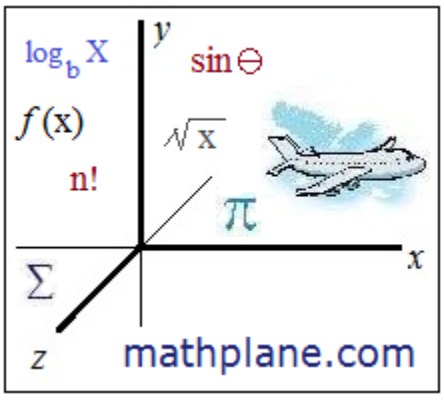
Radicals

Brief notes, quiz (w/solutions), and comic

Topics include prime factorization, rationalizing the denominator, perfect squares, word problems, conjugates, and more.



Simplifying Radicals

Strategy 1: Prime Factorization

$$\sqrt{700}$$

Factor (to primes) $\sqrt{7 \cdot 2 \cdot 5 \cdot 2 \cdot 5}$

Remove "pairs" $2 \cdot 5 \sqrt{7}$

Simplify $10\sqrt{7}$

Strategy 2: Using Perfect Squares

$$\sqrt{700}$$

Factor $\sqrt{7 \cdot 100}$

Remove Perfect Squares $10\sqrt{7}$

Rationalizing the Denominator

It's improper to have a radical in the denominator. So, to correct a fraction, simply rationalize the denominator.

Single term denominator: $\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \longrightarrow \frac{2\sqrt{5}}{5}$

multiply by 1 radical moves to numerator

Double term denominator: use the conjugate

$$\frac{3}{4+\sqrt{6}} \cdot \frac{4-\sqrt{6}}{4-\sqrt{6}} \longrightarrow \frac{12-3\sqrt{6}}{10}$$

$$\frac{12-3\sqrt{6}}{16+4\sqrt{6}-4\sqrt{6}-6}$$

Adding/Subtracting Roots

In algebra, to add or subtract, you must have "like terms".

$3x + 2y + 14 - x + 4y + 20 = 2x + 6y + 34$ Collect the x's, the y's, and the numbers...

$2x + 5y + 3xy + 6 + 7x - 4y = 9x + y + 3xy + 6$ Add/Subtract the x's, y's, xy's, and the numbers separately.

Adding/Subtracting radicals must have same root and radicand

Example: $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

$3x + 2x = 5x$

Each is a "square root" and each has a 2 under the radical

definititon: a radicand is the quantity under the radical

Example: $\sqrt{27} - \sqrt{3}$ The roots are both square roots, but the radicands are different..

$\sqrt{3 \cdot 9} - \sqrt{3}$ However, if we simplify, we see the terms are now the same!

$3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$

Multiplying/Dividing Roots

When multiplying variables in algebra, we collect terms and multiply everything together...

$(2a^2 b^4) \cdot (5a^3 b^3 c) = 10a^5 b^7 c$ We combined the mubers, a's, b's, and the c terms... And, all the terms are multiplied together...

Multiplying/Dividing Radicals: collect "like roots" and attach all the terms together

Example: numbers

$3\sqrt{5} \cdot 2\sqrt{6} = 3 \cdot 2 \cdot \sqrt{5} \cdot \sqrt{6} = 6\sqrt{30}$

square roots

Note: The radicands are different, but the roots are the same.. therefore, the terms can be combined!

Example: $3\sqrt[3]{5} \cdot 2\sqrt{6} = 6 \cdot \sqrt{6} \cdot \sqrt[3]{5}$

number, square root, cube root

Since the roots are different, they cannot be combined...

Additonal Radical Topic: rationalizing roots greater than 2

Example: Simplify/Rationalize the denominator: $\sqrt[3]{\frac{2}{3x}}$

If you simply multiply by $\frac{\sqrt[3]{3x}}{\sqrt[3]{3x}}$ the result is still not simplified!

$$\sqrt[3]{\frac{2}{3x}} \cdot \frac{\sqrt[3]{3x}}{\sqrt[3]{3x}} = \sqrt[3]{\frac{6x}{9x^2}}$$ (The denominator is irrational)

You need to multiply TWICE to get a perfect cube in the denominator!

$$\sqrt[3]{\frac{6x}{9x^2}} \cdot \frac{\sqrt[3]{3x}}{\sqrt[3]{3x}} = \sqrt[3]{\frac{18x^2}{27x^3}} = \boxed{\frac{\sqrt[3]{18x^2}}{3x}}$$

Example: Simplify the following: $\sqrt[5]{\frac{14x}{4x^2y^2}}$

First, reduce the fraction... $\sqrt[5]{\frac{14x}{4x^2y^2}} = \sqrt[5]{\frac{7}{2x\,y^2}}$

Then, rationalize the denominator...

Since the root is 5, multiply by the 4th power...

$$\frac{\sqrt[5]{7}}{\sqrt[5]{2xy^2}} \cdot \frac{\sqrt[5]{(2xy^2)^4}}{\sqrt[5]{(2xy^2)^4}} = \frac{\sqrt[5]{7\,(2xy^2)^4}}{\sqrt[5]{(2xy^2)^5}} =$$

$$\frac{\sqrt[5]{7\,(16x^4y^8)}}{2xy^2} = \frac{y\sqrt[5]{112\,x^4y^3}}{2xy^2} = \boxed{\frac{\sqrt[5]{112\,x^4y^3}}{2xy}}$$

Example: Simplify $\dfrac{5}{\sqrt[4]{x}+10}$

Multiply by the "conjugate over the conjugate"

$$\frac{5}{\sqrt[4]{x}+10}\cdot\frac{\sqrt[4]{x}-10}{\sqrt[4]{x}-10}=\frac{5\sqrt[4]{x}-50}{\sqrt[4]{x^2}+10\sqrt[4]{x}-10\sqrt[4]{x}-100}=\frac{5\sqrt[4]{x}-50}{\sqrt[4]{x^2}-100}$$

(Denominator is still irrational)
Try to rationalize denominator again...

$$\frac{5\sqrt[4]{x}-50}{\sqrt[4]{x^2}-100}\cdot\frac{\sqrt[4]{x^2}+100}{\sqrt[4]{x^2}+100}=\frac{5\sqrt[4]{x^3}+500\sqrt[4]{x}-50\sqrt[4]{x^2}-5000}{\sqrt[4]{x^4}+100\sqrt[4]{x^2}-100\sqrt[4]{x^2}-10000}=$$

$$\frac{5\sqrt[4]{x^3}+500\sqrt[4]{x}-50\sqrt[4]{x^2}-5000}{|x|-10000}\quad\text{or}\quad\frac{5x^{\frac{3}{4}}-50x^{\frac{2}{4}}+500x^{\frac{1}{4}}-5000}{|x|-10000}$$

Additional Topic: Why do you need an absolute value?

Example: Does $\sqrt{x^2}=x$?

Let's test points: If x = 3: $\sqrt{3^2}=3$, $\sqrt{9}=3$, $3=3$ ✓

But, if x = -3 $\sqrt{(-3)^2}=-3$, $\sqrt{9}=-3$, $3=-3$ ✗

However, if we include an absolute value sign:

$\sqrt{x^2}=|x|$

If x = 3: $\sqrt{3^2}=|3|$, $\sqrt{9}=|3|$, $3=|3|$ ✓

But, if x = -3 $\sqrt{(-3)^2}=|-3|$, $\sqrt{9}=|-3|$, $3=|-3|$ ✓

General Rule: If n is even, then $\sqrt[n]{x^n}=|n|$

Additional Topic: Simplifying radical fractions

Example: $\sqrt{\dfrac{54}{24}}$

approach 1: reduce the fraction $\sqrt{\dfrac{54}{24}}=\sqrt{\dfrac{9}{4}}=\dfrac{\sqrt{9}}{\sqrt{4}}=\dfrac{3}{2}$

approach 2: reduce radicals first $\dfrac{\sqrt{54}}{\sqrt{24}}=\dfrac{\sqrt{9}\cdot\sqrt{6}}{\sqrt{4}\cdot\sqrt{6}}=\dfrac{3\sqrt{6}}{2\sqrt{6}}=\dfrac{3}{2}$

Square Root Exercises

I. Simplifying

a) $\sqrt{16}$ b) $\sqrt{50}$ c) $\sqrt{27}$ d) $\sqrt{98}$ e) $\sqrt{63}$

f) $\sqrt{1100}$ g) $\sqrt{52}$ h) $\sqrt{72}$ i) $5\sqrt{8}$ j) $7\sqrt{28}$

II. Addition/Subtraction

a) $3\sqrt{2} + 4\sqrt{2}$ b) $7\sqrt{3} - \sqrt{3}$ c) $\sqrt{20} + \sqrt{45}$ d) $\sqrt{24} + \sqrt{54}$

e) $3\sqrt{8} + 7\sqrt{2}$ f) $4\sqrt{5} + 3\sqrt{28}$ g) $11\sqrt{10} - 2\sqrt{300}$ h) $2\sqrt{75} + 3\sqrt{300}$

III. Multiplication/Division

a) $\sqrt{6} \cdot \sqrt{12}$ b) $2\sqrt{3} \cdot \sqrt{18}$ c) $4\sqrt{6} \cdot 7\sqrt{2}$ d) $\dfrac{\sqrt{72}}{3}$

e) $\sqrt{\dfrac{200}{63}}$ f) $\sqrt{\dfrac{80}{90}}$ g) $\sqrt{5}\,(\sqrt{15} + \sqrt{60})$ h) $4\sqrt{3}\,(\sqrt{48} - \sqrt{3})$

i) $\sqrt{29.6} \cdot \sqrt{29.6}$ j) $\frac{1}{3}(\sqrt{5} + \sqrt{125})$ k) $\dfrac{9\sqrt{6} \cdot 2\sqrt{2}}{4\sqrt{12} \cdot 7\sqrt{3}}$

IV. Additional Questions

a) Find x

1) $x\sqrt{3} + 4x\sqrt{12} = 8\sqrt{48}$

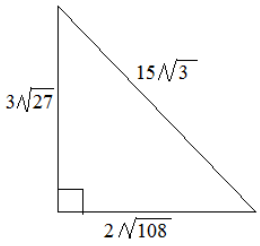
2) $x\sqrt{50} + 4\sqrt{2} = 8\sqrt{200}$

3) $\sqrt{2}(x + 3\sqrt{2}) = 18$

4) $3 - 2\sqrt{7} \cdot x = -5$

b) Find the midpoint of $(2,\ 4\sqrt{3})$ and $(8,\ 6\sqrt{12})$

c) Find the perimeter and area of the triangle:



d) Simplify the expressions

1) $\frac{\sqrt{6}}{2} + \frac{3\sqrt{6}}{8}$

2) $\sqrt{\frac{3}{2}} + 3\sqrt{\frac{1}{6}}$

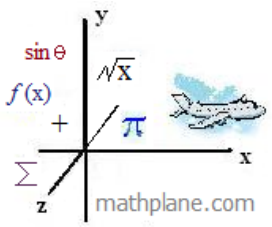
3) $\frac{\sqrt{5} - 1}{2 + 3\sqrt{10}}$

4) $\frac{\sqrt{6} + 2\sqrt{8}}{5\sqrt{2} + 3\sqrt{10}}$

5) $\frac{2\sqrt{3} + 4\sqrt{5}}{6\sqrt{7} - 8\sqrt{9}}$

6) $\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{3}}$

$\sqrt{\text{Radicals Quiz}}$



I. Simplify

a) $\sqrt{125}$

b) $\sqrt{56b^2}$

c) $\sqrt{68}$

d) $\sqrt{128ab^3}$

e) $\sqrt{99}$

II. True or False?

a) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$

c) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

III. Combine the terms

a) $\sqrt{5} + 3\sqrt{25} + 5\sqrt{125}$

b) $\sqrt{2} + \sqrt{4} + \sqrt{8} + \sqrt{16}$

c) $2\sqrt{49} - (\sqrt{64} + 14)$

Radicals Quiz (continued)

IV. Miscellaneous

a) List all perfect squares < 150

b) $3\sqrt{7} + 2\sqrt{28} - \sqrt{162} - \sqrt{2} =$

c) $3\sqrt{3} \cdot 6\sqrt{3} =$

d) $3\sqrt{3} + 6\sqrt{3} =$

V. Simplify (and, if necessary, rationalize the denominator)

a) $\sqrt{\frac{44}{144}}$

b) $\frac{(3\sqrt{7} + 8\sqrt{7})}{22}$

c) $\frac{3}{\sqrt{3}}$

d) $\frac{16}{\sqrt{17}}$

e) $\frac{3\sqrt{7} \cdot 8\sqrt{7}}{\sqrt{2} \cdot \sqrt{8}}$

f) $\sqrt{2}\,(3\sqrt{3} + 2\sqrt{2})$

Radicals Quick Quiz

Simplify:

1) $\sqrt{52}$

2) $\sqrt{\frac{80}{90}}$

3) $\frac{\sqrt{225}}{5}$

4) $3\sqrt{8} + 7\sqrt{2}$

5) $4\sqrt{3}\,(\sqrt{48} - \sqrt{3})$

Solve:

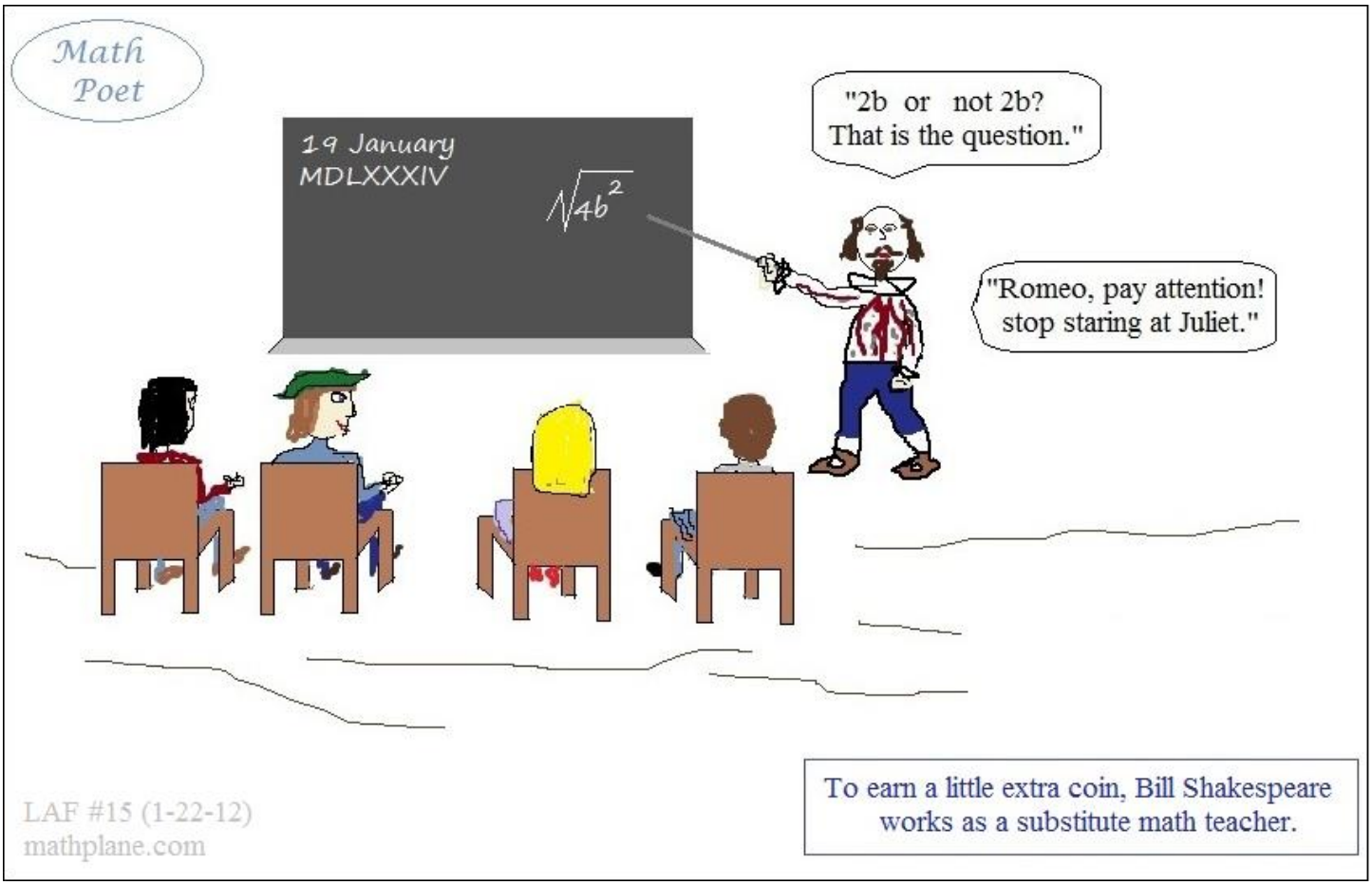
6) $\sqrt{4x - 27} - 1 = 4$

7) $\sqrt{x + 2} = x$

8) $\sqrt{x + 7} + 5 = x$

Graph:

9) $y = 3\sqrt{x + 2}$


Math Poet
19 January
MDLXXXIV
$\sqrt{4b^2}$
"2b or not 2b? That is the question."
"Romeo, pay attention! stop staring at Juliet."
LAF #15 (1-22-12)
mathplane.com
To earn a little extra coin, Bill Shakespeare works as a substitute math teacher.

ANSWERS-→

Square Root Exercises

SOLUTIONS

I. Simplifying

a) $\sqrt{16}$

4

Note: 4 x 4 = 16

b) $\sqrt{50}$

$\sqrt{2 \cdot 25} = \sqrt{2} \cdot \sqrt{25}$

$5\sqrt{2}$

c) $\sqrt{27}$

$\sqrt{3 \cdot 9} = \sqrt{3} \cdot \sqrt{9}$

$3\sqrt{3}$

d) $\sqrt{98}$

$\sqrt{2 \cdot 49} = \sqrt{2} \cdot \sqrt{49}$

$7\sqrt{2}$

e) $\sqrt{63}$

$\sqrt{9 \cdot 7} = \sqrt{9} \cdot \sqrt{7}$

$3\sqrt{7}$

f) $\sqrt{1100}$

$\sqrt{11 \cdot 100} = \sqrt{11} \cdot \sqrt{100}$

$10\sqrt{11}$

g) $\sqrt{52}$

$\sqrt{2 \cdot 2 \cdot 13}$

$2\sqrt{13}$

h) $\sqrt{72}$

$\sqrt{2 \cdot 36} = \sqrt{2} \cdot \sqrt{36}$

$6\sqrt{2}$

i) $5\sqrt{8}$

$5\sqrt{2 \cdot 2 \cdot 2} = 5 \cdot 2\sqrt{2}$

$10\sqrt{2}$

j) $7\sqrt{28}$

$7\sqrt{2 \cdot 2 \cdot 7}$

$7 \cdot 2\sqrt{7}$

$14\sqrt{7}$

II. Addition/Subtraction

a) $3\sqrt{2} + 4\sqrt{2}$

$7\sqrt{2}$

Note: $3x + 4x = 7x$

b) $7\sqrt{3} - \sqrt{3}$

$6\sqrt{3}$

It's acceptable to add/subtract "like radicals"

c) $\sqrt{20} + \sqrt{45}$

simplify

$2\sqrt{5} + 3\sqrt{5}$

add like terms..

$5\sqrt{5}$

d) $\sqrt{24} + \sqrt{54}$

$2\sqrt{6} + 3\sqrt{6}$

$5\sqrt{6}$

e) $3\sqrt{8} + 7\sqrt{2}$

$3 \cdot 2\sqrt{2} + 7\sqrt{2}$

$6\sqrt{2} + 7\sqrt{2}$

$13\sqrt{2}$

f) $4\sqrt{5} + 3\sqrt{28}$

$4\sqrt{5} + 3 \cdot 2\sqrt{7}$

$4\sqrt{5} + 6\sqrt{7}$

cannot combine these terms

g) $11\sqrt{10} - 2\sqrt{300}$

$11\sqrt{10} - 2 \cdot 10\sqrt{3}$

$11\sqrt{10} - 20\sqrt{3}$

h) $2\sqrt{75} + 3\sqrt{300}$

$2 \cdot 5\sqrt{3} + 3 \cdot 10\sqrt{3}$

$10\sqrt{3} + 30\sqrt{3}$

$40\sqrt{3}$

III. Multiplication/Division

a) $\sqrt{6} \cdot \sqrt{12}$

$\sqrt{72} = 6\sqrt{2}$

or,

$\sqrt{6} \cdot \sqrt{6 \cdot 2} = 6\sqrt{2}$

b) $2\sqrt{3} \cdot \sqrt{18}$

$2\sqrt{3} \cdot 3\sqrt{2}$

multiply the numbers and the radicals separately

$6\sqrt{6}$

c) $4\sqrt{6} \cdot 7\sqrt{2}$

$28\sqrt{12}$

$28 \cdot 2\sqrt{3}$

$56\sqrt{3}$

d) $\dfrac{\sqrt{72}}{3}$

$\dfrac{6\sqrt{2}}{3}$

$2\sqrt{2}$

e) $\sqrt{\dfrac{200}{63}}$

$\dfrac{\sqrt{200}}{\sqrt{63}} = \dfrac{10\sqrt{2}}{3\sqrt{7}}$

f) $\sqrt{\dfrac{80}{90}}$

$\sqrt{\dfrac{8}{9}} = \dfrac{2\sqrt{2}}{3}$

g) $\sqrt{5}\,(\sqrt{15} + \sqrt{60})$

$\sqrt{5} \cdot 3\sqrt{15}$

$15\sqrt{3}$

h) $4\sqrt{3}\,(\sqrt{48} - \sqrt{3})$

$4\sqrt{3}\,(4\sqrt{3} - \sqrt{3})$

$4\sqrt{3} \cdot 3\sqrt{3}$

$12 \cdot 3 = 36$

i) $\sqrt{29.6} \cdot \sqrt{29.6}$

29.6

if you multiply a square root by itself, you get the number!

j) $\frac{1}{3}(\sqrt{5} + \sqrt{125})$

$\frac{1}{3}(\sqrt{5} + 5\sqrt{5})$

$\frac{1}{3}(6\sqrt{5}) = 2\sqrt{5}$

k) $\dfrac{9\sqrt{6} \cdot 2\sqrt{2}}{4\sqrt{12} \cdot 7\sqrt{3}}$

$\dfrac{18\sqrt{12}}{28\sqrt{36}} = \dfrac{9\sqrt{1}}{14\sqrt{3}} = \dfrac{9\sqrt{3}}{42} = \dfrac{3\sqrt{3}}{14}$

IV. Additional Questions

a) Find x

1) $x\sqrt{3} + 4x\sqrt{12} = 8\sqrt{48}$

$x\sqrt{3} + 8x\sqrt{3} = 32\sqrt{3}$

$9x\sqrt{3} = 32\sqrt{3}$

$x = \boxed{\frac{32}{9}}$

2) $x\sqrt{50} + 4\sqrt{2} = 8\sqrt{200}$

$5x\sqrt{2} + 4\sqrt{2} = 80\sqrt{2}$

$5x + 4 = 80$

$x = \boxed{\frac{76}{5}}$

3) $\sqrt{2}(x + 3\sqrt{2}) = 18$

$\sqrt{2}x + 6 = 18$

$\sqrt{2}x = 12$

$x = \boxed{6\sqrt{2}}$

4) $3 - 2\sqrt{7} \cdot x = -5$

$-2\sqrt{7}x = -8$

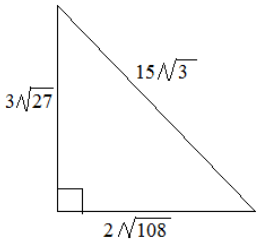
$\sqrt{7}x = 4$

$x = \boxed{\frac{4\sqrt{7}}{7}}$

b) Find the midpoint of $(2, 4\sqrt{3})$ and $(8, 6\sqrt{12})$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $\frac{2+8}{2} = 5$ $\frac{4\sqrt{3} + 12\sqrt{3}}{2} = 8\sqrt{3}$ $\boxed{(5, 8\sqrt{3})}$

c) Find the perimeter and area of the triangle:



Perimeter: $9\sqrt{3} + 15\sqrt{3} + 12\sqrt{3}$

$= \boxed{36\sqrt{3}}$

Area: $\frac{1}{2}$(base)(height)

$\frac{1}{2}(12\sqrt{3})(9\sqrt{3}) = (6\sqrt{3})(9\sqrt{3}) = 54 \cdot 3 = \boxed{162}$

d) Simplify the expressions

1) $\frac{\sqrt{6}}{2} + \frac{3\sqrt{6}}{8}$

$\frac{4\sqrt{6}}{8} + \frac{3\sqrt{6}}{8}$

$\boxed{\frac{7\sqrt{6}}{8}}$

2) $\sqrt{\frac{3}{2}} + 3\sqrt{\frac{1}{6}}$

$\frac{3}{\sqrt{6}} + \frac{3}{\sqrt{6}}$

$\frac{6}{\sqrt{6}} = \boxed{\sqrt{6}}$

3) $\frac{\sqrt{5} - 1}{2 + 3\sqrt{10}} \cdot \frac{2 - 3\sqrt{10}}{2 - 3\sqrt{10}}$

$\frac{2\sqrt{5} - 2 - 15\sqrt{2} + 3\sqrt{10}}{4 - 90}$

$\boxed{\frac{-2\sqrt{5} + 2 + 15\sqrt{2} - 3\sqrt{10}}{86}}$

4) $\frac{\sqrt{6} + 2\sqrt{8}}{5\sqrt{2} + 3\sqrt{10}} \cdot \frac{5\sqrt{2} - 3\sqrt{10}}{5\sqrt{2} - 3\sqrt{10}}$

$\frac{5\sqrt{12} - 3\sqrt{60} + 10\sqrt{16} - 6\sqrt{80}}{50 - 90}$

$\boxed{\frac{10\sqrt{3} - 6\sqrt{15} + 40 - 24\sqrt{5}}{-40}}$

5) $\frac{2\sqrt{3} + 4\sqrt{5}}{6\sqrt{7} - 8\sqrt{9}} \cdot \frac{6\sqrt{7} + 24}{6\sqrt{7} + 24}$

$\frac{12\sqrt{21} + 48\sqrt{3} + 24\sqrt{35} + 96\sqrt{5}}{252 - 576}$

$\boxed{\frac{12\sqrt{21} + 48\sqrt{3} + 24\sqrt{35} + 96\sqrt{5}}{-324}}$

6) $\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{3}}$

$\frac{3\sqrt{3}}{\sqrt{6}} + \frac{5\sqrt{2}}{\sqrt{6}}$

$\frac{3\sqrt{3} + 5\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$\boxed{\frac{9\sqrt{2} + 10\sqrt{3}}{6}}$

$\sqrt{\text{Radicals Quiz}}$

SOLUTIONS

I. Simplify

a) $\sqrt{125}$ $\quad \sqrt{5 \cdot 25} = 5\sqrt{5}$

b) $\sqrt{56b^2}$ $\quad \sqrt{4 \cdot 2 \cdot 7 \cdot b \cdot b} = 2|b|\sqrt{14}$

c) $\sqrt{68}$ $\quad \sqrt{2 \cdot 2 \cdot 17} = 2\sqrt{17}$

d) $\sqrt{128ab^3}$ $\quad \sqrt{2 \cdot 64 \cdot a \cdot b \cdot b^2} = 8b\sqrt{2ab}$ (note: b cannot be negative)

factor — remove perfect squares

e) $\sqrt{99}$ $\quad \sqrt{9 \cdot 11} = 3\sqrt{11}$

II. True or False?

a) $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ False... EX: $a = 4 \quad b = 16$ (an exception: $a = b = 0$)

$\sqrt{20} \neq \sqrt{4} + \sqrt{16}$

b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ True...

c) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ True...

III. Combine the terms

a) $\sqrt{5} + 3\sqrt{25} + 5\sqrt{125}$ $\quad \sqrt{5} + 3\sqrt{5 \cdot 5} + 5\sqrt{5 \cdot 25} =$

$1\sqrt{5} + 3 \cdot 5 + 25\sqrt{5} = \boxed{15 + 26\sqrt{5}}$

b) $\sqrt{2} + \sqrt{4} + \sqrt{8} + \sqrt{16}$ $\quad \sqrt{2} + 2 + 2\sqrt{2} + 4 = \boxed{6 + 3\sqrt{2}}$

c) $2\sqrt{49} - (\sqrt{64} + 14)$ $\quad 2 \cdot 7 - (8 + 14) = 14 - 22$

$\boxed{= -8}$

Solutions

Radicals Quiz (continued)

IV. Miscellaneous

a) List all perfect squares < 150 1, 2, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144

b) $3\sqrt{7} + 2\sqrt{28} - \sqrt{162} - \sqrt{2} =$ $3\sqrt{7} + 4\sqrt{7} - 9\sqrt{2} - \sqrt{2} = \boxed{7\sqrt{7} - 10\sqrt{2}}$

c) $3\sqrt{3} \cdot 6\sqrt{3} =$ $3 \cdot 6 \cdot \sqrt{3} \cdot \sqrt{3} \boxed{= 54}$

d) $3\sqrt{3} + 6\sqrt{3} = \boxed{9\sqrt{3}}$

V. Simplify (and, if necessary, rationalize the denominator)

a) $\sqrt{\frac{44}{144}}$ $\frac{2\sqrt{11}}{12} = \boxed{\frac{\sqrt{11}}{6}}$

b) $\frac{(3\sqrt{7} + 8\sqrt{7})}{22}$ $\frac{11\sqrt{7}}{22} = \boxed{\frac{\sqrt{7}}{2}}$

c) $\frac{3}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{\sqrt{3}}\right) =$ $\frac{3\sqrt{3}}{3} \boxed{= \sqrt{3}}$

d) $\frac{16}{\sqrt{17}} \cdot \left(\frac{\sqrt{17}}{\sqrt{17}}\right) = \boxed{\frac{16\sqrt{17}}{17}}$

e) $\frac{3\sqrt{7} \cdot 8\sqrt{7}}{\sqrt{2} \cdot \sqrt{8}}$ $\frac{24 \cdot 7}{\sqrt{16}} = \boxed{42}$

f) $\sqrt{2}\,(3\sqrt{3} + 2\sqrt{2})$

distribute: $3\sqrt{6} + 2\sqrt{4} = \boxed{4 + 3\sqrt{6}}$

Radicals Quick Quiz

Simplify:

ANSWERS

1) $\sqrt{52}$

$\sqrt{2 \cdot 2 \cdot 13}$ $\boxed{2\sqrt{13}}$

2) $\sqrt{\frac{80}{90}}$

$\sqrt{\frac{8}{9}} = \boxed{\frac{2\sqrt{2}}{3}}$

3) $\frac{\sqrt{225}}{5}$

$\frac{15}{5} = \boxed{3}$

4) $3\sqrt{8} + 7\sqrt{2}$

$3 \cdot 2\sqrt{2} + 7\sqrt{2}$

$6\sqrt{2} + 7\sqrt{2}$

$\boxed{13\sqrt{2}}$

5) $4\sqrt{3}\,(\sqrt{48} - \sqrt{3})$

$4\sqrt{3}\,(4\sqrt{3} - \sqrt{3})$

$4\sqrt{3} \cdot 3\sqrt{3}$

$12 \cdot 3 = \boxed{36}$

Solve:

6) $\sqrt{4x - 27} - 1 = 4$

$\sqrt{4x - 27} = 5$

(square both sides)

$4x - 27 = 25$

$4x = 52$

$\boxed{x = 13}$

7) $\sqrt{x + 2} = x$

$x + 2 = x^2$

$x^2 - x - 2 = 0$ $\boxed{x = 2}$

$(x - 2)(x + 1) = 0$

$x = \cancel{-1}, 2$ (check answers)

$\sqrt{(-1) + 2} = (-1)$ NO (extraneous)

$\sqrt{(2) + 2} = (2)$ YES

8) $\sqrt{x + 7} + 5 = x$

(isolate radical)

$\sqrt{x + 7} = x - 5$

(square both sides)

$x + 7 = x^2 - 10x + 25$

$x^2 - 11x + 18 = 0$

$(x - 2)(x - 9) = 0$

$x = \cancel{2}, 9$ $\boxed{x = 9}$

(check answers)

$\sqrt{(2) + 7} + 5 = (2)$

$3 + 5 \neq 2$

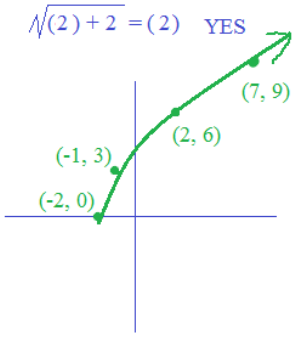
$\sqrt{(9) + 7} + 5 = (9)$

$4 + 5 = 9$

Graph:

9) $y = 3\sqrt{x + 2}$

x	$\sqrt{x}$		
-2	0	0	0
-1	1	1	3
2	4	2	6
7	9	3	9
14	16	4	12



Example: A circle is centered at the origin.

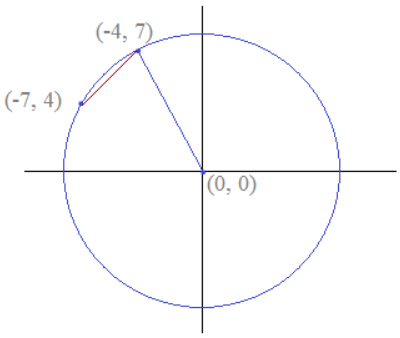
If point P (-4, 7) lies on the circle, what is the length of the radius?

To find the length of the radius, we use the distance formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_2 - y_2)^2}$$

radius (distance) = $\sqrt{(-4 - 0)^2 + (7 - 0)^2} = \sqrt{16 + 49} = \sqrt{65}$

approx. 8.06



If point Q (-7, 4) lies on the circle, what is the length of chord $\overline{PQ}$?

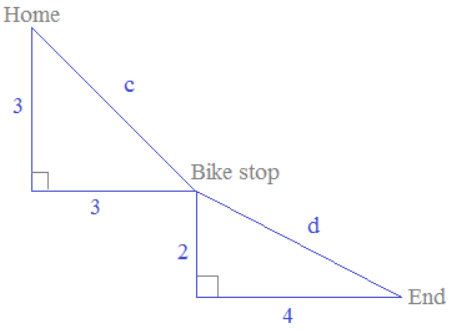
chord (distance) = $\sqrt{(-4 - -7)^2 + (7 - 4)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

approx. 4.24

Example: A boy leaves home and rides his bike 3 miles due South. Then, he turns and rides 3 miles due East. He stops, leaves his bike, and runs 2 miles due South. Then, he turns and runs 4 miles due East. On the return home, he runs directly to his bike. Then, he rides directly home. What is the total distance traveled?

Since the boy's path creates 2 right triangles, we can use the Pythagorean Theorem to get the unknown distances.

Pythagorean Theorem: $a^2 + b^2 = c^2$



$3^2 + 3^2 = c^2$

$18 = c^2$

$\sqrt{c^2} = \sqrt{18}$

$c = \sqrt{18} = 3\sqrt{2}$

$2^2 + 4^2 = d^2$

$20 = d^2$

$d = \sqrt{20} = 2\sqrt{5}$

Total distance traveled: $3 + 3 + 2 + 4 + 2\sqrt{5} + 3\sqrt{2}$

these radicals cannot be combined b/c not like terms

$= 12 + 2\sqrt{5} + 3\sqrt{2}$ or approx. 20.71

Example: The path of a punted football has the following function:

$$h(d) = -.05d^2 + 2d + 1$$

where d is the horizontal distance traveled (in yards) and $h(d)$ is the height.

How far does the football travel before it hits the ground?

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The graph of this function is an upside down parabola. Since the x-axis (or d-axis) represents ground height of 0, we need to find the d-intercepts...

$0 = -.05d^2 + 2d + 1$

a = -.05
b = 2
c = 1

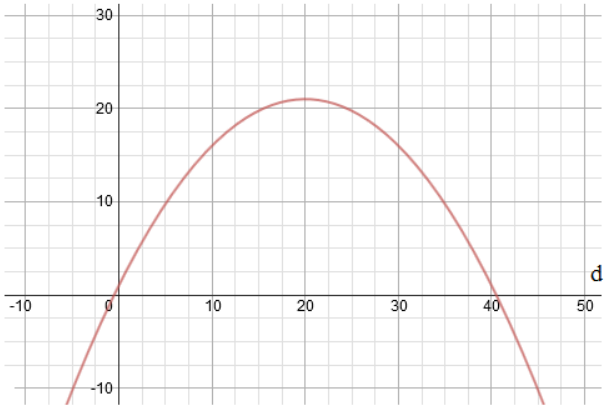
$$d = \frac{-2 \pm \sqrt{(2)^2 - 4(-.05)(1)}}{2(-.05)} = \frac{-2 \pm \sqrt{4.2}}{-.1}$$

multiply by -10/-10

$$\frac{20 \mp 10\sqrt{4.2}}{1}$$

(Since the football presumably went forward, we can eliminate the negative answer)

$d = 20 + 10\sqrt{4.2}$ or approx. 40.49



Finding square roots of numbers that aren't perfect squares (without a calculator)

1) Estimate - Get close by finding 2 perfect squares that your number is between.

2) Divide - Divide your number by one of those square roots.

3) Average - Take the average of the result and the root.

4) Repeat - Use the result of step 3 to repeat steps 2 and 3, until you get a number accurate enough for you.

Example: Calculate the square root of 10 to two decimal places.

1) $3^2 = 9$
$4^2 = 16$
So, $\sqrt{10}$ will be between 3 and 4

2) Since 10 is closer to 9, we'll use the square root of 9.

10 divided by 3 = $3.33\overline{3}$

3) Find the average of 3.000 and $3.33\overline{3}$

$(3.333 + 3)/2 = 3.1667$

4) (repeat step 2)
10 divided by 3.1667 = 3.1579

(repeat step 3)
$(3.1579 + 3.1667)/2 = 3.1623$

Check the answer: 3.1623 x 3.1623 = 10.0001

Example: Calculate $\sqrt{71}$ (without a calculator)

1) 64 and 81 are perfect squares near 71.

2) $\sqrt{64} = 8$ $\quad \frac{71}{8} = 8.875$

3) Average of 8 & 8.875 is $\frac{(8 + 8.875)}{2} = 8.4375$ $\quad$ 8.4375 x 8.4375 = 71.1914

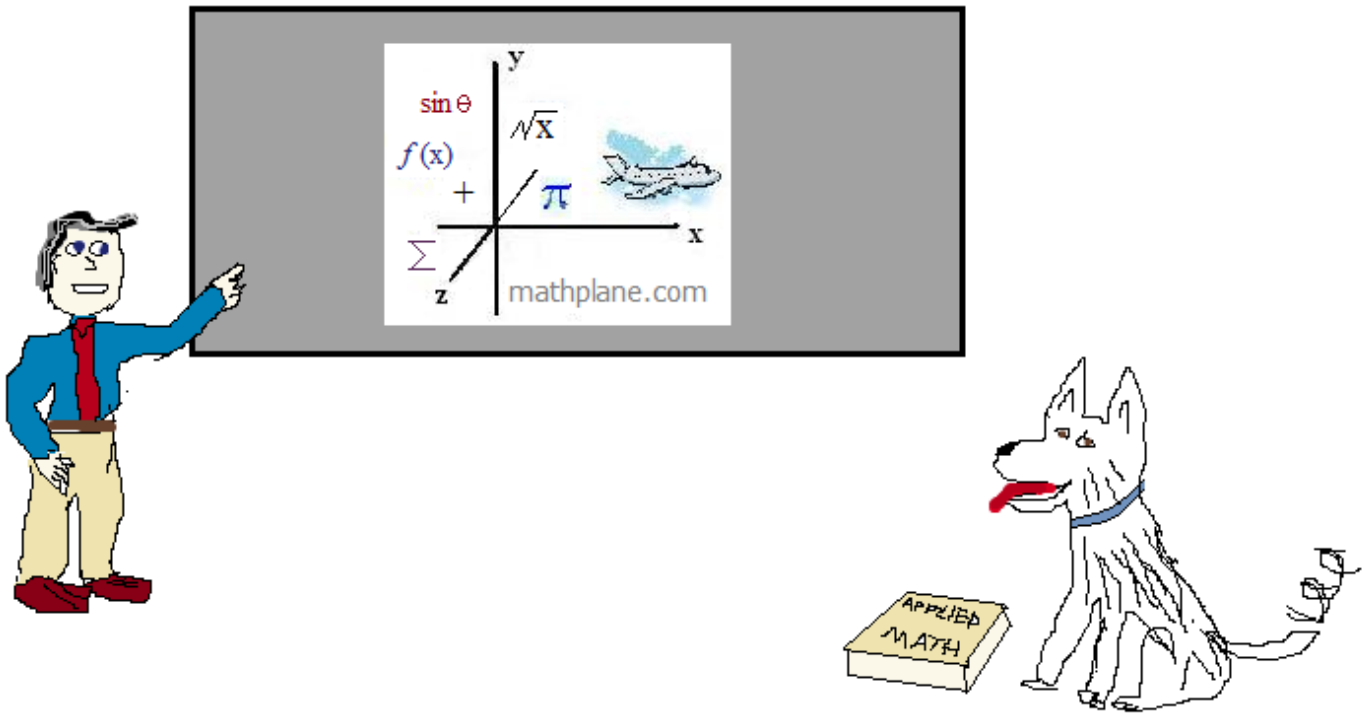
4) (repeat) $\frac{71}{8.4375} = 8.4148$

Average of 8.4148 & 8.4375 is 8.42615 $\quad$ 8.4262 x 8.4262 = 71.0008

Thanks for visiting. (Hope this quiz helped!)

If you have questions, suggestions, or requests, let us know.

Cheers,



Also, at TeachersPayTeachers and mathplane.ORG

Two more questions:

Simplify:

$$\sqrt{45} + 2\sqrt{20} + \frac{1}{2}\sqrt{500}$$

Simplify

$$\frac{5\sqrt{6}}{2\sqrt{3}}$$

Solution on next page…

Simplify:

$$\sqrt{45} + 2\sqrt{20} + \frac{1}{2}\sqrt{500}$$

$$\sqrt{5 \cdot 9} + 2\sqrt{5 \cdot 4} + \frac{1}{2}\sqrt{5 \cdot 100}$$

$$\sqrt{5} \cdot \sqrt{9} + 2 \cdot \sqrt{5} \cdot \sqrt{4} + \frac{1}{2} \cdot \sqrt{5} \cdot \sqrt{100}$$

$$\sqrt{5} \cdot 3 + 2 \cdot \sqrt{5} \cdot 2 + \frac{1}{2} \cdot \sqrt{5} \cdot 10$$

$$3\sqrt{5} + 4\sqrt{5} + 5\sqrt{5}$$

$$\boxed{12\sqrt{5}}$$

Simplify

$$\frac{5\sqrt{6}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} =$$

$$\frac{5\sqrt{18}}{2\sqrt{9}} =$$

$$\frac{5\sqrt{2 \cdot 9}}{2 \cdot 3} =$$

$$\frac{15\sqrt{2}}{6} =$$

$$\boxed{\frac{5\sqrt{2}}{2}}$$

Hidden Answer

Hint: "What is a square root?"

Letter Key:

0	1	2	3	4	5	6	7	8	9
D	I	C	A	S	L	E	R	M	T

1) $\sqrt[3]{27} - \sqrt[3]{8} =$ ☐ → ____

2) $\sqrt{16} + \sqrt{25} =$ ☐ → ____

3) $\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{27}}{\sqrt{45}} =$ ☐ → ____

4) $\sqrt{48} =$ ☐$\sqrt{3}$ → ____

5) $\sqrt{28} + \sqrt{7} =$ $3\sqrt{\square}$ → ____

6) $\frac{\sqrt{225}}{5} =$ ☐ → ____

7) $\sqrt{500} =$ 1☐$\sqrt{5}$ → ____

8) $\frac{\left(\sqrt{72} + \sqrt{2}\right)}{7\sqrt{2}} =$ ☐ → ____

9) $\frac{\left(\sqrt{24} \cdot 2\sqrt{6}\right)}{12} =$ ☐ → ____

10) $\frac{\left(\sqrt{16} + \sqrt{4}\right)}{\sqrt{4}} =$ ☐ → ____

11) The number of perfect squares between 5 & 50 ☐ → ____

SOLUTIONS-→

Hidden Answer

Hint: "What is a square root?"

Letter Key:

0	1	2	3	4	5	6	7	8	9
D	I	C	A	S	L	E	R	M	T

1) $\sqrt[3]{27} - \sqrt[3]{8} = 3 - 2 = 1$

2) $\sqrt{16} + \sqrt{25} = 4 + 5 = 9$

IT IS RADICAL

3) $\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{27}}{\sqrt{45}} = \frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{3\sqrt{3}}{3\sqrt{5}} = 1$

4) $\sqrt{48} = \sqrt{3 \cdot 16} = 4\sqrt{3}$

5) $\sqrt{28} + \sqrt{7} = \sqrt{4 \cdot 7} + 1\sqrt{7} = 3\sqrt{7}$

6) $\frac{\sqrt{225}}{5} = \frac{15}{5} = 3$

7) $\sqrt{500} = \sqrt{100 \cdot 5} = 10\sqrt{5}$

8) $\frac{(\sqrt{72} + \sqrt{2})}{7\sqrt{2}} = \frac{(6\sqrt{2} + \sqrt{2})}{7\sqrt{2}} = 1$

9) $\frac{(\sqrt{24} \cdot 2\sqrt{6})}{12} = \frac{(2\sqrt{6} \cdot 2\sqrt{6})}{12} = \frac{24}{12} = 2$

10) $\frac{(\sqrt{16} + \sqrt{4})}{\sqrt{4}} = \frac{(4+2)}{2} = 3$

11) The number of perfect squares between 5 & 50 ---> Five

Perfect squares: 1 4 [9 16 25 36 49] 64 81 ...

1	---->	I	
9	---->	T	
1	---->	I	
4 $\sqrt{3}$	---->	S	
3 $\sqrt{7}$	---->	R	
3	---->	A	
1 0 $\sqrt{5}$	---->	D	
1	---->	I	
2	---->	C	
3	---->	A	
5	---->	L	

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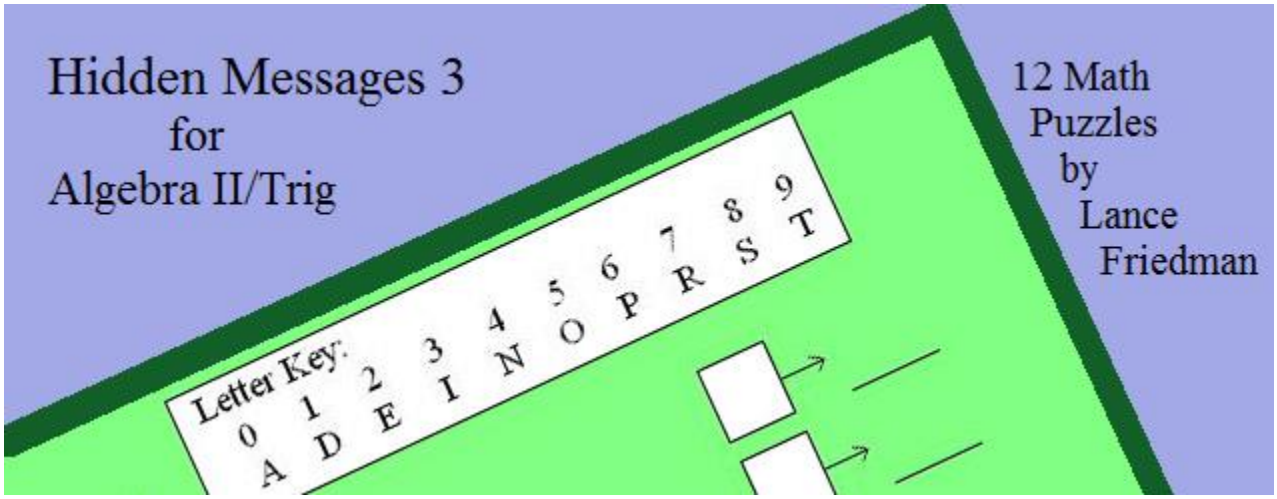

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