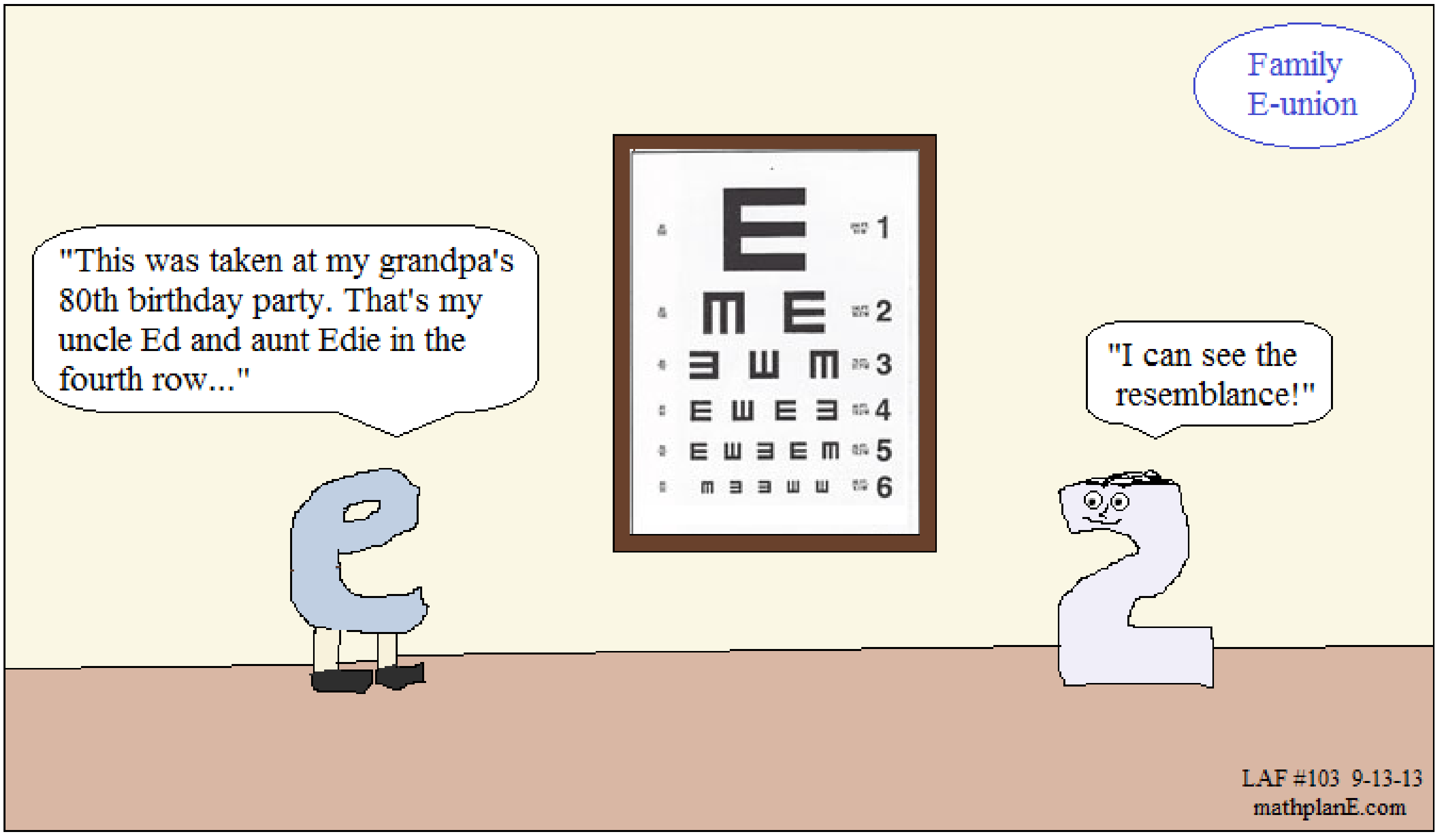
2) $f(x) = x^2 e^x$

3) $g(t) = t^2 2^t$

4) What is the equation of the line tangent to $y = 2^{-x}$ at $(0, 1)$?

(optional: sketch a graph containing
the function and tangent line)




Family E-union
"This was taken at my grandpa's 80th birthday party. That's my uncle Ed and aunt Edie in the fourth row..."
"I can see the resemblance!"
LAF #103 9-13-13
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SOLUTIONS-→

Calculus: Logarithms, ln, and e

I. Logarithm Review

1) Answer: (no calculator)

$\ln 1 =$ $\log_e 1 = x$

$e^x = 1$

$\boxed{x = 0}$

$\ln e =$ $\log_e e = y$

$e^y = e$

$\boxed{y = 1}$

$\ln 0 =$ $\log_e 0 = z$

$e^z = 0$

No solution (logarithms cannot be zero or negative)

2) $\log 4 = .602$ $\log 3 = .477$

Find: (no calculator)

$\log 12 = \log(3 \cdot 4) = \log 3 + \log 4 = .477 + .602 = \boxed{1.079}$

$\log 400 = \log(4 \cdot 10) = \log 4 + \log 10 = .602 + 1 = \boxed{1.602}$

$\log(.75) = \log \frac{3}{4} = \log 3 + \log 4 = .477 - .602 = \boxed{-.125}$

3) Solve for x:

A) $\log_5 x + \log_5 (x-4) = 1$

$\log_5 x(x-4) = 1$

$5^1 = x(x-4)$

$5 = x^2 - 4x$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$\boxed{x = 5}, \cancel{-1}$ log (-1) does not exist

B) $3^x = 8$

$\log 3^x = \log 8$

$x \log 3 = \log 8$

$x = \frac{\log 8}{\log 3}$

$= \frac{.903}{.477} \boxed{= 1.89}$

quick check:

$3^{1.89} = 8$ ✓

C) $2^{6-x} = 4^{2+x}$

$2^{6-x} = (2^2)^{2+x}$

$2^{6-x} = 2^{4+2x}$

$6 - x = 4 + 2x$

$\boxed{x = \frac{2}{3}}$

quick check:

$2^{5.33} = 4^{2.67}$

$40.3 = 40.3$ ✓

Challenge: $4^x - 2^{x+1} = 3$

$4^x - 2^{x+1} - 3 = 0$

$(2^2)^x - (2^x)(2^1) - 3 = 0$

$(2^x)^2 - (2^x)(2^1) - 3 = 0$

Let $y = 2^x$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$y = -1, 3$

therefore,

$2^x = -1$ and 3

-1 is extraneous!

$2^x = 3$

approximately 1.585

$2^x = 3$	$2^x = -1$
X log 2 = log 3	x log 2 = log (-1)
X = Log3/log 2	x = log (-1)/ log (2)
X = 1.5849625	x does not exist

Check:

$4^{1.585} - 2^{2.585} =$

$9 - 6 = 3$ ✓

II. Calculus: e and ln

Calculus: Logarithms, ln, and e

1) Find $\frac{dy}{dx}$

for $y = e^{u}$ "derivative of exponent times itself"

for $y = \ln(u)$ "derivative over itself"

$y = e^{2x}$

$\boxed{2e^{2x}}$

$y = -e^{-x}$

$(-1)(-e^{-x})$

$\boxed{e^{-x} \text{ or } \frac{1}{e^{x}}}$

$y = e^{2}$

$\boxed{0}$

e^{2} is a constant

$y = \ln(2x + 4)$

$\boxed{\frac{2}{2x + 4} \text{ or } \frac{1}{x + 2}}$

$y = \ln(3)$

$\boxed{0}$

ln(3) is a constant

$y = \ln(x + 3)^{2}$

$2(\ln(x + 3))^{1} \cdot \frac{1}{(x + 3)}$

$\boxed{\frac{2\ln(x + 3)}{(x + 3)}}$

$y = \ln((x + 3)^{2})$

$\frac{2(x + 3)}{(x + 3)^{2}}$

$\boxed{\frac{2}{x + 3}}$

$y = \frac{2}{e^{3x}}$

$2 \cdot e^{-3x}$

$\boxed{-6e^{-3x} \text{ or } \frac{-6}{e^{3x}}}$

2) What is the equation of the line tangent to $y = e^{2x - 3}$ at the point $(\frac{3}{2}, 1)$?

Optional: graph your result

To find the equation of a line, we need a point and the slope.

Point: (3/2, 1)

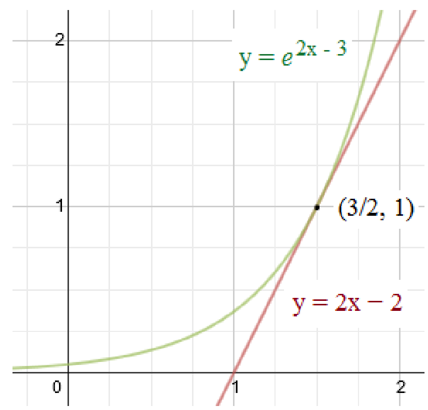
Slope: rate of change at x = 3/2

$y' = 2 \cdot e^{2x - 3}$

at (3/2, 1)

$y' = 2 \cdot e^{0} = 2$

$\boxed{\text{tangent line: } y - 1 = 2(x - \frac{3}{2})}$



3) What is the equation of the normal to $y = \ln(x - 2) + 4$ at the point (3, 4) ?

Optional: graph your result

The normal is perpendicular to the tangent line

Point: (3, 4)

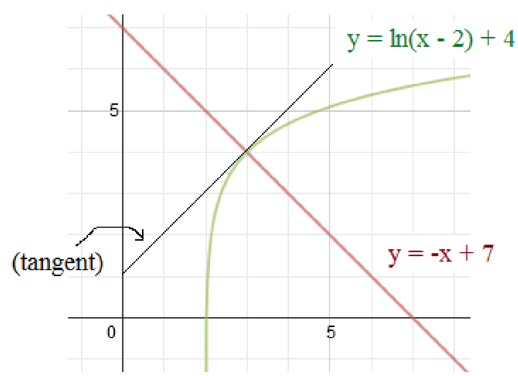
Find $\frac{dy}{dx}$ to get instantaneous rate of change (i.e. slope)

$\boxed{\text{normal line: } y - 4 = -1(x - 3)}$

$$\frac{dy}{dx} = \frac{1}{x - 2} + 0$$

slope at (3, 4) is 1 (tangent slope)

therefore, opposite reciprocal is -1 (normal slope)



4) $\int e^{2x}\, dx$

$\frac{1}{2}\int 2e^{2x}\, dx$

$\boxed{\frac{1}{2}e^{2x} + C}$

$\int \frac{3x}{3x^2+2}\, dx$

$\frac{1}{2}\int \frac{2 \cdot 3x}{3x^2+2}\, dx$

$\boxed{\frac{1}{2}\ln(3x^2+2) + C}$

$\int \frac{2}{3x+3}\, dx$

$\int \frac{2}{3(x+1)}\, dx$

$\frac{2}{3}\int \frac{1}{(x+1)}\, dx = \boxed{\frac{2}{3}\ln(x+1) + C}$

or $\ln(x+1)^{\frac{2}{3}} + C$

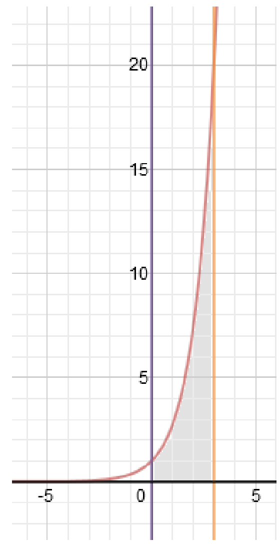
5) What is the area of the region above the x-axis that is bounded by the y-axis, x = 3, and e^x ?

A quick sketch will show the enclosed region (and its boundaries)
The endpoints of the integral will be x = 0 and x = 3

and, the upper boundary will be $y = e^x$
and the lower boundary will be y = 0

$$\int_0^3 e^x\, dx = e^x\Big|_0^3 = e^3 - e^0 \approx \boxed{19.09}$$

$$\boxed{\int_0^3 e^x\, dx - \int_0^3 0\, dx}$$



6) What is the area of the region bounded by $y = \ln(x) + 2$, y = 2, and x = 5 ?
(Use Calculator)

First, draw a sketch

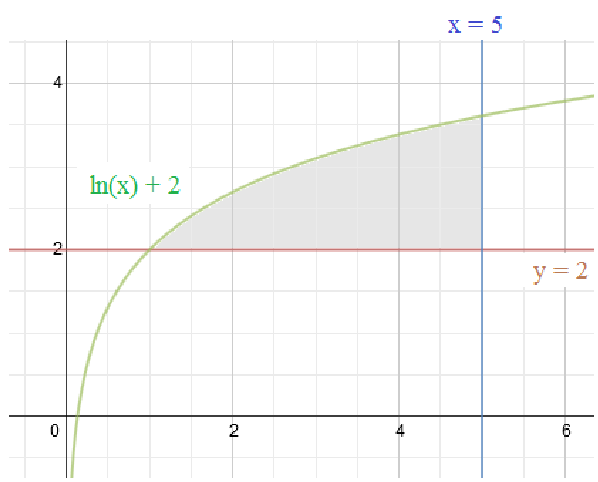
note: a region with boundaries y = ln(x), the x-axis, and x = 5 would have the same area. (a downward shift of 2)

Second, establish boundaries of integral
(i.e. interval of the *integrand*)

The right boundary is x = 5
The left boundary is x = 1
because

$y = \ln(x) + 2$
$y = 2$

find intersection:
(set equations equal)
$2 = \ln(x) + 2$
$0 = \ln(x)$
$x = 1$



Then, evaluate the definite integral

$$\underbrace{\int_1^5 \ln(x) + 2\, dx}_{\text{upper boundary}} - \underbrace{\int_1^5 2\, dx}_{\text{lower boundary}} = \int_1^5 \ln(x)\, dx = x\ln(x) - x\Big|_1^5 = 5\ln(5) - 5 - (0 - 1)$$

$\boxed{\approx 4.047}$

7) Find the equation of the line that is tangent to $f(x) = 3x^2 - \ln x$ at (1, 3)

(Optional: Use a graphing calculator to confirm your answer)

Point: (1, 3)

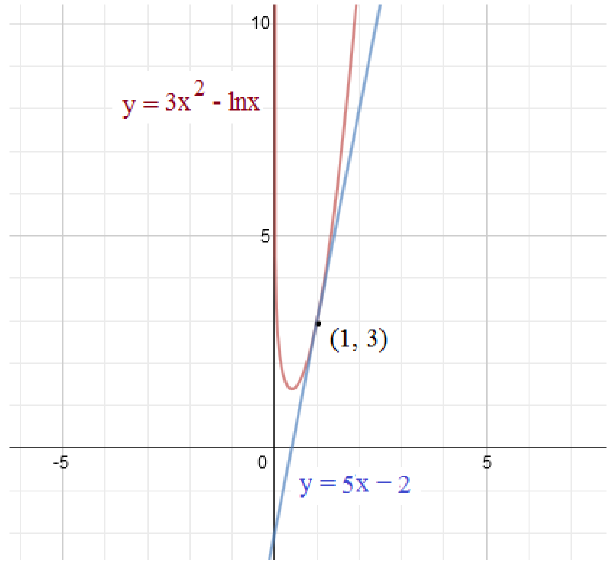
Slope: find instantaneous rate of change (derivative)

$$f'(x) = 6x - \frac{1}{x}$$

then, slope at (1, 3) is $f'(1) = 5$

Equation of tangent line:

$y - 3 = 5(x - 1)$ or $y = 5x - 2$

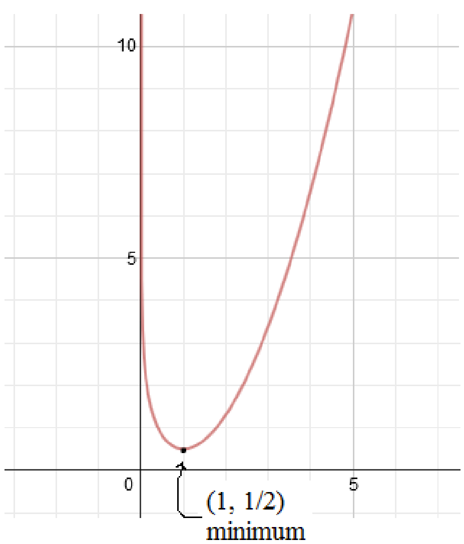


8) $y = \dfrac{x^2}{2} - \ln x$

What are the extrema?

Points of inflection?

(Optional: Use a graphing calculator to check your answers)



To find extrema (max. or min), set first derivative equal to zero.

$y' = x - \dfrac{1}{x}$ $\qquad x - \dfrac{1}{x} = 0$ multiply both sides by x

$x^2 - 1 = 0$ factor

$(x + 1)(x - 1) = 0$

$x = \cancel{-1}$ and 1 $\qquad$ since ln(-1) does not exist, -1 is extraneous!

at x = 0, derivative is < 0 (decreasing)
at x = 2, derivative is > 0 (increasing)

x = 1 is a minimum

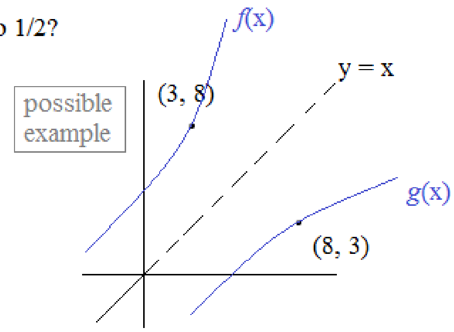
$y'' = 1 + \dfrac{1}{x^2}$ $\qquad 1 + \dfrac{1}{x^2} = 0$

$\dfrac{1}{x^2} = -1$

No solution, so there is no point of inflection!

III. Inverses and derivatives

1) $f(x)$ and $g(x)$ are one to one inverses.
If the slope of $f(x)$ at (3, 8) is 2, where is the slope of $g(x)$ equal to 1/2?



Since $f(x)$ and $g(x)$ are inverses, they reflect over $y = x$.

the rate of change (slopes) of reflection points will be reciprocals...

the reflection point of (3, 8) is $\boxed{(8, 3)}$

2) $f(x) = x^3 - x - 6$

What is $f^{-1\prime}(0)$?

We need to find two parts: 1) the value of $f^{-1}(0)$ If $x^3 - x - 6 = 0$

$x = 2$

2) $f'(x)$ $f' = 3x^2 - 1$

$$\boxed{f^{-1\prime}(a) = \frac{1}{f'(f^{-1}(a))}}$$

$$\frac{1}{3(2)^2 - 1} = \boxed{\frac{1}{11}}$$

3) $h(x) = \ln(x) + 4$

a) What is the inverse of $h(x)$?

$y = \ln(x) + 4$ "switch x and y"

$x = \ln(y) + 4$

$\ln(y) = x - 4$ "solve for y" $\boxed{h^{-1}(x) = e^{x-4}}$

$\log_e(y) = (x - 4)$

$y = e^{x-4}$

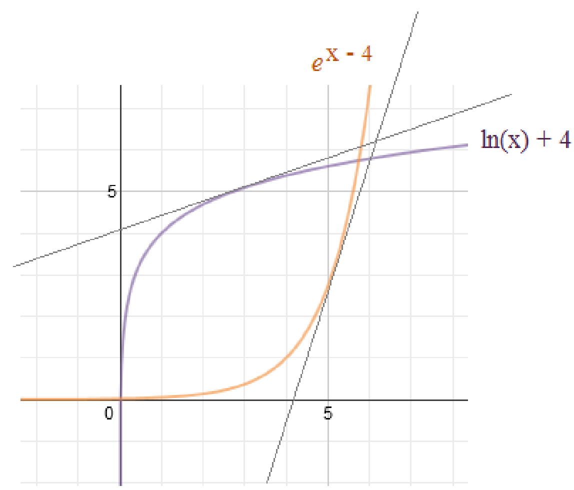
b) $h(3) = \ln(3) + 4$ ($\approx$ 5.1) What is the slope at $h(3)$?

$h'(x) = \frac{1}{x} + 0$ $h'(3) = \boxed{\frac{1}{3}}$

$h^{-1}(5.1) \approx 3$ What is the slope at $h^{-1}(5.1)$?

$h^{-1\prime}(x) = e^{x-4}$ $h^{-1\prime}(5.1) = e^{5.1-4} = e^{1.1} = \boxed{3}$

c) Graph $h(x)$ and $h^{-1}(x)$



Sketch the tangent lines at (3, 5.1) and (5.1, 3)

What are the equations of the tangent lines?

$y - 5.1 = \frac{1}{3}(x - 3)$ and $y - 3 = 3(x - 5.1)$

$\boxed{y = \frac{1}{3}x + 4.1 \qquad y = 3x - 12.3}$

IV. Exponential Functions

SOLUTIONS

Find the first derivatives:

1) $g(x) = 2^{x+3}$

using logarithmic differentiation:

$y = 2^{x+3}$

$\ln y = \ln(2^{x+3})$

$\ln y = (x+3)\ln 2$

$\frac{1}{y} \cdot y' = (1+0)\ln 2 + 0(x+3)$

$y' = y\ln 2$

$\boxed{y' = 2^{x+3} \cdot \ln 2}$

using the definition/formula:

$u = x + 3$

$\frac{du}{dx} = 1 \qquad \frac{d}{dx}(2^{x+3}) = (1)(2^{x+3})\ln 2$

$a = 2$

$$\boxed{\frac{d}{dx}(a^u) = \frac{du}{dx}(a^u)\ln a}$$

2) $f(x) = x^2 e^x$

product rule:

$f'(x) = \underbrace{2x}_{u'}\underbrace{(e^x)}_{v} + \underbrace{e^x}_{v'}\underbrace{(x^2)}_{u}$

$\boxed{= xe^x(x+2)}$

3) $g(t) = t^2 2^t$

$g'(t) = 2t \cdot 2^t + 2^t(\ln 2) \cdot t^2$

$\boxed{= 2^t\left((\ln 2)t^2 + 2t\right)}$

4) What is the equation of the line tangent to $y = 2^{-x}$ at $(0, 1)$?

(optional: sketch a graph containing the function and tangent line)

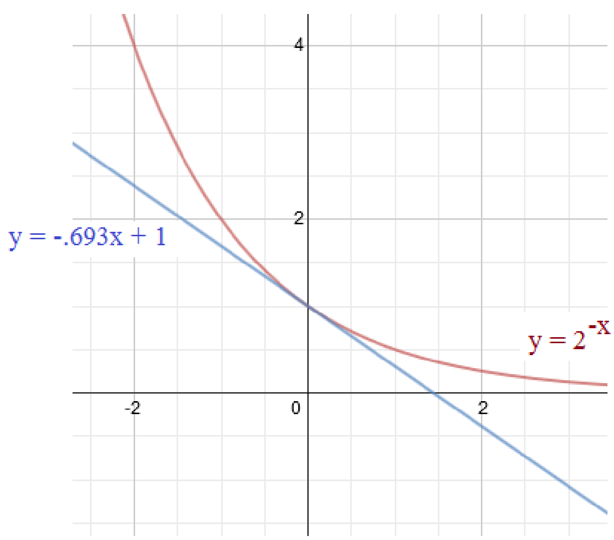
To find the equation of a line, we need the

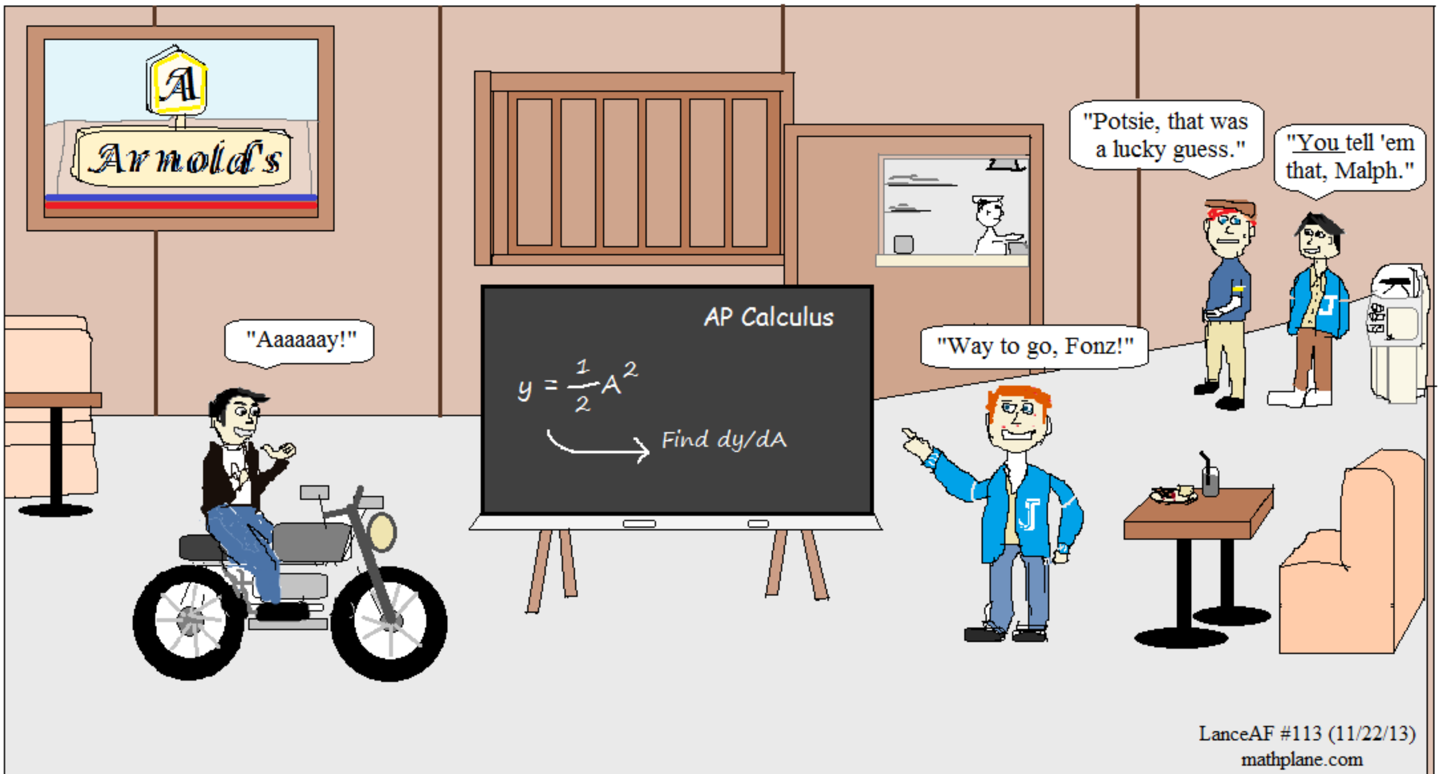
point: $(0, 1)$

slope: $y' = (-1)(2^{-x})(\ln 2)$

at $x = 0$, the slope is $-(\ln 2) \approx -.693$

$\boxed{\text{tangent line: } y = -.693x + 1}$





Happy Days

Despite Richie's help, Fonzie dropped out of Calculus.
(... although he did have some success with velocity and acceleration!)

Implicit Differentiation and Logarithm extras---→

Implicit differentiation and natural log

Example: Find $\frac{dy}{dx}$: $x^2 + 3\ln y + y^2 = 10$

$$2x + 3 \cdot \frac{1}{y}\,\frac{dy}{dx} + 2y\,\frac{dy}{dx} = 0$$

$$\frac{3}{y}\,\frac{dy}{dx} + 2y\,\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx}\left(\frac{3}{y} + 2y\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{\dfrac{3+2y^2}{y}} = \boxed{\frac{-2xy}{3+2y^2}}$$

Example: Find the equation of the line tangent to $x + y - 1 = \ln(x^2 + y^2)$ at $(1, 0)$

To find equation of a line, we need slope and a point.

Point: $(1, 0)$

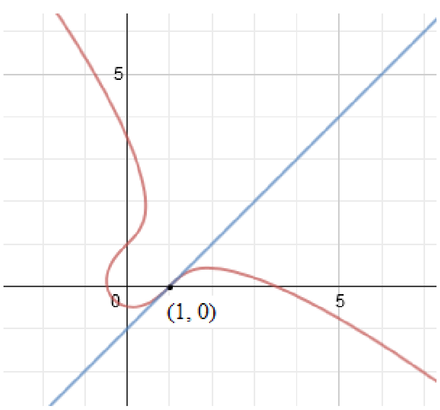
Slope: Take the derivative and evaluate the point of tangency

Implicit differentiation: $$1 + (1)\frac{dy}{dx} - 0 = \frac{2x + (2y)\frac{dy}{dx}}{(x^2 + y^2)}$$ cross-multiply

$$(x^2 + y^2) + (x^2 + y^2)\frac{dy}{dx} = 2x + (2y)\frac{dy}{dx}$$ collect dy/dx to one side

$$(x^2 + y^2) - 2x = (2y)\frac{dy}{dx} - (x^2 + y^2)\frac{dy}{dx}$$ factor out the dy/dx

$$\frac{(x^2 + y^2) - 2x}{(2y) - (x^2 + y^2)} = \frac{dy}{dx}$$



To find slope at point of tangency, substitute $(1, 0)$ for (x, y)

$$\frac{(1+0) - 2}{0 - (1+0)} = 1$$

Equation of the tangent line: $y - 0 = 1(x - 1)$

or

$$\boxed{y = x - 1}$$

Derivatives of logarithms (other than e)

Example: $f(x) = 5^x$ find $f'(x)$

$$\frac{d}{dx}(a^x) = (a^x)\ln a$$

(the base a is a constant)

using the definition:

$$5^x \cdot \ln 5$$

using logarithmic differentiation:

$y = 5^x$

$\ln y = \ln 5^x$ log of both sides

$\ln y = x \cdot \ln 5$ logarithm power rule

$\frac{1}{y} \cdot y' = (1)\ln 5 + 0(x)$ derivative

$y' = y\ln 5$

$y' = 5^x \cdot \ln 5$ substitution

Example: $y = 3^{x^2}$ find $\frac{dy}{dx}$

$$\frac{d}{dx}(a^u) = \frac{du}{dx}(a^u)\ln a$$

(the base a is a constant and u is a function)

using the definition:

$u = x^2$

$a = 3$

$\frac{du}{dx} = 2x$

$$\frac{dy}{dx}3^{x^2} = (2x) \cdot 3^{x^2} \cdot \ln 3$$

using logarithmic differentiation:

$y = 3^{x^2}$

$\ln y = \ln 3^{x^2}$

$\ln y = x^2 \ln 3$

$\ln y = 1.098x^2$

$\frac{1}{y} \cdot y' = 2.196x$ $\frac{1}{y} \cdot y' = 2x(\ln 3)$

$y' = 2.196xy$ $y' = 2x(\ln 3)y$

$y' = 2.196x\,(3^{x^2})$ $y' = 2x \cdot \ln 3 \cdot 3^{x^2}$

Example: What is the derivative of e^x ?

$y = e^x$ $y' = e^x \cdot \ln(e)$

$y' = e^x \cdot 1 = e^x$

Comparing Logarithmic Differentiation

Example: $y = (2x+3)^2\,(x^2+1)$

$$\ln y = \ln\left[(2x+3)^2\,(x^2+1)\right]$$

$$\ln y = \ln(2x+3)^2 + \ln(x^2+1)$$

$$\ln y = 2\ln(2x+3) + \ln(x^2+1)$$

$$\frac{1}{y}\,\frac{dy}{dx} = 2\,\frac{2}{(2x+3)} + \frac{2x}{(x^2+1)}$$

$$\frac{1}{y}\,\frac{dy}{dx} = \frac{4}{(2x+3)} + \frac{2x}{(x^2+1)}$$

$$\frac{dy}{dx} = \left[\frac{4}{(2x+3)} + \frac{2x}{(x^2+1)}\right]y$$

$$\frac{dy}{dx} = \left[\frac{4}{(2x+3)} + \frac{2x}{(x^2+1)}\right]\left[(2x+3)^2\,(x^2+1)\right]$$

$$\overset{f}{(2x+3)^2}\;\overset{g}{(x^2+1)}$$

$$\underbrace{2(2x+3)^1\cdot 2}_{f'}\cdot\underbrace{(x^2+1)}_{g} + \underbrace{2x}_{g'}\cdot\underbrace{(2x+3)^2}_{f}$$

$$4(2x+3)(x^2+1) + 2x(2x+3)^2$$

The left uses logarithms and implicit differentiation...

The right uses power rule, product rule, and chain rule...

Example:

$$y = \sqrt[4]{\frac{(x-2)^3\,(x^2+1)}{(2x+5)^3}}$$

$$\ln y = \ln\left[\frac{(x-2)^3\,(x^2+1)}{(2x+5)^3}\right]^{\frac{1}{4}}$$

$$\ln y = \frac{1}{4}\ln\left[\frac{(x-2)^3\,(x^2+1)}{(2x+5)^3}\right]$$

$$\ln y = \frac{1}{4}\left[\ln(x-2)^3 + \ln(x^2+1) + \ln(2x+5)^3\right]$$

$$\ln y = \frac{1}{4}\left[3\ln(x-2) + \ln(x^2+1) + 3\ln(2x+5)\right]$$

$$\frac{1}{y}\,\frac{dy}{dx} = \frac{1}{4}\left[\frac{3}{(x-2)} + \frac{2x}{(x^2+1)} + \frac{3\cdot 2}{(2x+5)}\right]$$

$$\frac{dy}{dx} = \frac{1}{4}\left[\frac{3}{(x-2)} + \frac{2x}{(x^2+1)} + \frac{3\cdot 2}{(2x+5)}\right]y$$

$$\frac{dy}{dx} = \frac{1}{4}\left[\frac{3}{(x-2)} + \frac{2x}{(x^2+1)} + \frac{3\cdot 2}{(2x+5)}\right]\left[\frac{(x-2)^3\,(x^2+1)}{(2x+5)^3}\right]^{\frac{1}{4}}$$

Calculus: Logarithm Extras

$$\frac{d}{dx}(a^x) = (a^x)\ln a$$

(the base a is a constant)

Derivative of an exponential function (other than e^x)

Example: Find the equation of the line tangent to $y = 4^x$ at (1, 4)

To determine the equation of a line,
we need a point and the slope.

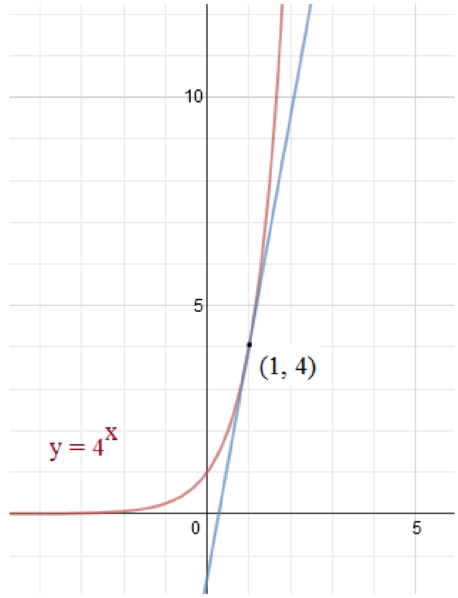
Point: (1, 4)

Slope: Find the derivative

$$y' = 4^x \cdot \ln(4)$$

and, at $x = 1$ the slope is $4^{(1)} \cdot \ln(4) \approx 5.545$

$$\boxed{y - 4 = 5.545(x - 1)}$$



Logarithmic Differentiation: Using logarithms and implicit differentiation to find a derivative.

Example: $y = x^{\sin x}$ Find $\frac{dy}{dx}$

Since there is a variable in the term AND the exponent, it cannot be directly differentiated.

But, we can take the natural log of both sides...

$$\ln(y) = \ln(x^{\sin x})$$

Logarithm properties: power rule $\quad \ln(y) = \sin x \cdot \ln(x)$

Implicit differentiation $\quad \frac{1}{y}\frac{dy}{dx} = \cos x \cdot \ln(x) + \sin x \cdot \frac{1}{x}$

multiply both sides by y
(to isolate the dy/dx) $\quad \frac{dy}{dx} = y\left(\cos x(\ln(x)) + \frac{\sin x}{x}\right)$

substitute the y with the original terms $\quad \boxed{\frac{dy}{dx} = x^{\sin x}\left(\cos x(\ln(x)) + \frac{\sin x}{x}\right)}$

Using partial fractions to find the integral

Example: $\int \frac{8x+5}{x^2+3x-10}\,dx$

$(x+5)(x-2)$

$$\frac{A}{(x+5)} + \frac{B}{(x-2)} = \frac{8x+5}{x^2+3x-10}$$

$$\frac{A(x-2)}{(x+5)(x-2)} + \frac{B(x+5)}{(x+5)(x-2)} = \frac{8x+5}{x^2+3x-10}$$

$$A(x-2) + B(x+5) = 8x+5$$

$$Ax - 2A + Bx + 5B = 8x + 5$$

$$(A+B)(x) - 2A + 5B = 8x + 5$$

Then, we know $A + B = 8$

$-2A + 5B = 5$

$$\begin{array}{r} 2A + 2B = 16 \\ -2A + 5B = 5 \\ \hline 7B = 21 \\ B = 3 \end{array}$$

then, $A = 5$

$A = 5$
$B = 3$

$$\boxed{\frac{5}{(x+5)} + \frac{3}{(x-2)}}$$

$$\int \frac{8x+5}{x^2+3x-10}\,dx = \int \frac{5}{(x+5)}\,dx + \int \frac{3}{(x-2)}\,dx$$

$$\boxed{5\ln|x+5| + 3\ln|x-2| + C}$$

Example: $\int \frac{-6x^2+3x+5}{x^3-x}\,dx$

$x(x^2-1) = x(x+1)(x-1)$

$$\frac{-6x^2+3x+5}{x^3-x} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)}$$

$$= \frac{(x+1)(x-1)A}{x(x+1)(x-1)} + \frac{x(x-1)B}{x(x+1)(x-1)} + \frac{x(x+1)C}{x(x+1)(x-1)}$$

$$-6x^2+3x+5 = (x+1)(x-1)A + x(x-1)B + x(x+1)C$$

$$Ax^2 - 1A + Bx^2 - Bx + Cx^2 + Cx$$

(regroup the terms)

$$(A+B+C)x^2 + (-B+C)x + A(-1)$$

$$-6x^2 + 3x + 5$$

"Express method"

Let x = 0 (to eliminate B and C)

$0 + 0 + 5 = -1A + 0B + 0C$

$A = -5$

Let x = -1 (to eliminate A and C)

$-6 + (-3) + 5 = 0A + 2B + 0C$

$B = -2$

Let x = 1 (to eliminate A and B)

$-6 + 3 + 5 = 0A + 0B + 2C$

$C = 1$

$A + B + C = -6$

$-B + C = 3$

$-A = 5$

$A = -5$

$$\begin{array}{r} -5 + B + C = -6 \\ -B + C = 3 \\ \hline 2C = 2 \\ C = 1 \end{array}$$

$-B + C = 3$

$-B + 1 = 3$

$B = -2$

$$\frac{-6x^2+3x+5}{x^3-x} = \boxed{\frac{-5}{x} + \frac{-2}{(x+1)} + \frac{1}{(x-1)}}$$

$$\int \frac{-5}{x} + \frac{-2}{(x+1)} + \frac{1}{(x-1)}\,dx$$

$$\boxed{-5\ln|x| + -2\ln|x+1| + \ln|x-1| + C}$$

Find the derivative (using logrithmic differentiation):

1) $y = \sqrt{\dfrac{x^2 + 1}{x^2 + 2}}$

2) $y = \dfrac{x\sqrt{x^2 + 4}}{x + 1}$

Find the derivative (using logrithmic differentiation):

1) $y = \sqrt{\dfrac{x^2+1}{x^2+2}}$

(take the natural log of each side; then, use log properties to rewrite)

$$\ln y = \ln\left(\frac{x^2+1}{x^2+2}\right)^{1/2} \Rightarrow \ln y = \frac{1}{2}\left[\ln(x^2+1) - \ln(x^2+2)\right]$$

(take derivative; use implicit differentiation)

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{2x}{(x^2+1)} - \frac{2x}{(x^2+2)}\right]$$

(simplify and substitue for y)

$$\frac{dy}{dx} = \left[\frac{x}{(x^2+1)} - \frac{x}{(x^2+2)}\right] y$$

$$\frac{dy}{dx} = \left[\frac{x}{(x^2+1)} - \frac{x}{(x^2+2)}\right]\sqrt{\frac{x^2+1}{x^2+2}}$$

2) $y = \dfrac{x\sqrt{x^2+4}}{x+1}$

(take the natural log of each side; then, use log properties to rewrite)

$$\ln y = \ln x + \ln(x^2+4)^{\frac{1}{2}} - \ln(x+1)$$

$$\ln y = \ln x + \frac{1}{2}\ln(x^2+4) - \ln(x+1)$$

(take derivative; use implicit differentiation)

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\frac{2x}{(x^2+4)} - \frac{1}{x+1}$$

$$\frac{dy}{dx} = \left[\frac{1}{x} + \frac{x}{(x^2+4)} - \frac{1}{x+1}\right] y$$

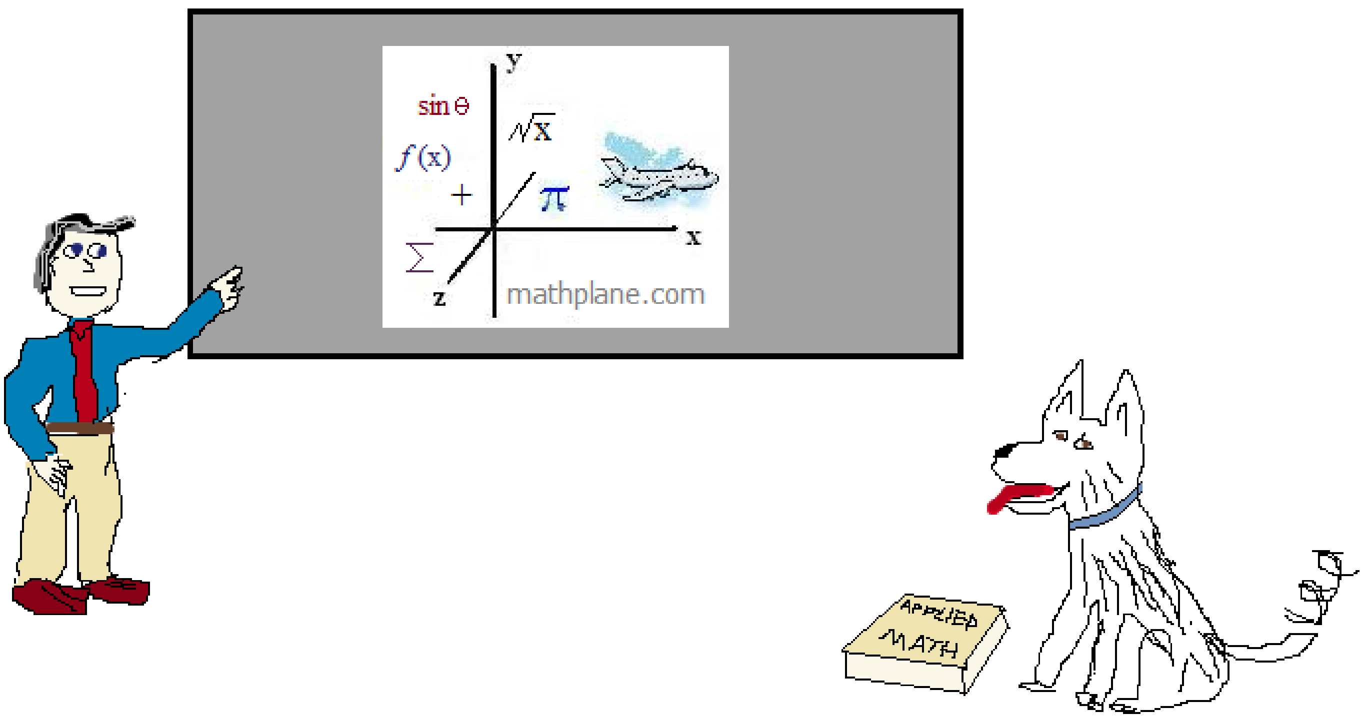
(simplify and substitue for y)

$$\frac{dy}{dx} = \left[\frac{1}{x} + \frac{x}{(x^2+4)} - \frac{1}{x+1}\right]\frac{x\sqrt{x^2+4}}{x+1}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy



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