Geometry

Parallel Lines Cut by Transversals

(Definitions, Examples, Applications & Proofs)

Includes Notes, Practice Quiz, and Solutions

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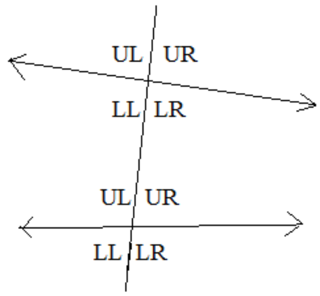
Introduction: Two lines cut by a transversal

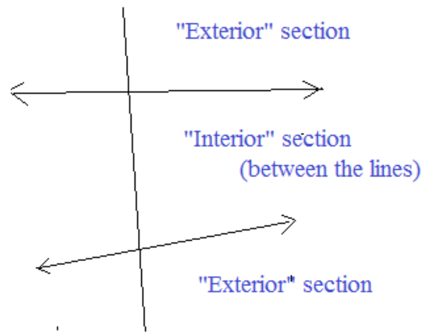
"Corresponding angles":
Angles in the same relative position



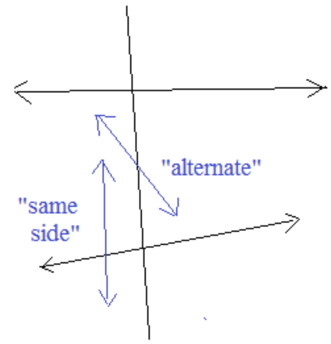
Example:

UL upper left angle (top) corresponds to
UL upper left angle (bottom)

"Interior angles"
Angles between the two lines



"Alternate angles":
Angles on opposite sides of the transversal



Parallel Lines Cut by a Transversal

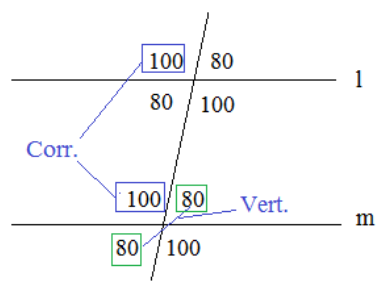
If two parallel lines are cut by a transversal, then....

Examples:

l ∥ m

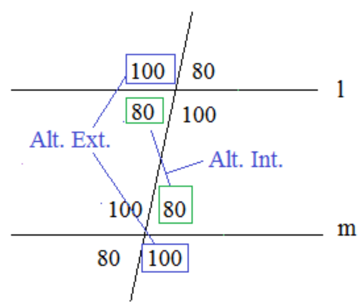
Corresponding Angles Congruent

Vertical Angles Congruent



Alternate Interior Angles Congruent

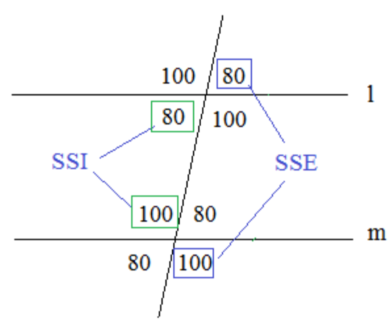
Alternate Exterior Angles Congruent



Note: The converse of each theorem is true.
EX: If alternate interior angles are congruent, then the lines are parallel

Same Side Interior Angles Supplementary

Same Side Exterior Angles Supplementary



Examples: In the diagram A || B and C || D.

Identify the pairs of corresponding angles that include ∠1

1 & 3 are corresponding angles
1 & 9 are corresponding angles

Which pair of alternate interior angles include ∠3?

6 and 3 are alternate interior angles

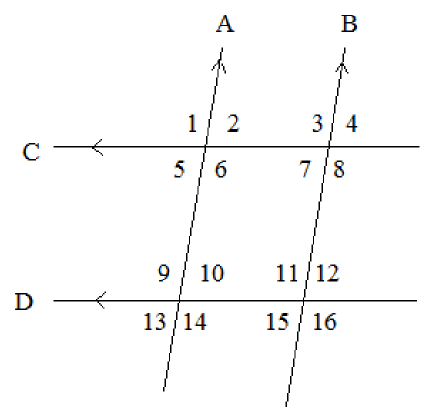
Which pairs of alternate interior angles include ∠10?

10 & 15 are alternate interior angles
10 & 5 are alternate interior angles

List all the angles that are congruent to ∠13

13 & 10 vertical angles
13 & 15 corresponding angles
13 & 5 corresponding angles
13 & 2 alternate exterior angles
13 & 12 alternate exterior angles

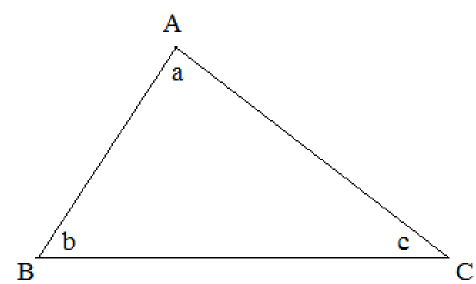
also, 7 and 4 are congruent to 13
(transitive property)



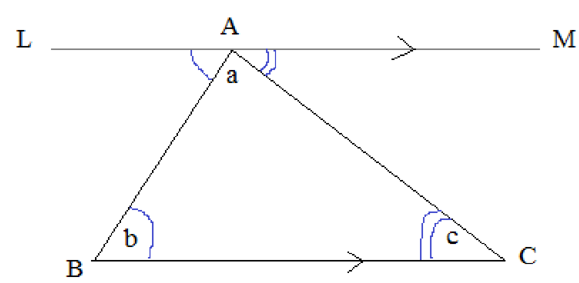
Proving the sum of interior angles of a triangle is 180 degrees (using Parallel Lines Cut by a Transversal)

Given: △ABC

Prove: m∠a + m∠b + m∠c = 180



Statements	Reasons
1. Triangle ABC	1. Given
2. Auxiliary $\overline{LM}$ is line parallel to BC through A	2. (parallel postulate) A line can be drawn through a point parallel to a given line.
3. ∠LAB + ∠a + ∠MAC = 180°	3. (angle addition postulate &) measure of a straight angle is 180 degrees
4. ∠b = ∠LAB ∠c = ∠MAC	4. if parallel lines cut by a transversal, alternate interior angles congruent
5. ∠a + ∠b + ∠c = 180°	5. substitution

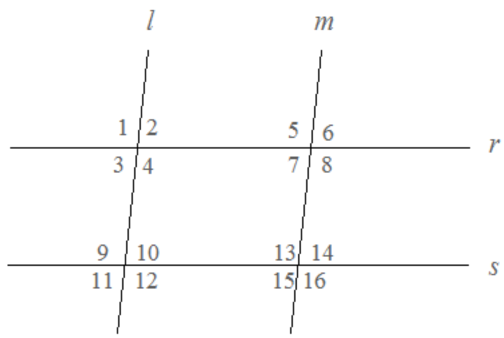


Using Parallel Lines Cut by Transversals: Theorems and Converses

Example: Given: $l \parallel m$ and $r \parallel s$

Prove: $1 \cong 16$

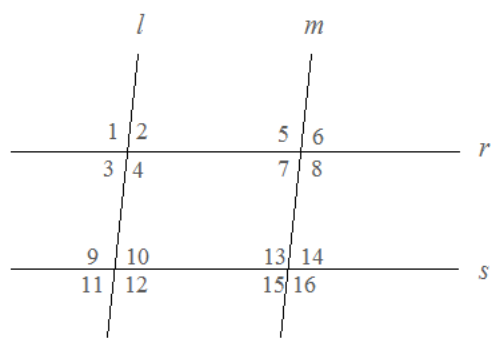
Statements	Reasons
1) $l \parallel m$	1) Given
2) $\angle 1 \cong \angle 5$	2) If parallel lines cut by transversal, then corresponding angles congruent
3) $\angle 5 \cong \angle 8$	3) Vertical angles are congruent
4) $\angle 1 \cong \angle 8$	4) Transitive property
5) $r \parallel s$	5) Given
6) $\angle 8 \cong \angle 16$	6) Corresponding Angles
7) $\angle 1 \cong \angle 16$	7) Transitive property



Example: Given: $l \parallel m$ and $r \parallel s$

Prove: Angles 9 and 6 are supplementary

Statements	Reasons
1) $r \parallel s$	1) Given
2) $\angle 9 \cong \angle 4$	2) If parallel lines cut by transversal then alternate interior angles congruent
3) Angles 4 and 2 are supplementary	3) Definition of Supplementary angles
4) $\angle 9$ supp. to $\angle 2$	4) Substitution property
5) $l \parallel m$	5) Given
6) $\angle 2 \cong \angle 6$	6) If parallel lines cut by transversal, then corresponding angles congruent
7) Angles 9 and 6 are supplementary	7) Substitution property

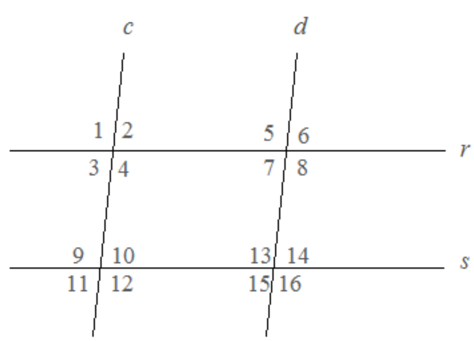


Example: Given: $r \parallel s$

$\angle 7 = \angle 10$

Prove: $c \parallel d$

Statements	Reasons
1) $r \parallel s$	1) Given
2) $\angle 2 = \angle 10$	2) If parallel lines cut by transversal, then corresponding angles congruent
3) $\angle 7 = \angle 10$	3) Given
4) $\angle 2 = \angle 7$	4) Transitive property (or Substitution)
5) $c \parallel d$	5) (converse of alt. interior angles) If 2 lines cut by a transversal form congruent alternate interior angles, then the 2 lines are parallel



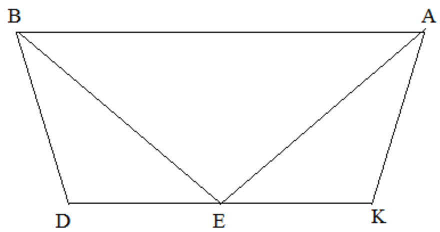
NOTE: Although c and d look parallel, their angles cannot be considered congruent/supplementary UNTIL they are proven to be parallel!!

For example, angles 1 and 5 are not considered congruent UNTIL *c* and *d* are proven parallel...

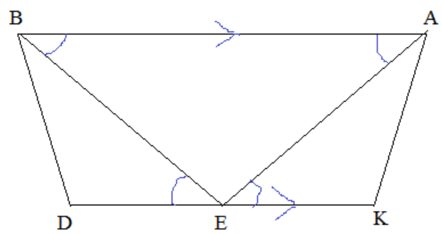
Example: Given: $\overline{AB} \parallel \overline{EC}$

$\angle AEK = \angle BED$

Prove: $\triangle ABE$ is isosceles



Recognizing the alternate interior angles...

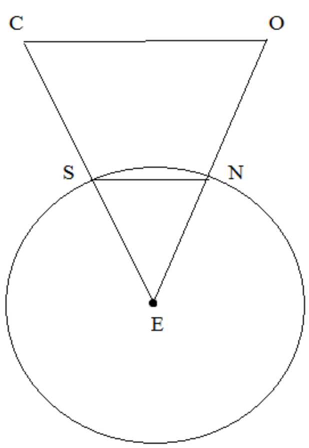


Statements	Reasons
1) $\overline{AB} \parallel \overline{EC}$	1) Given
2) $\angle AEK = \angle BED$	2) Given
3) $\angle BED = \angle EBA$ $\angle AEK = \angle BAE$	3) If parallel lines cut by transversal, then alternate angles are congruent
4) $\angle EBA = \angle BAE$	4) Transitive property
5) $\triangle ABE$ is isosceles	5) If base angles are congruent, then triangle is isosceles

Example: Given: Circle E

$\triangle COE$ is scalene

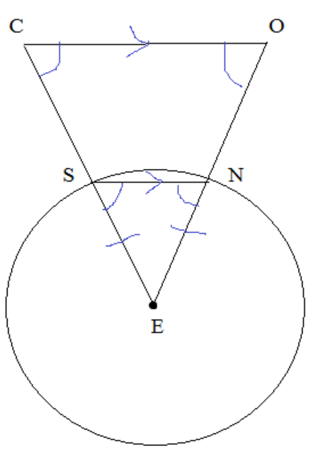
Prove: $\angle C \neq \angle ESN$



C = ESN or C $\neq$ ESN

Statements	Reasons
1) Circle E	1) Given
2) $\triangle COE$ is scalene	2) Given
3) $\angle C = \angle ESN$	3) Assume for Contradiction
4) $\overline{ES} = \overline{EN}$	4) All radii are congruent
5) $\overline{CO} \parallel \overline{SN}$	5) If corresponding angles are congruent, then lines are parallel
6) $\angle O = \angle ENS$	6) If lines are parallel, then corresponding angles are congruent
7) $\angle ESN = \angle ENS$	7) If congruent sides, then congruent angles
8) $\angle O = \angle C$	8) Transitive property
9) $\triangle COE$ is isosceles	9) If base angles are congruent, then triangle is isosceles

However, 2) and 9) contradict each other



Recognizing the corresponding angles...

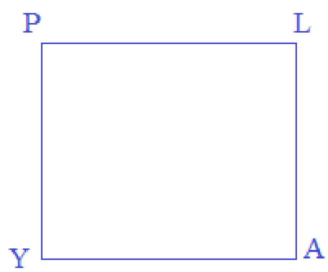
Geometry Applications:

EXAMPLE: If quadrilateral PLAY has angles

P: 59 degrees
L: 37 degrees
A: 143 degrees
Y: 121 degrees

Which sides are parallel? Sketch the figure.

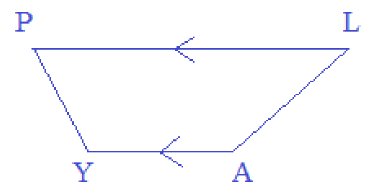
Since the quadrilateral is PLAY,
the figure will have consecutive vertices
P - L - A - Y



If parallel lines are cut by a transversal,
then *same side interior angles are supplementary.*

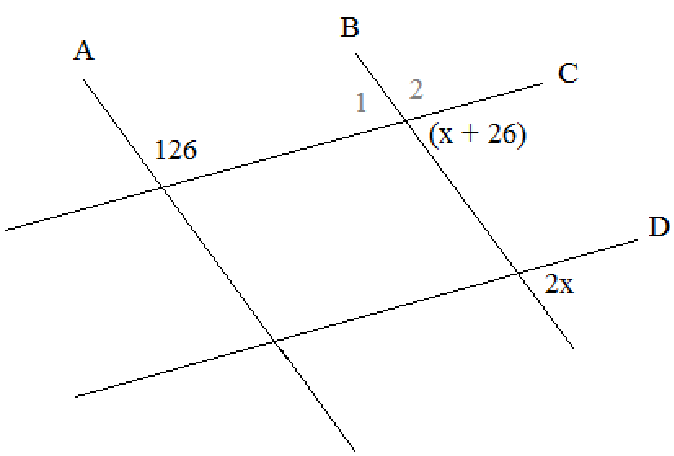
Since $\angle L$ and $\angle A$ are supplementary
and $\angle P$ and $\angle Y$ are supplementary

$\overline{PL} \parallel \overline{YA}$



Trapezoid

EXAMPLE: If C and D are parallel, are A and B parallel?



Since C || D, then

$(x + 26) = 2x$ corresponding angles

$x = 26$

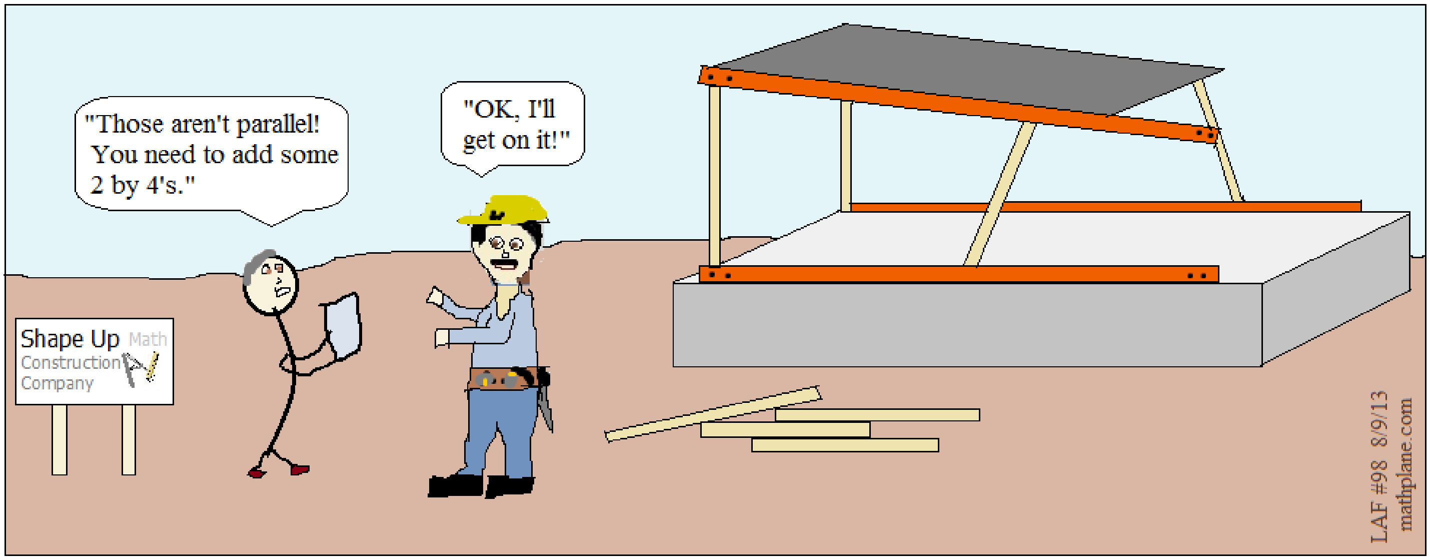
Because $x = 26$,

$x + 26 = 52$...

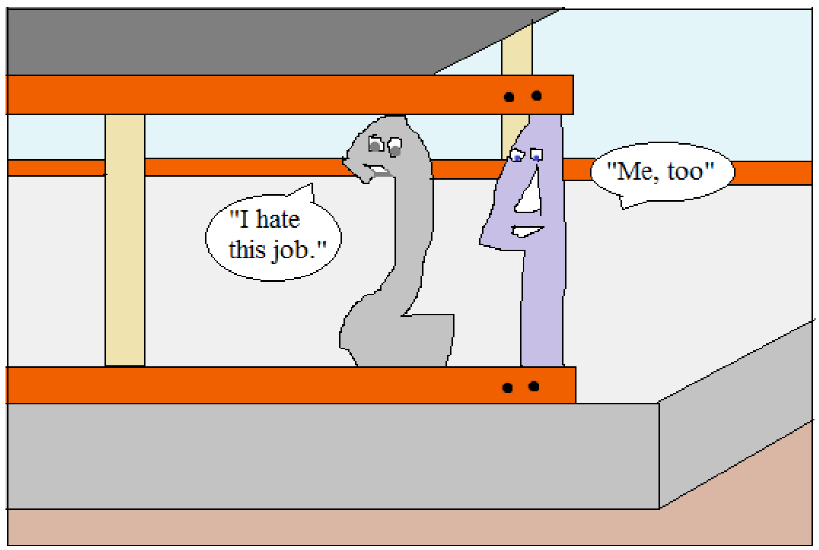
so, $\angle 1 = 52$ vertical angles

$126 + 52 = 178$

If same side interior angles $\neq 180$,
then lines A and B are NOT parallel



The Math Guy misunderstood
the Architect's suggestion...



Building
Materials

Practice Exercises -→

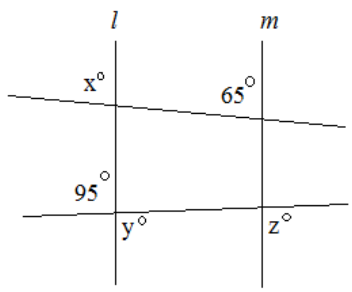
Parallel Lines Cut by Transversals

I. Determine the following:

1) x =

y =

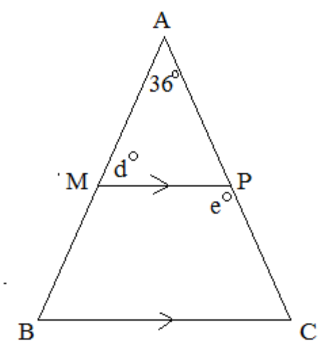
z =



$l \parallel m$

2) d =

e =

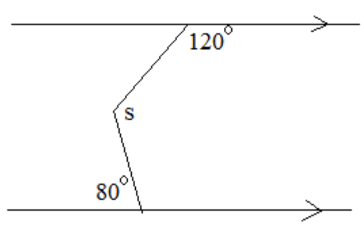


△ABC is Isosceles triangle

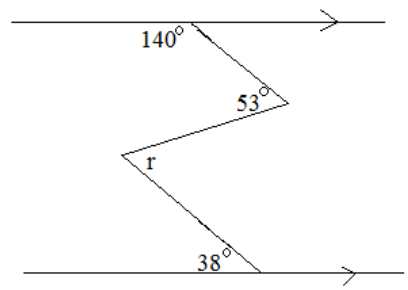
$\overline{AC} \cong \overline{AB}$

MP ∥ BC

3) m∠s =



4) m∠r =

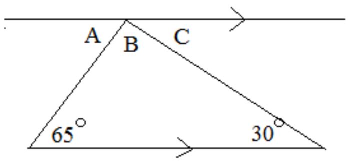


Parallel lines cut by Transversals

5) $\angle A =$

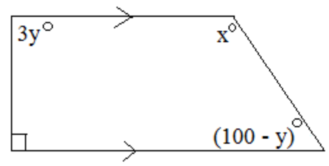
$\angle B =$

$\angle C =$



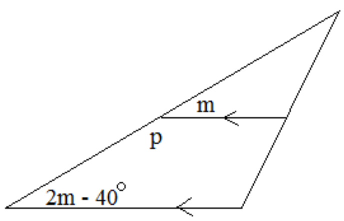
6) x =

y =



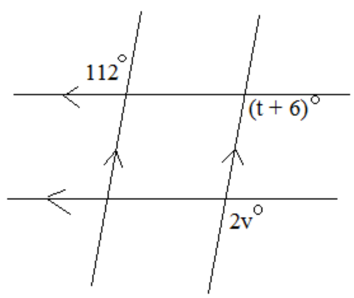
7) m =

p =



8) t =

v =



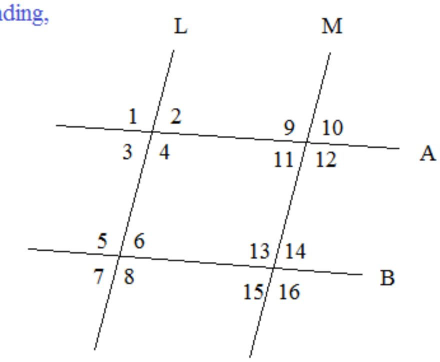
9) Use information to determine which lines (if any) must be parallel:

a) $1 \cong 9$ since 1 and 9 are corresponding, L ∥ M

b) $4 \cong 8$

c) $2 \cong 3$

d) $10 \cong 7$



e) $6 \cong 15$

f) $9 \cong 16$

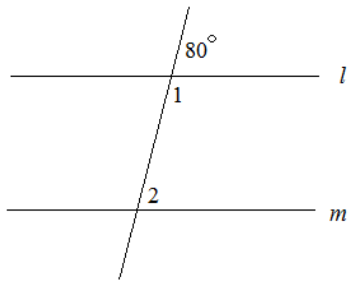
g) $5 \cong 9$

h) $13 \cong 14$

Parallel lines cut by Transversals

10) Answer and identify the relevant theorem or postulate:
(assume $l \parallel m$)

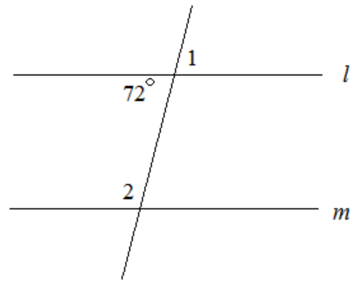
a)



1 = 100 supplemetary

2 = 80 corresponding angles

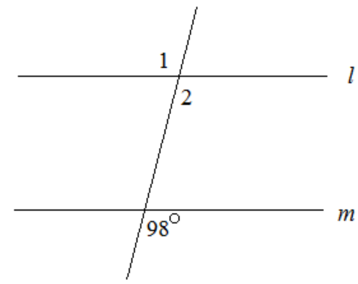
b)



1 =

2 =

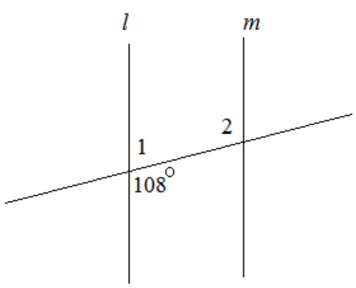
c)



1 =

2 =

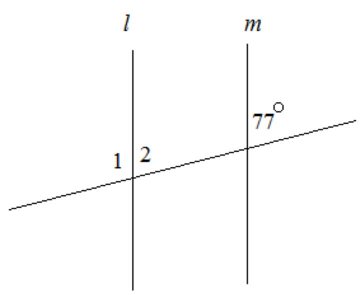
d)



1 =

2 =

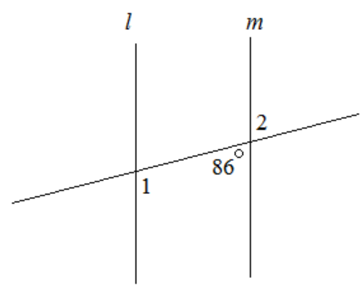
e)



1 =

2 =

f)

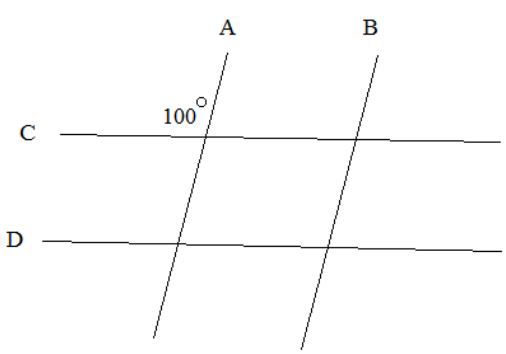


1 =

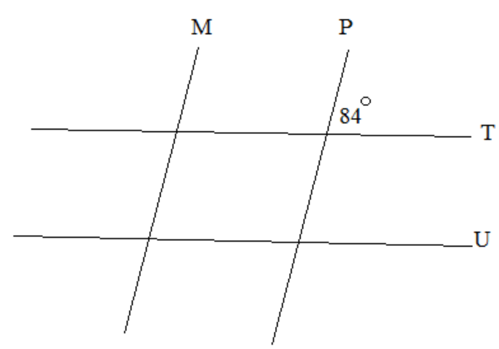
2 =

11) Fill in the possible angles from the given information...

a) $A \parallel B$ and $C \parallel D$



b) $T \parallel U$



Parallel Lines Cut by Transversals

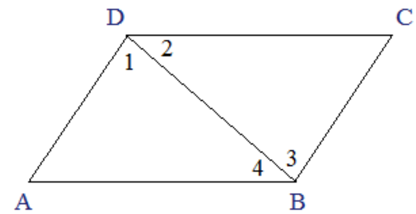
II. Answer or prove the following:

1) Given $\overline{AD} \parallel \overline{BC}$

$m\angle 1 = 5.8x + 2.2$
$m\angle 2 = 4x$
$m\angle 3 = 6.4x - 4.4$
$m\angle 4 = 42$

Find $m\angle 1 =$

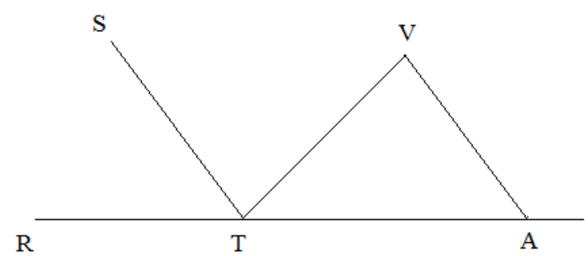
Are $\overline{DC}$ and $\overline{AB}$ parallel segments?



2) Given: $\overline{ST}$ bisects $\angle RTV$

$\overline{ST} \parallel \overline{VA}$

Prove: $\triangle VAT$ is isosceles

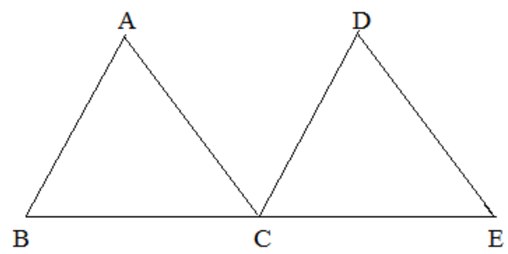


Statements	Reasons

3) Given: $\overline{AB} \parallel \overline{CD}$; $\overline{AB} \cong \overline{CD}$
C is the midpoint of $\overline{BE}$

Prove: $\overline{AC} \parallel \overline{DE}$

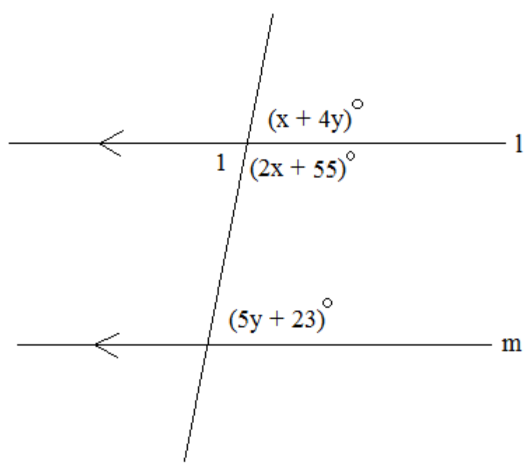
Parallel lines cut by a transversal



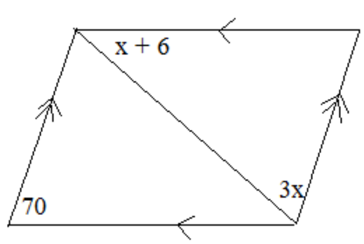
Statements	Reasons

4) Given: 1 ∥ m

Find: measure of angle 1



5) find x:

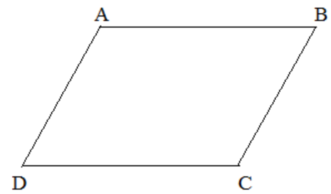


6) Given: $\overline{AB} \parallel \overline{CD}$

$\overline{AB} \cong \overline{CD}$

Prove: $\overline{AD} \parallel \overline{BC}$

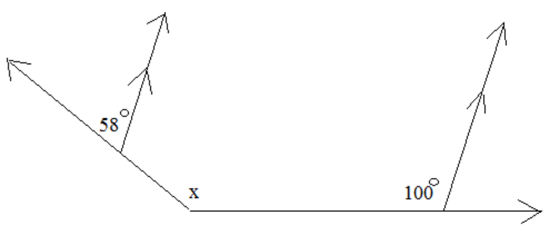
(Hint: Use an auxilary line segment)



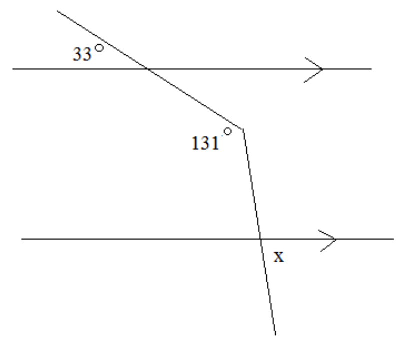
Statements	Reasons

7) "Crook Problems"

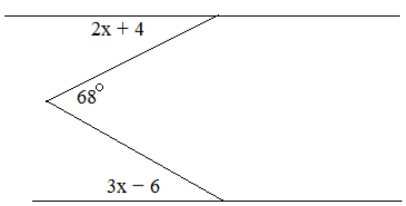
a)



b)



c)



Parallel Lines Cut by Transversals

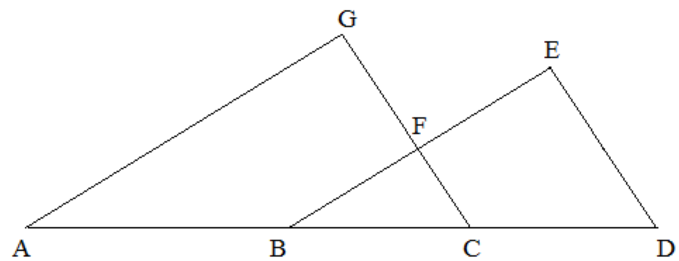
III. Proofs

1) Given: $\overline{AB} \cong \overline{CD}$ $\overline{AG} \cong \overline{BE}$

$\overline{AG} \parallel \overline{BE}$

Prove: $\overline{GC} \parallel \overline{ED}$

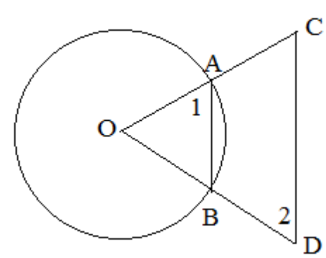
(Hint: This proof uses a "detour")



Statements	Reasons

2) Given: ⊙ O $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \parallel \overline{CD}$



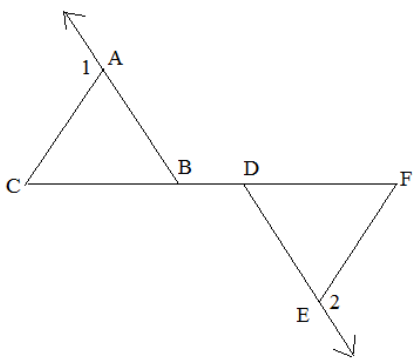
Statements	Reasons

3) Given: $\overline{AB} \parallel \overline{DE}$

$\overline{AB} \cong \overline{DE}$

$\angle 1 \cong \angle 2$

Prove: $\overline{CD} \cong \overline{FB}$

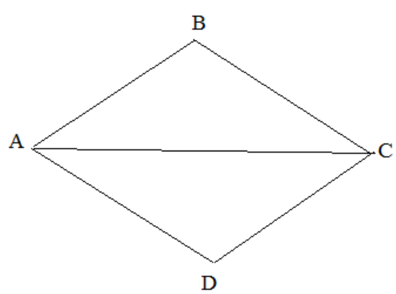


Statements	Reasons

4) Given: $\overline{AB} = \overline{CD}$

$\overline{BC} = \overline{AD}$

Prove $\overline{AB} \parallel \overline{CD}$

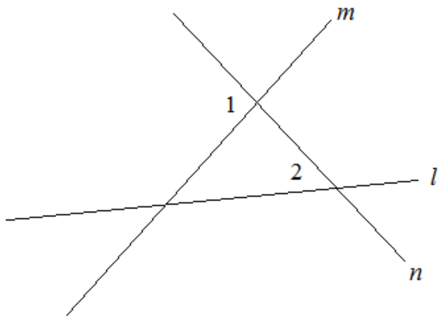


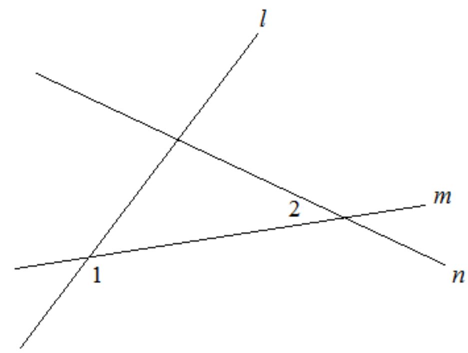
Statements	Reasons

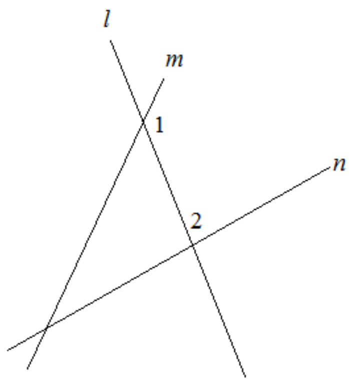
IV. **Challenge Questions

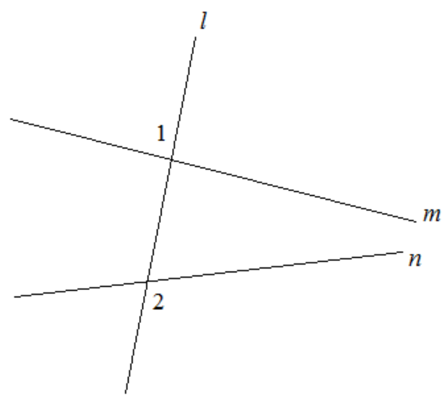
Parallel Lines Cut by Transversals

1) In each group, identify the transversal. Then, describe the angle pairs.

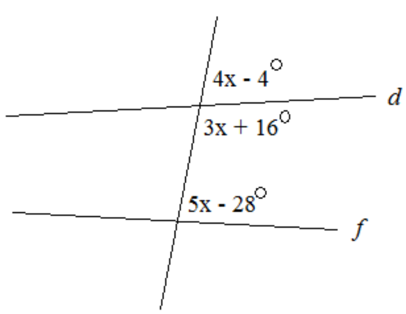


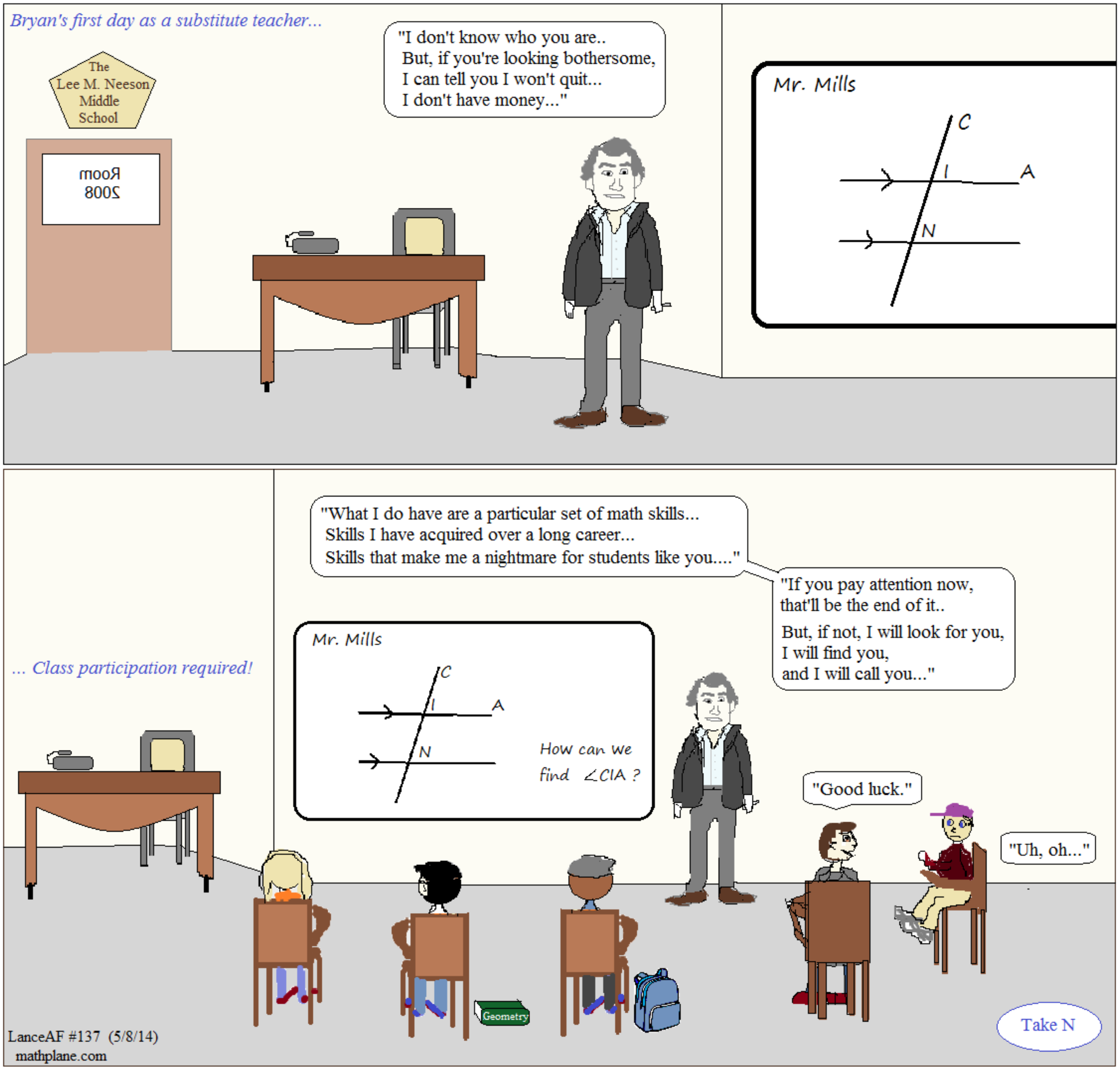






2) Is *d* parallel to *f*?




Bryan's first day as a substitute teacher...
The Lee M. Neeson Middle School
Room 2008
"I don't know who you are..
But, if you're looking bothersome,
I can tell you I won't quit...
I don't have money..."
Mr. Mills
C
I
A
N
... Class participation required!
"What I do have are a particular set of math skills...
Skills I have acquired over a long career...
Skills that make me a nightmare for students like you...."
"If you pay attention now,
that'll be the end of it..
But, if not, I will look for you,
I will find you,
and I will call you..."
Mr. Mills
C
I
A
N
How can we find ∠CIA ?
"Good luck."
"Uh, oh..."
Geometry
Take N
LanceAF #137 (5/8/14)
mathplane.com

SOLUTIONS-→

Parallel Lines Cut by Transversals

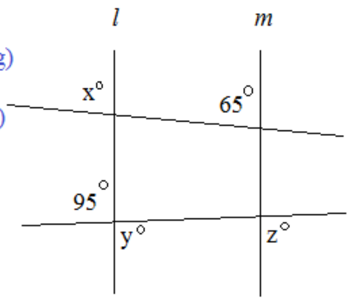
SOLUTIONS

I. Determine the following:

1) x = 65 (corresponding)

y = 95 (vertical angles)

z = 95 (y and z are corresponding angles)



$l \parallel m$

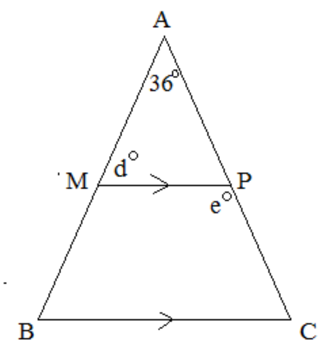
2) d = 72

e = 108

since angle A is 36, angle B and C are 72 and 72 (isosceles and sum is 180 degrees)

Since B is 72, AMP is 72 (corresponding angles)

Since C is 72, APM is 72.. then, MPC is 108 (supplementary angles)

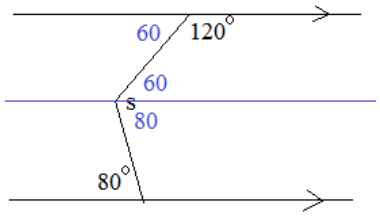


$\triangle$ ABC is Isosceles triangle

$\overline{AC} \cong \overline{AB}$

MP ∥ BC

3) $m\angle s = 140^\circ$



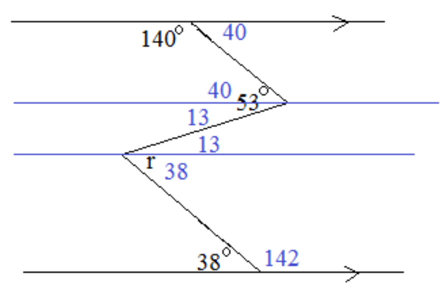
Draw an auxilary parallel line through the vertex of middle angle...

then, using supplementary angles: 60
alternate interior angles: 60 and 80
and
addition postulate,
$s = 60 + 80 = 140^\circ$

4) $m\angle r = 51^\circ$

Draw auxilary parallel lines.. Then, use supplementary angles, interior angles, and addition...

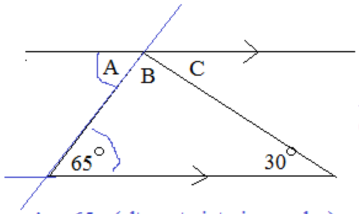
140 + 40 = 180
38 + 142 = 180
53 - 40 = 13
13 + 38 = 51



5) $\angle A =$ 65

$\angle B =$ 85

$\angle C =$ 30



A + B + C = 180
(straight angle)

A = 65 (alternate interior angles)

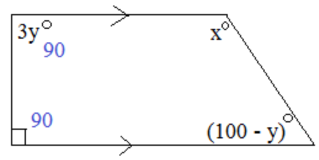
C = 30 (alternate interior angles)

B = 85 (angles in triangle add up to 180)

6) x = 110

y = 30

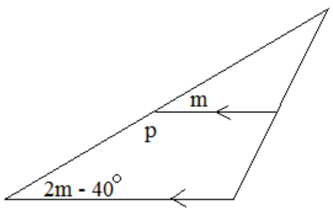
x + (100 - y) = 180
x + (70) = 180
x = 110



3y + 90 = 180 (same side interior angles are supplementary)
3y = 90
y = 30

7) m = 40

p = 140



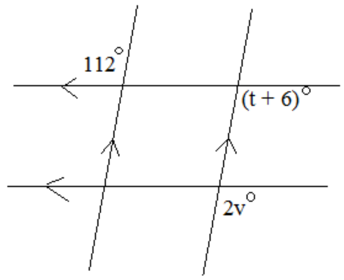
2m - 40 = m (corresponding angles)
m = 40

m + p = 180 (supplementary)
so, p = 140

8) t = 106

v = 56

t + 6 = 112 (alternate exterior angles)
t = 106



2v = 112 (corresponding angles)
v = 56

9) Use information to determine which lines (if any) must be parallel:

a) $1 \cong 9$ since 1 and 9 are corresponding, L ∥ M

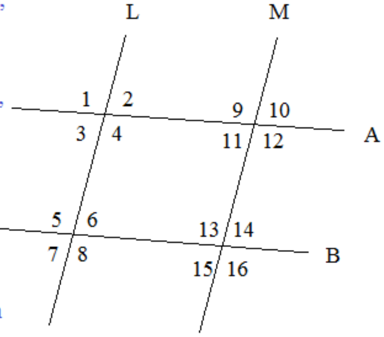
b) $4 \cong 8$ since 4 and 8 are corresponding, A ∥ B

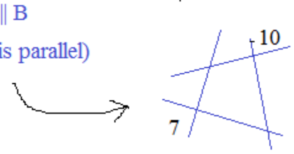
L and M may or may not be ∥

c) $2 \cong 3$ 2 and 3 are vertical angles, so we don't know if any lines are parallel...

d) $10 \cong 7$ If 10 and 7 are congruent, then L ∥ M AND A ∥ B

(Or, NEITHER is parallel)





e) $6 \cong 15$ since 6 and 15 are alternate interior angles, then L ∥ M

f) $9 \cong 16$ 9 and 16 are alternate exterior angles, so A ∥ B

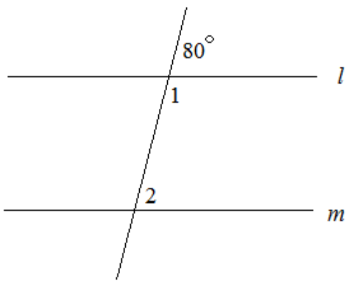
g) $5 \cong 9$ If 5 = 9, then L ∥ M AND A ∥ B (Or, NEITHER see d))

h) $13 \cong 14$ Since 13 = 14, both must be right angles... However, that doesn't determine if the lines are parallel...

10) Answer and identify the relevant theorem or postulate: SOLUTIONS

(assume $l \parallel m$)

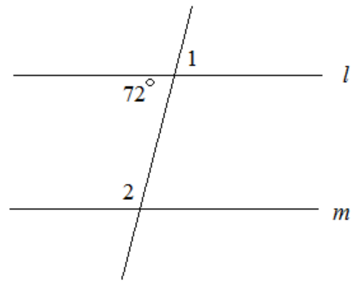
a)



1 = 100 supplemetary

2 = 80 corresponding angles

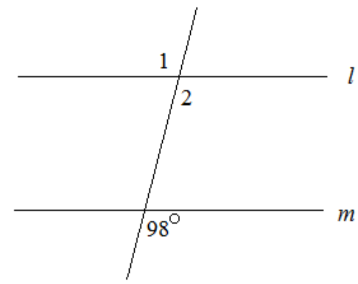
b)



1 = 72 vertical angles

2 = 108 same side interior (supp.)

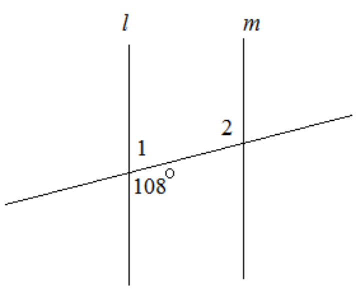
c)



1 = 98 alternate exterior

2 = 98 corresponding angles

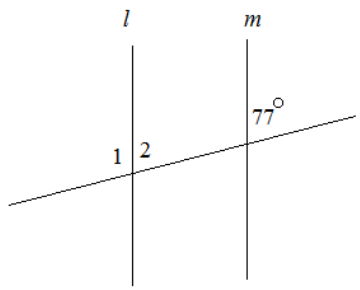
d)



1 = 72 supplementary angles

2 = 108 alternate interior angles

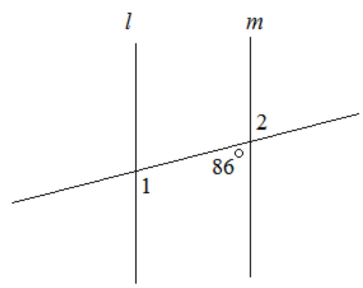
e)



1 = 103 same side exterior

2 = 77 corresponding angles

f)

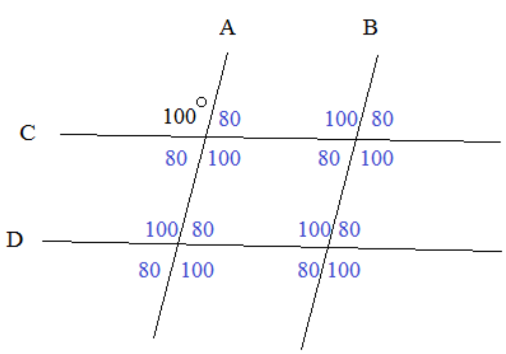


1 = 94 same side interior (supp.)

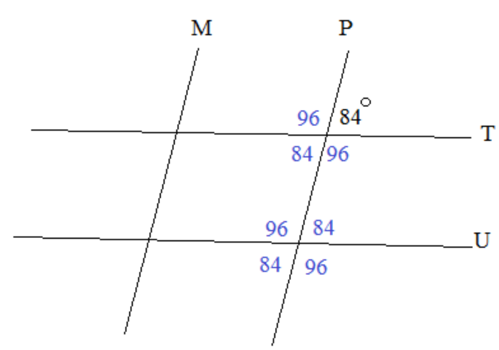
2 = 86 vertical angles

11) Fill in the possible angles from the given information...

a) A || B and C || D



b) T || U



**since we don't know if M || P, some of the angles cannot be determined!

Parallel Lines Cut by Transversals

SOLUTIONS

II. Answer or prove the following:

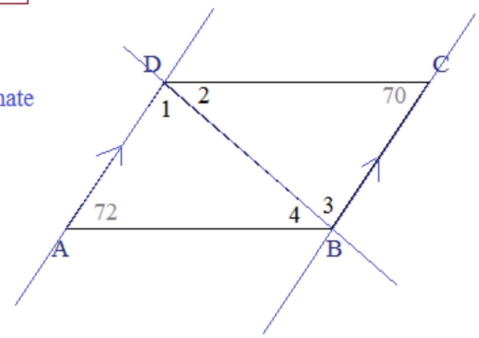
1) Given $\overline{AD} \parallel \overline{BC}$

$m\angle 1 = 5.8x + 2.2$
$m\angle 2 = 4x$
$m\angle 3 = 6.4x - 4.4$
$m\angle 4 = 42$

Find $m\angle 1 =$ 66°

Are $\overline{DC}$ and $\overline{AB}$ parallel segments? NO

1 and 3 are alternate interior angles..



Since AD and BC are parallel, angles 1 and 3 are congruent:

$5.8x + 2.2 = 6.4x - 4.4$

$6.6 = .6x$

$x = 11$

angle 1: $5.8(11) + 2.2 = 66$ ✓
angle 3: $6.4(11) - 4.4 = 66$ ✓

the measure of angle 4 is 42 degrees...
the measure of angle 2 is $4(11) = 44$ degrees

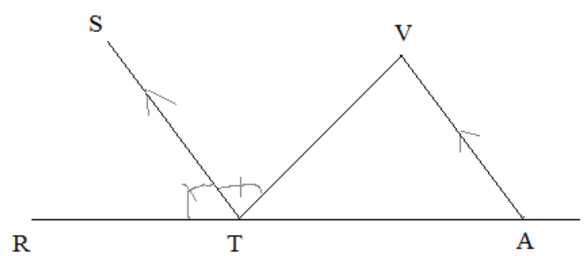
Since $\angle 4 \neq \angle 2$,

then $\overline{AB}$ and $\overline{DC}$ are not parallel

2) Given: $\overline{ST}$ bisects $\angle RTV$

$\overline{ST} \parallel \overline{VA}$

Prove: $\triangle VAT$ is isosceles

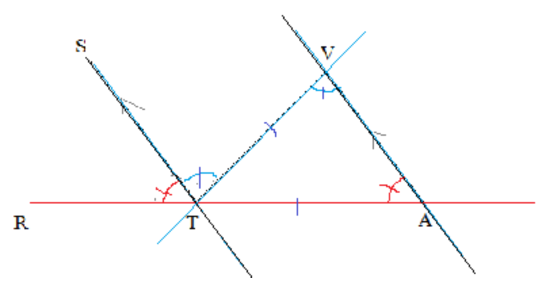


Statements	Reasons
1. $\overline{ST}$ bisects $\angle RTV$	1. Given
2. $\angle RTS \cong \angle STV$	2. Definition of angle bisector
3. $\overline{ST} \parallel \overline{VA}$	3. Given
4. $\angle TAV \cong \angle RTS$	4. If parallel lines cut by transversal, then corresponding $\angle$s congruent
5. $\angle STV \cong \angle TVA$	5. If parallel lines cut by transversal, then alternate interior angles congruent
6. $\angle TAV \cong \angle TVA$	6. Transitive property (from 4, 2, and 5.)
7. $\overline{TV} \cong \overline{TA}$	7. "sides-angles" (if 2 angles of triangle are congruent, then their opposite sides are $\cong$)
8. $\triangle VAT$ is isosceles	8. Definition of isosceles triangle

Label the diagram

What are we trying to find?
2 sides of $\triangle VAT$ that are the same... (or, 2 angles that are congruent)

Strategy: Use parallel lines cut by transversal to identify congruent angles

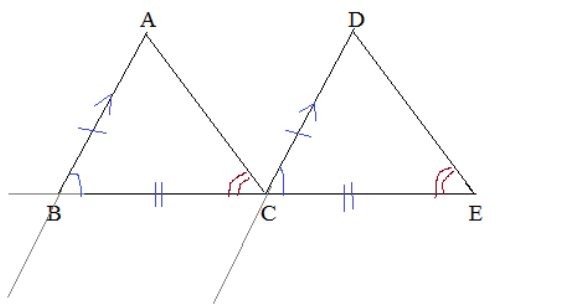


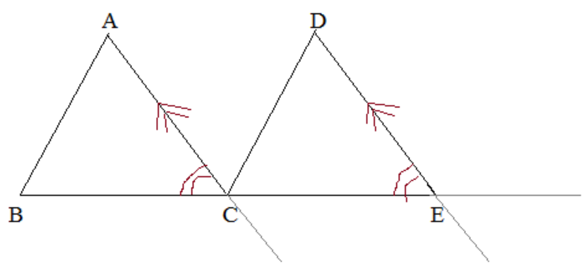
3) Given: $\overline{AB} \parallel \overline{CD}$; $\overline{AB} \cong \overline{CD}$
C is the midpoint of $\overline{BE}$

SOLUTIONS

Parallel lines cut by a transversal

Prove: $\overline{AC} \parallel \overline{DE}$

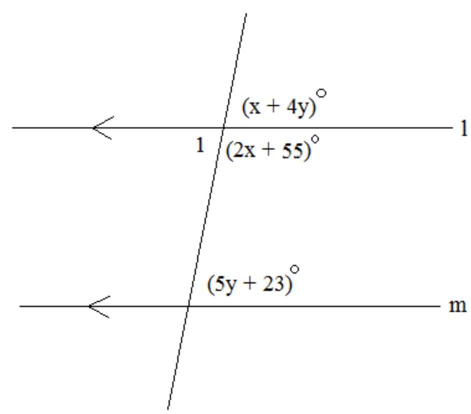




Statements	Reasons
1) $\overline{AB} \parallel \overline{CD}$	1) Given
2) $\angle B \cong \angle DCE$	2) If parallel lines cut by transversal then corresponding angles $\cong$
3) $\overline{AB} \cong \overline{CD}$	3) Given
4) C is midpoint of $\overline{BE}$	4) Given
5) $\overline{BC} \cong \overline{CE}$	5) Definition of midpoint
6) $\triangle ABC = \triangle DCE$	6) Side-Angle-Side (SAS) (3, 2, 5)
7) $\angle ACB = \angle E$	7) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8) $\overline{AC} \parallel \overline{DE}$	8) If corresponding angles are $\cong$, then lines are parallel (converse of above theorem)

4) Given: $l \parallel m$

Find: measure of angle 1



Since lines are parallel,

$x + 4y = 5y + 23$

(corresponding angles)

$(2x + 55) + (5y + 23) = 180$

(same side interiors are supplementary)

$x - y = 23$

$2x + 5y = 102$

$-2x + 2y = -46$

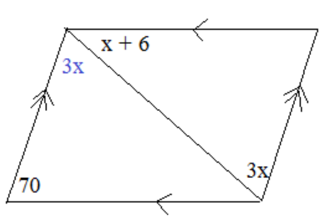
$7y = 56$

$y = 8$

$x = 31$

angle 1 = 63 degrees

5) find x:



$3x + x + 6 + 70 = 180$

$4x = 104$

$x = 26$

to check: plug in x = 26...

angles are 78 + 32 = 110
and 70

same side angles add up to 180...

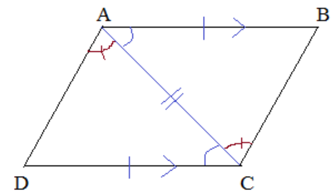
SOLUTIONS

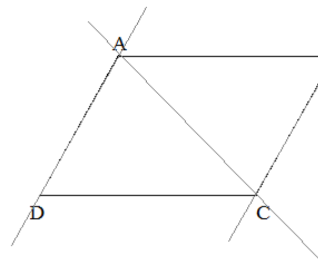
6) Given: $\overline{AB} \parallel \overline{CD}$

$\overline{AB} \cong \overline{CD}$

Prove: $\overline{AD} \parallel \overline{BC}$

(Hint: Use an auxilary line segment)



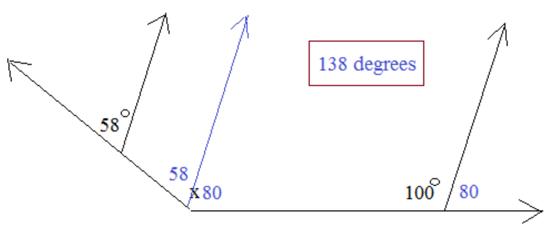


Statements	Reasons
1) $\overline{AB} = \overline{CD}$	1) Given
2) $\overline{AC}$ is a line segment	2) Auxilary line (2 points make a line)
3) $\overline{AB} \parallel \overline{CD}$	3) Given
4) $\angle BAC = \angle DCA$	4) If ∥ lines cut by transversal, then alternate interior angles congruent
5) $\overline{AC} \cong \overline{AC}$	5) Reflexive Property
6) $\triangle BAC \cong \triangle DCA$	6) Side-Angle-Side (SAS) (1, 4, 5)
7) $\angle ACB = \angle CAD$	7) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8) $\overline{AD} \parallel \overline{BC}$	8) If alternate interior angles are congruent, then lines are parallel

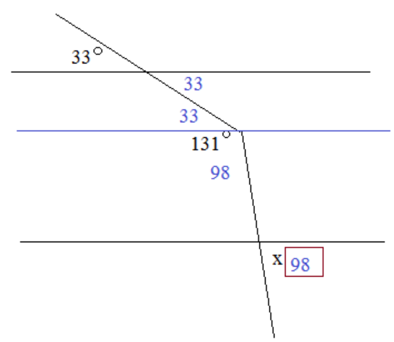
7) "Crook Problems"

a)

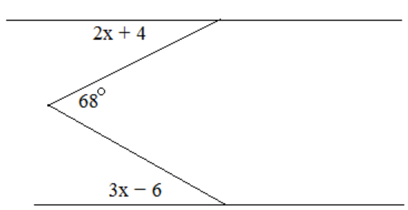
corresponding angles



b)



c)



5x - 2 = 68

5x = 70

x = 14

Parallel Lines Cut by Transversals

SOLUTIONS

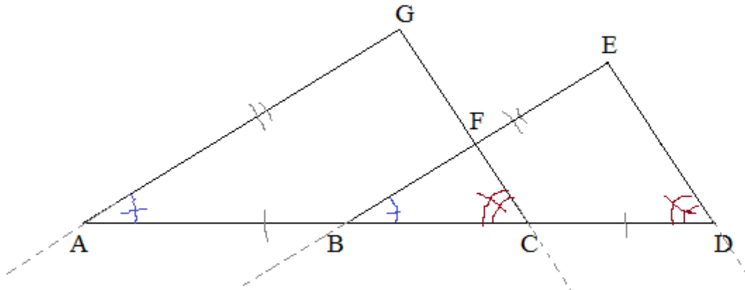
III. Proofs

1) Given: $\overline{AB} \cong \overline{CD}$ $\overline{AG} \cong \overline{BE}$

$\overline{AG} \parallel \overline{BE}$

Prove: $\overline{GC} \parallel \overline{ED}$

(Hint: This proof uses a "detour")



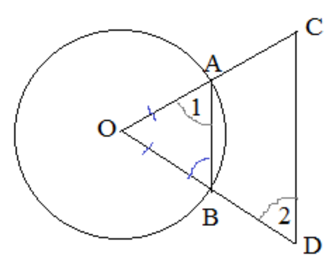
Label the figure and observe that there are 2 overlapping triangles.

Strategy: Using SAS, prove that the triangles are congruent. Then, CPCTC will prove that corresponding angles C and D are congruent

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{BC} \cong \overline{BC}$	2. Reflexive property
3. $\overline{AC} \cong \overline{BD}$	3. Addition postulate
4. $\overline{AG} \cong \overline{BE}$	4. Given
5. $\overline{AG} \parallel \overline{BE}$	5. Given
6. $\angle GAC = \angle EBD$	6. If parallel lines cut by transversal, then corresponding angles congruent
7. $\triangle GAC = \triangle EBD$	7. Side-Angle-Side (statements 4, 6, 3)
8. $\angle ACG = \angle BDE$	8. CPCTC
9. $\overline{GC} \parallel \overline{ED}$	9. If the corresponding angles of 2 lines (cut by a transversal) are congruent, then the lines are parallel

2) Given: ⊙ O $\angle 1 \cong \angle 2$

Prove: $\overline{AB} \parallel \overline{CD}$



label the diagram -- we see 1 and 2 are congruent...

Statements	Reasons
1. Circle with center O	1. Given
2. $\overline{OA} = \overline{OB}$	2. All radii of circle are congruent
3. $\angle OBA \cong \angle 1$	3. "Sides-Angles" (If 2 sides of △ are congruent, then opposite ∠s congruent)
4. $\angle 1 = \angle 2$	4. Given
5. $\angle OBA = \angle 2$	5. Transitive property (from statements 3, 4)
6. $\overline{AB} \parallel \overline{CD}$	6. If the corresponding angles of 2 lines (cut by transversal) are congruent, then the lines are parallel

Strategy: Verify that angle B is congruent to angle D (or angle C is congruent to angle A), because corresponding angles ⟶ parallel lines cut by transversal

3) Given: $\overline{AB} \parallel \overline{DE}$

$\overline{AB} \cong \overline{DE}$

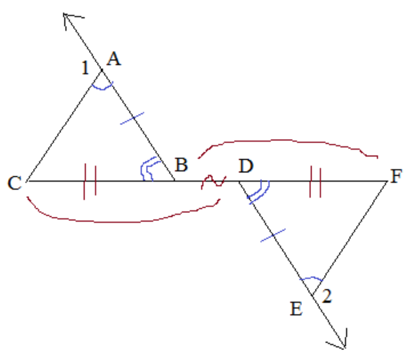
$\angle 1 \cong \angle 2$

Prove: $\overline{CD} \cong \overline{FB}$

SOLUTIONS

Parallel Lines Cut by Transversals

Strategy: use the given statements to help prove that the triangles are congruent.. Then, use sides in triangles to ultimately get congruent segments...

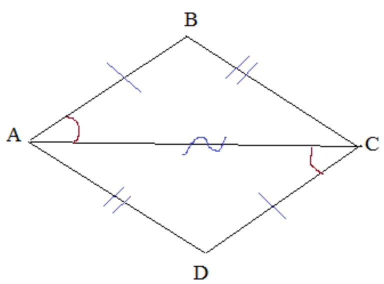


Statements	Reasons
1) $\overline{AB} \cong \overline{DE}$	1) Given
2) $\angle 1 \cong \angle 2$	2) Given
3) $\angle 1$ and $\angle BAC$ are supplementary $\angle 2$ and $\angle DEF$ are supplementary	3) Definition of supplementary angles (adjacent angles that form straight angle)
4) $\angle BAC \cong \angle DEF$	4) Congruent supplements (If angles are supplementary to congruent angles, then they are congruent.)
5) AB $\parallel$ DE	5) Given
6) $\angle ABC \cong \angle EDF$	6) If parallel lines cut by transversal, then alternate exterior angles are congruent
7) $\triangle ABC \cong \triangle EDF$	7) ASA (Angle-Side-Angle) 4, 1, 6
8) $\overline{CB} \cong \overline{FD}$	8) CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
9) $\overline{BD} \cong \overline{BD}$	9) Reflexive Property
10) $\overline{CD} \cong \overline{FB}$	10) Addition Property (If segment (BD) is added to congruent segments, then the sums are congruent)

4) Given: $\overline{AB} = \overline{CD}$

$\overline{BC} = \overline{AD}$

Prove $\overline{AB} \parallel \overline{CD}$



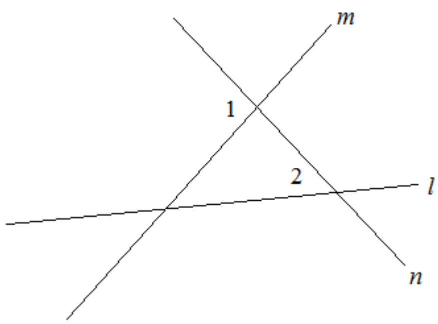
Statements	Reasons
1) $\overline{AB} = \overline{CD}$	1) Given
2) $\overline{BC} = \overline{AD}$	2) Given
3) $\overline{AC} = \overline{AC}$	3) Reflexive Property
4) $\triangle ABC = \triangle CDA$	4) SSS (Side-Side-Side)
5) $\angle BAC = \angle DCA$	5) CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
6) $\overline{AB} \parallel \overline{CD}$	6) If alternate interior angles are congruent, then the lines are parallel

Note: angles DAC and BCA are irrelevant, because they would prove BC $\parallel$ AD

IV. **Challenge Questions

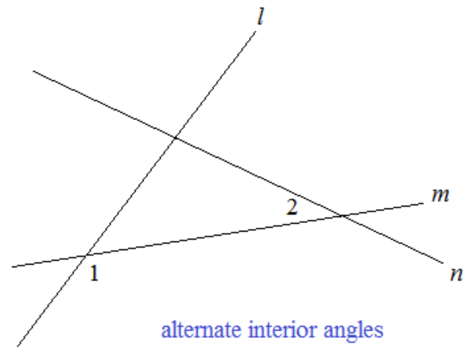
1) In each group, identify the transversal. Then, describe the angle pairs.

SOLUTIONS



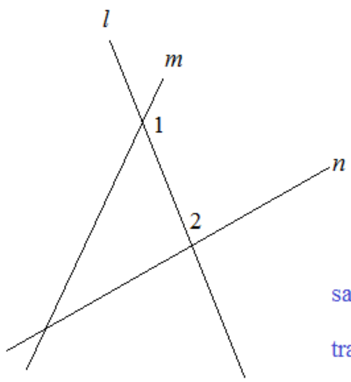
corresponding angles

transversal *n*



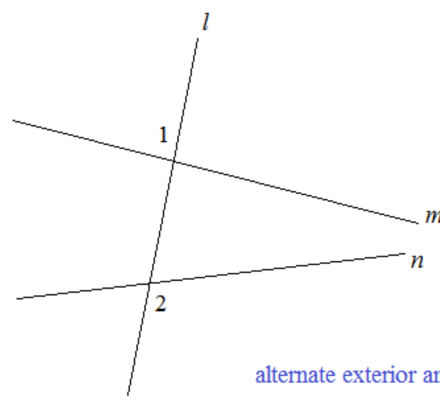
alternate interior angles

transversal *m*



same side interior

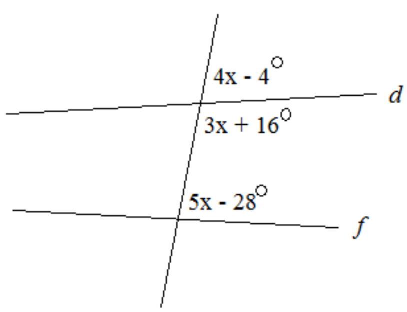
transversal *l*



alternate exterior angles

transversal *l*

2) Is *d* parallel to *f*?



$4x - 4 + 3x + 16 = 180$ (supplementary angles)

$x = 24$

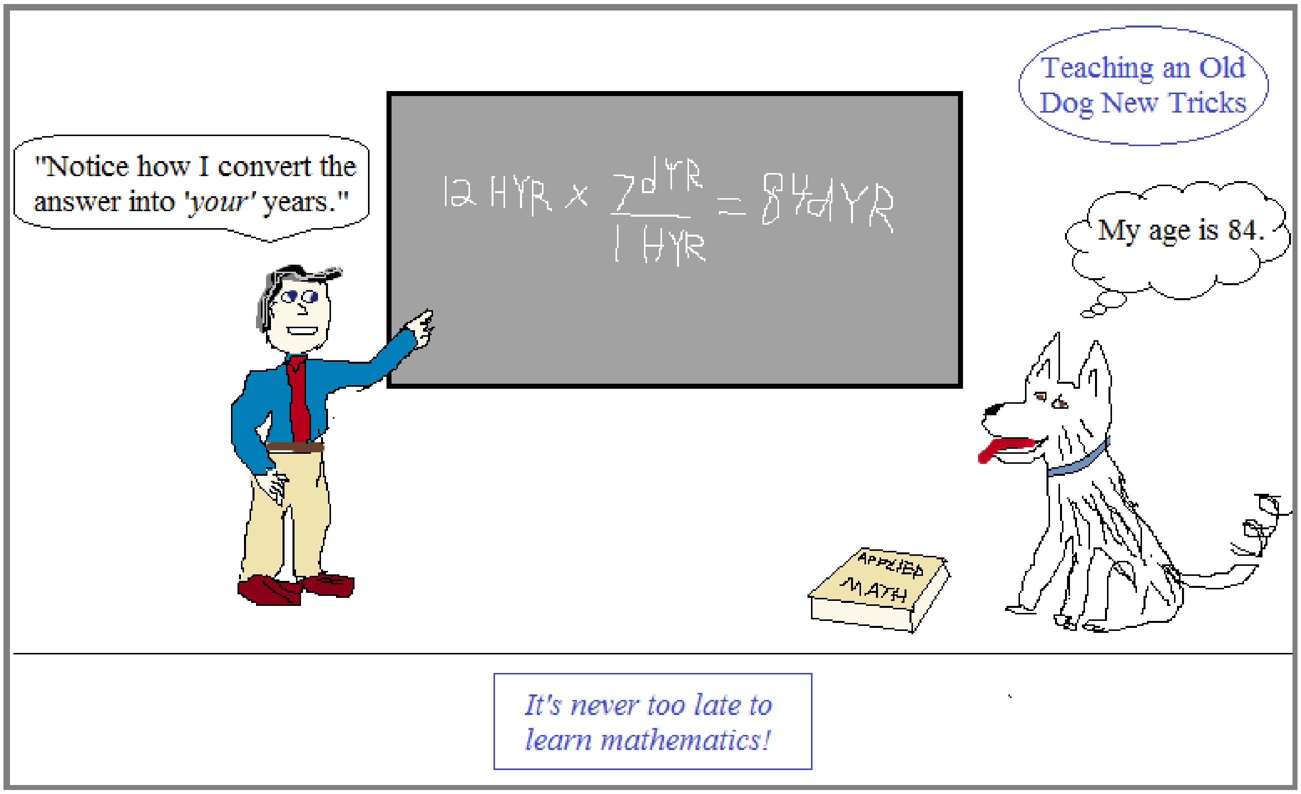
since x must be 24, angles are 92, 88, and 92

corresponding angles congruent.... d || f

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers.



ONE MORE…..

Which letters in the alphabet illustrate congruent corresponding angles?

Supplementary same side interior angles?

Congruent alternate interior angles?

Parallel Lines, Transversals, and the Alphabet

Consider the alphabet:

Which letters illustrate congruent alternate interior angles?

Which letters demonstrate same-side interior angles are supplementary?

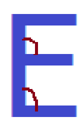
Which letters have congruent corresponding angles?

(Depending on the font, upper & lower case, and other factors,) here are some possibilities:

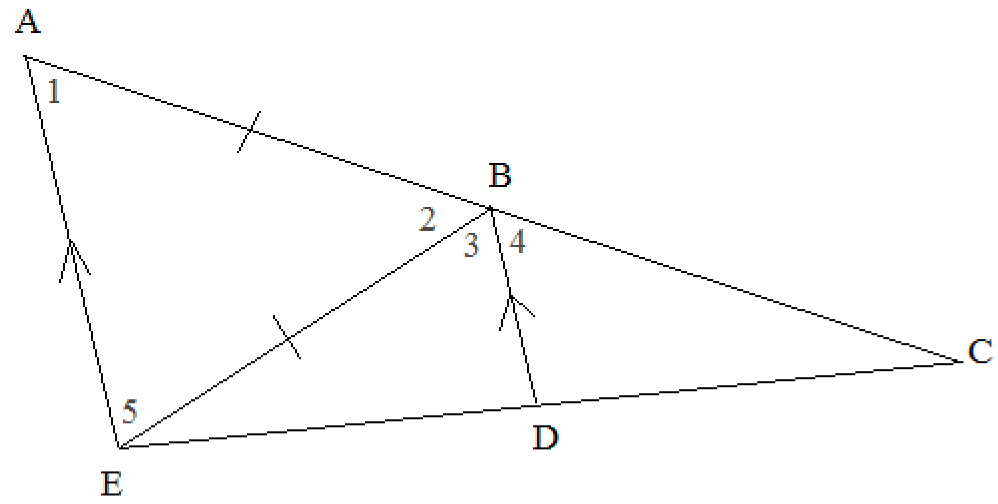
Congruent Alternate Interior:

Same-Side interior (supplementary):

Congruent Corresponding:



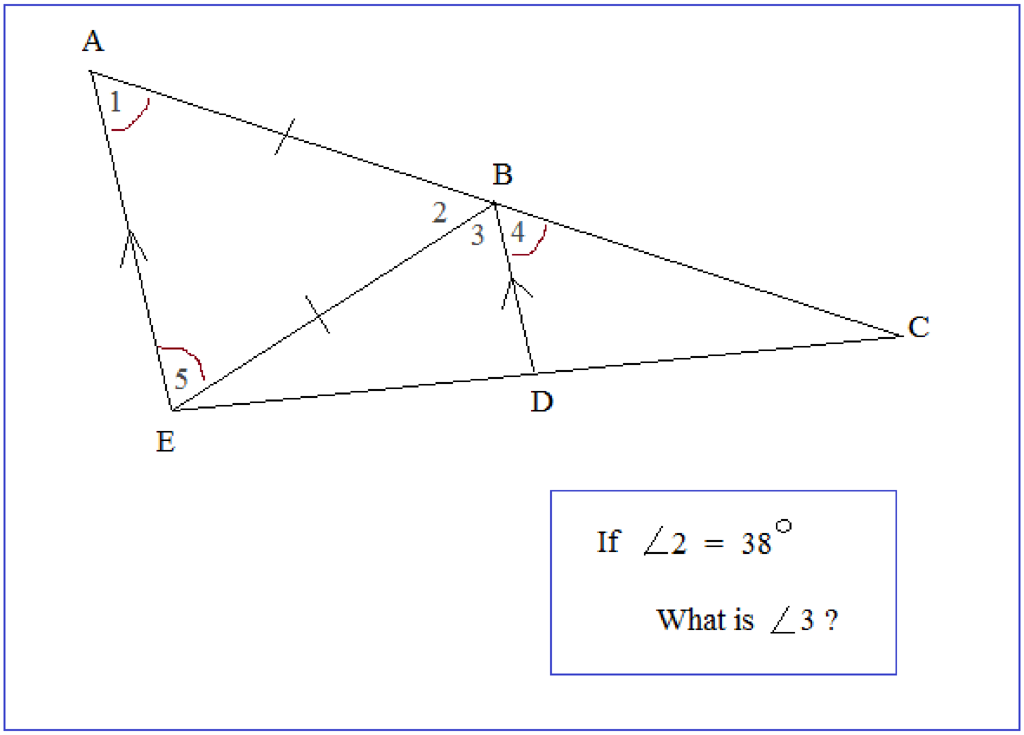
And, of course, the letter
with vertical angles:



If $\angle 2 = 38^{\circ}$

What is $\angle 3$?

SOLUTION -→



Answer: Since $\overline{AB} = \overline{EB}$, angles 1 and 5 are also congruent. (if 2 congruent sides in triangle, then opposite angles are congruent)

$1 + 5 + 2 = 180^\circ$ (angle sum of triangle)

$1 + 5 + 38^\circ = 180^\circ$

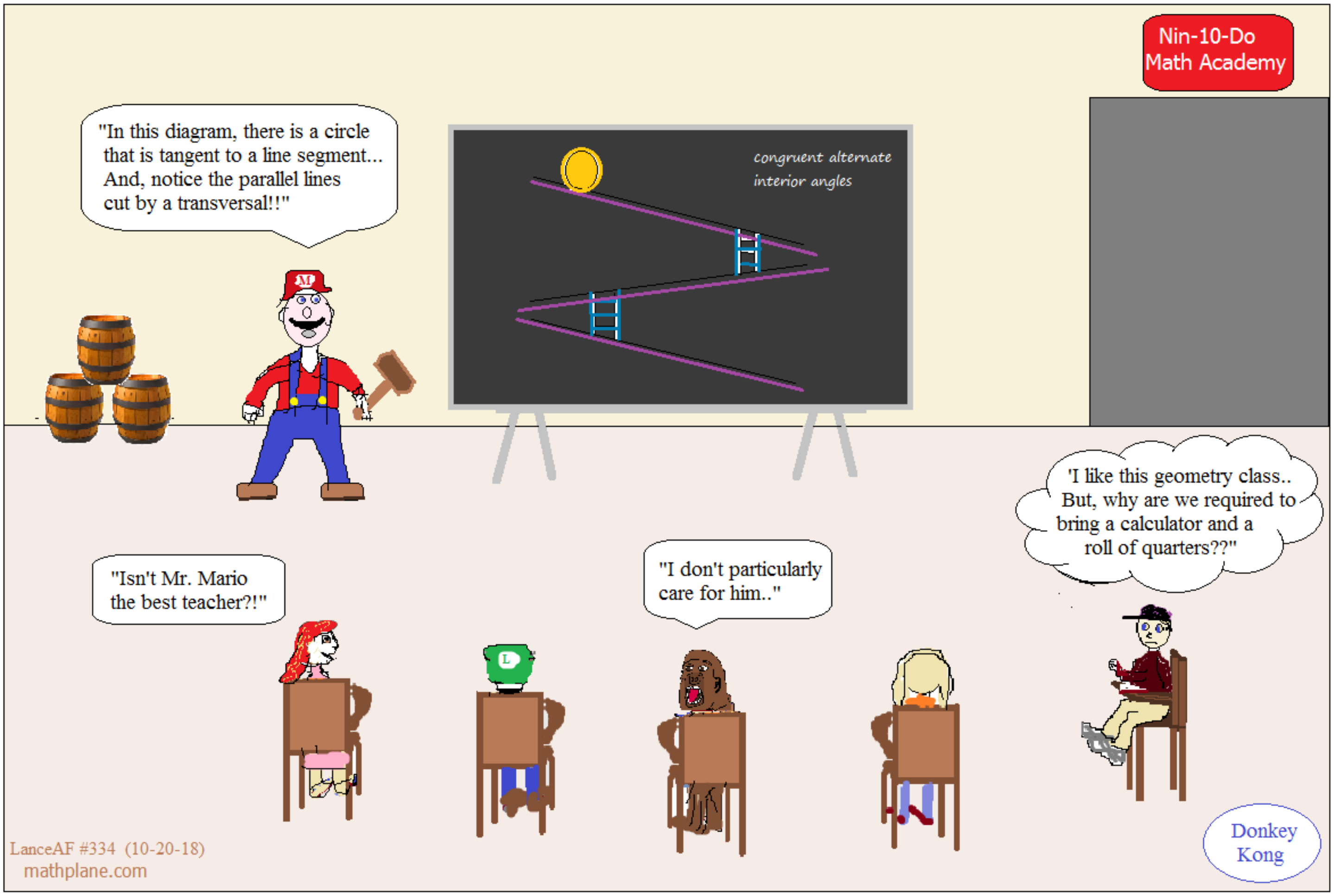
Angles 1 and 5 must be 71° each

Since AE || BD, angles 1 and 4 are congruent. (if parallel lines cut by transversal, corresponding angles are congruent)

$2 + 3 + 4 = 180^\circ$ (3 adjacent angles form a straight angle - 180 degrees)

$38^\circ + 3 + 71^\circ = 180^\circ$

$3 = 71^\circ$


Nin-10-Do
Math Academy
"In this diagram, there is a circle that is tangent to a line segment... And, notice the parallel lines cut by a transversal!!"
congruent alternate interior angles
'I like this geometry class.. But, why are we required to bring a calculator and a roll of quarters??"
"Isn't Mr. Mario the best teacher?!"
"I don't particularly care for him.."
Donkey Kong
LanceAF #334 (10-20-18)
mathplane.com