Medians

Altitudes

Parts & Centers of Triangles

Perpendicular Bisectors

Angle Bisectors

Includes definitions, illustrations, and notes...

And, practice test (& Solutions)

mathplane.com

Triangle: Median

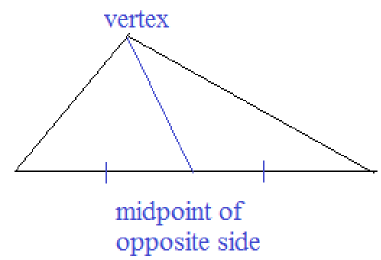
What is it?

A line segment from a vertex to the midpoint of the opposite side.

How to draw it:

--- Start at a *vertex*

--- Draw a line to the *midpoint* of the *opposite side*

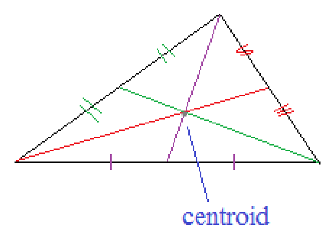


Notes:

--- Every triangle has 3 medians

--- The medians meet at a point inside the triangle (The "centroid")

--- The median *bisects the area* of the triangle



"Centroid 2/3 Theorem":

The centroid is the 'center of gravity'.
If a triangle were made of solid material, then it would balance on the centroid!

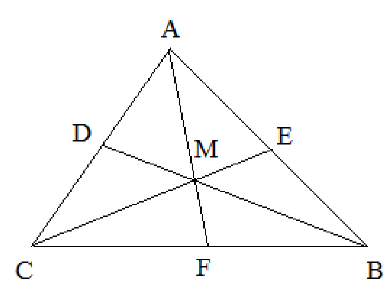
Example: The centroid "divides triangle ABC into three balanced triangles: AMC, AMB, CMB"

Point M is 2/3 distance from each vertex to the opposite side.

If $\overline{AF} = 12$, then $\overline{AM} = 8$

If $\overline{CM} = 7$, then $\overline{ME} = 3.5$

(and, CE = 10.5)



Triangle: Altitude

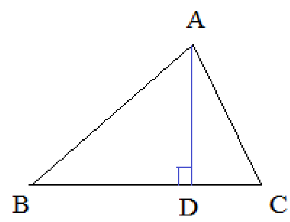
What is it?

A perpendicular line segment that connects a vertex to the (opposite) base.

How to draw it:

--- Start at a *vertex*

--- Drop a line *straight* to the *opposite side (base)*



$\overline{AD}$ is an altitude

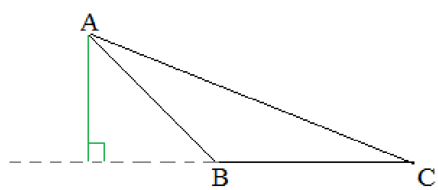
$\angle ADB = \angle ADC = 90^\circ$

Notes:

--- Every triangle has 3 altitudes

--- An *obtuse* triangle has an altitude that connects the vertex to an 'imaginary base'

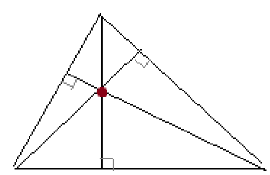
--- The altitudes are concurrent (meet at) a common point (The "orthocenter")



Obtuse triangle ABC

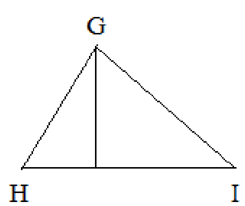
$\angle ABC > 90$

Altitude from A to base $\overline{BC}$ lies outside the triangle

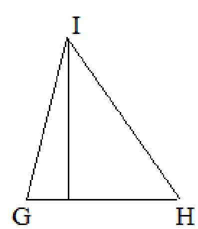


3 altitudes meet at the orthocenter

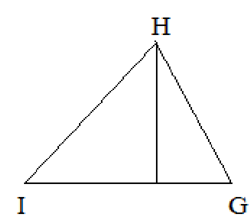
--- Altitude is the "height", depending on which side you consider the base



Triangle GHI
(Base HI)



Triangle GHI
(Base GH)



Triangle GHI
(Base GI)

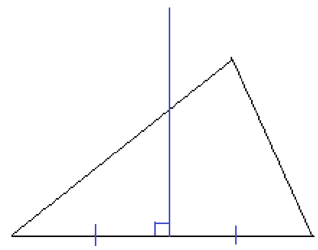
Triangle: Perpendicular Bisector

What is it?

A line, segment, or ray that is *perpendicular* to a triangle side *at the midpoint*.

How to draw it:

--- Start at a side
--- Go to the midpoint
--- Draw a perpendicular line (or segment or ray)

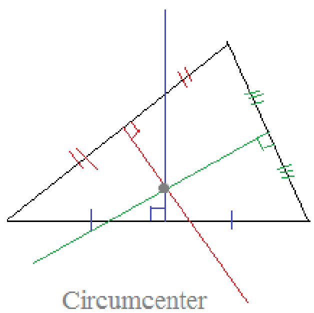


Perpendicular — 90 degree angle

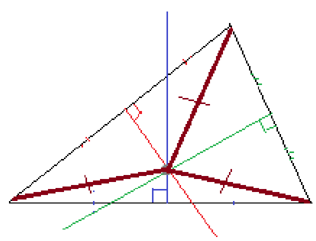
Bisector — 2 congruent segments

Notes:

--- Every triangle has 3 ⊥ bisectors
--- The 3 perpendicular bisectors are concurrent at a point in the middle (The "circumcenter")
--- The circumcenter is equidistant from the vertices of the triangle



Circumcenter



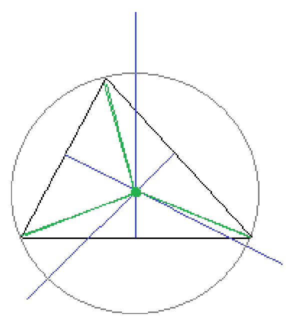
3 congruent segments meet at the circumcenter

"Circumcenter and Circumscribed Circle":

Construct 3 perpendicular bisectors

Connect the circumcenter to each vertex of the triangle (this creates 3 congruent segments)

**The circumcenter becomes the center of a circle. And, the 3 congruent segments are radii of the circle!!



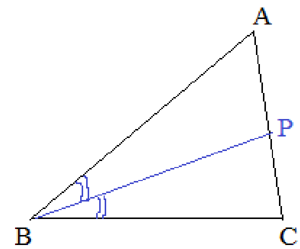
Triangle: Angle Bisector

What is it?

A line segment from the vertex that cuts that angle in half.

How to draw it:

--- Start at a vertex
--- Bisect that angle
--- Extend the line segment to the opposite side



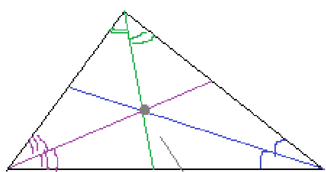
Angle Bisector

2 congruent angles

$\angle CBP = \angle ABP = (1/2)\angle ABC$

Notes:

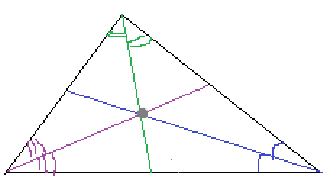
--- Every triangle has 3 angle bisectors
--- The three angle bisectors meet at a point inside the triangle (The "incenter")



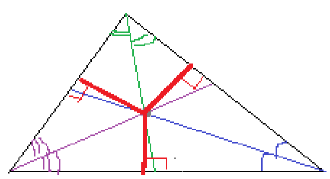
"Incenter and Inscribed Circle":

The incenter is equidistance from the 3 *sides* of the triangle.

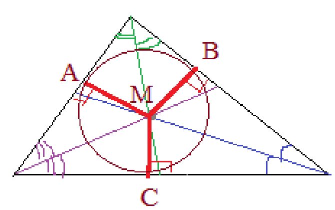
**The incenter is the center of the inscribed circle (in the triangle)



angle bisectors establish the incenter.



perpendicular line segments from sides to incenter (the segments are congruent!)



Inscribed circle

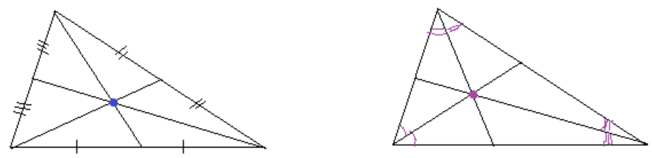
$\overline{AM} = \overline{BM} = \overline{CM}$

(can be verified by AAS)

Triangle Observations

1) In most cases, angle bisectors and medians meet at different points.
(i.e. the *centroid* and *incenter* are usually different points inside a triangle)

Example:



centroid   Incenter

(Note the slight difference is location)

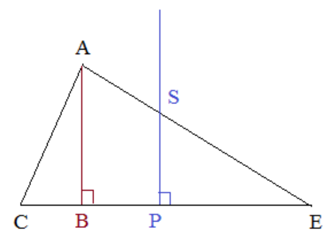
2) Altitudes and Perpendiculars are different.

--- Altitudes 'start at' the *vertex*
--- Perpendicular Bisectors 'start at' the *side*

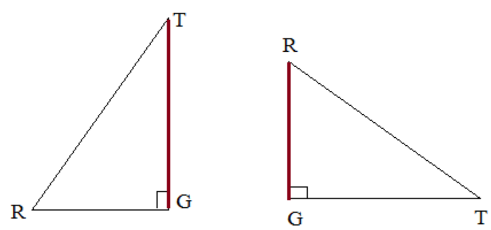
Altitude $\overline{AB}$

Perpendicular Bisector $\overline{PS}$
($\overline{PC} = \overline{PE}$)

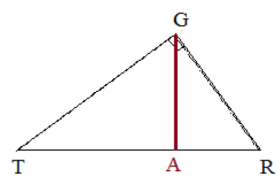
AB || PS
(AB is parallel to PS)



3) In a *right triangle*, the legs are 2 (of the 3) altitudes.



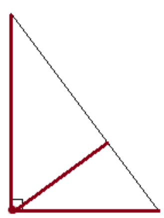
Right Triangle RGT is rotated.
$\overline{RG}$ and $\overline{GT}$ are the legs. $\overline{RT}$ is the hypotenuse.



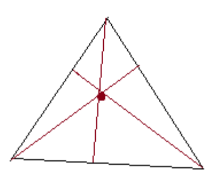
Altitudes of △RGT
$\overline{GT}$
$\overline{RG}$
$\overline{GA}$

4) Orthocenters may lie inside, outside, or on a triangle.

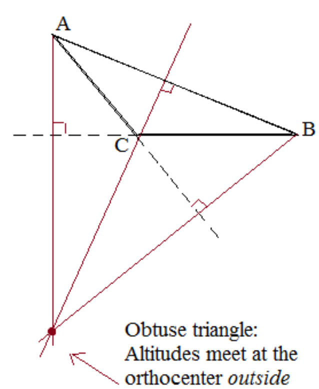
Examples:



Right triangle:
Altitudes meet at the vertex
opposite the hypotenuse.

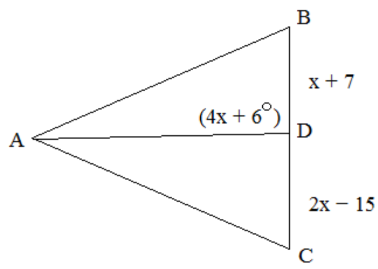


Acute triangle:
Orthocenter is *inside*
the triangle.



Obtuse triangle:
Altitudes meet at the
orthocenter *outside*
the triangle!

Example:



If $\overline{AD}$ is a median, what is the length of $\overline{BC}$?

If AD is a median, then BD = CD

x + 7 = 2x - 15
x = 22

BD = CD = 29

so, BC = 58

If $\overline{AD}$ is an altitude, what is the length of $\overline{BC}$?

If AD is an altitude, then ADB is a right angle...

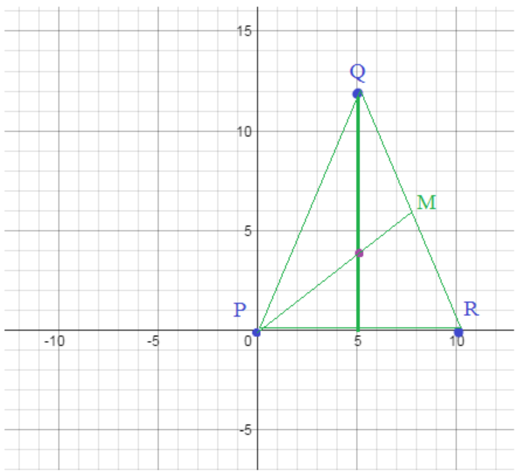
4x + 6 = 90
x = 21

BD = x + 7 = 28

CD = 2x - 15 = 27

So, BC = 55

Example:



P Q R
Points: (0, 0) (5, 12) (10, 0)

Find centroid of PQR

We can see it's an isosceles triangle with base PR...

So, one of the medians is a segment from (5,12) to (5, 0)

**since the centroid lies 2/3 of the way down the median, we know it's at (5, 4)

And, we can verify it... How?

Draw a second median from P to the midpoint of $\overline{QR}$...

midpoint of $\overline{QR}$ is (7.5, 6)

midpoint formula: $\left(\frac{x_1 + x_2}{2} \quad \frac{y_1 + y_2}{2}\right)$

The equation of line $\overline{PM}$ is:

slope: 6/7.5 or 4/5...
y-intercept: (0, 0)

y = (4/5)x

and, the intersection of the medians is

x = 5 and y = 4

median of a triangle:
a segment drawn from a vertex to the midpoint of the opposite side..

centroid:
the intersection of the 3 medians...

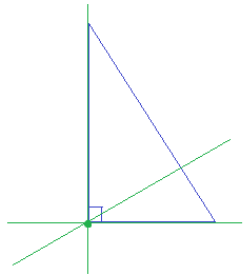
Example: Triangle ABC has vertices (0, 0), (9, 0), (0, 12)

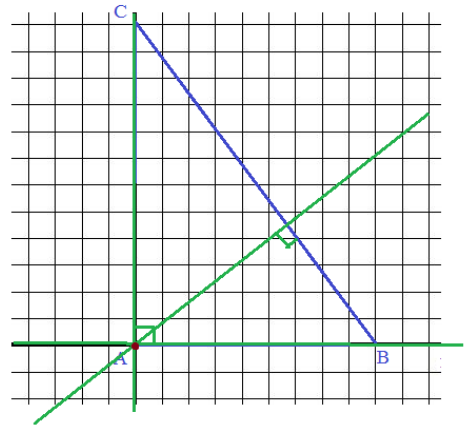
Find the coordinates of the

a) orthocenter

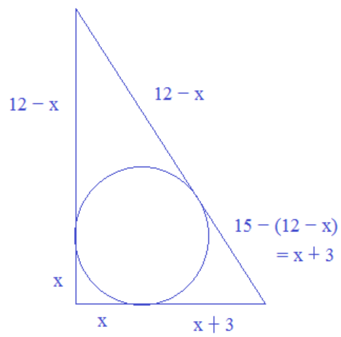
Orthocenter of a right triangle is at the vertex of the right angle!

(0, 0)





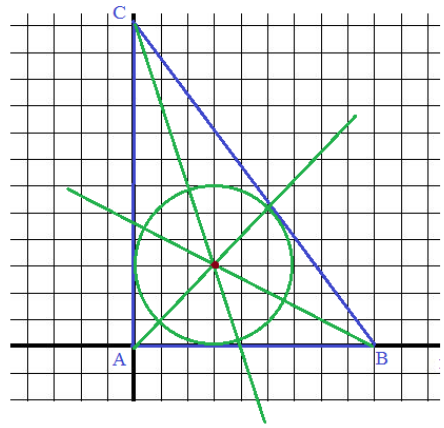
b) incenter



$2x + 3 = 9$

so, $x = 3$

Incenter: (3, 3)



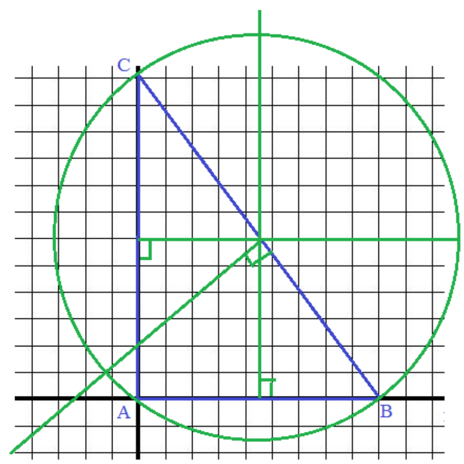
c) circumcenter

perpendicular bisectors:

AB: $x = 4.5$

AC: $y = 6$

Circumcenter: (4.5, 6)



d) centroid

median from C to $\overline{AB}$:

(0, 12) to (4.5, 0) (4.5, 0) is the midpoint of $\overline{AB}$

slope is -12/4.5 = -8/3 $y = (-8/3)x + 12$

median from A to $\overline{BC}$:

(0, 0) to (4.5, 6) (4.5, 6) is the midpoint of $\overline{BC}$

slope is 6/4.5 = 4/3 $y = (4/3)x$

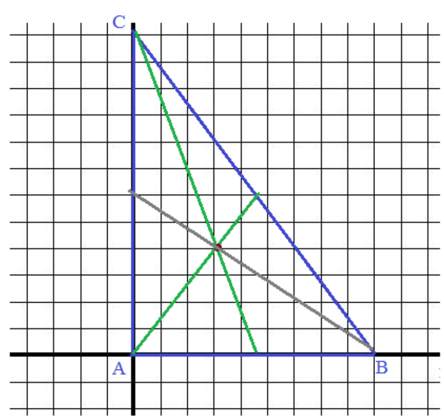
And, the intersection of the lines:

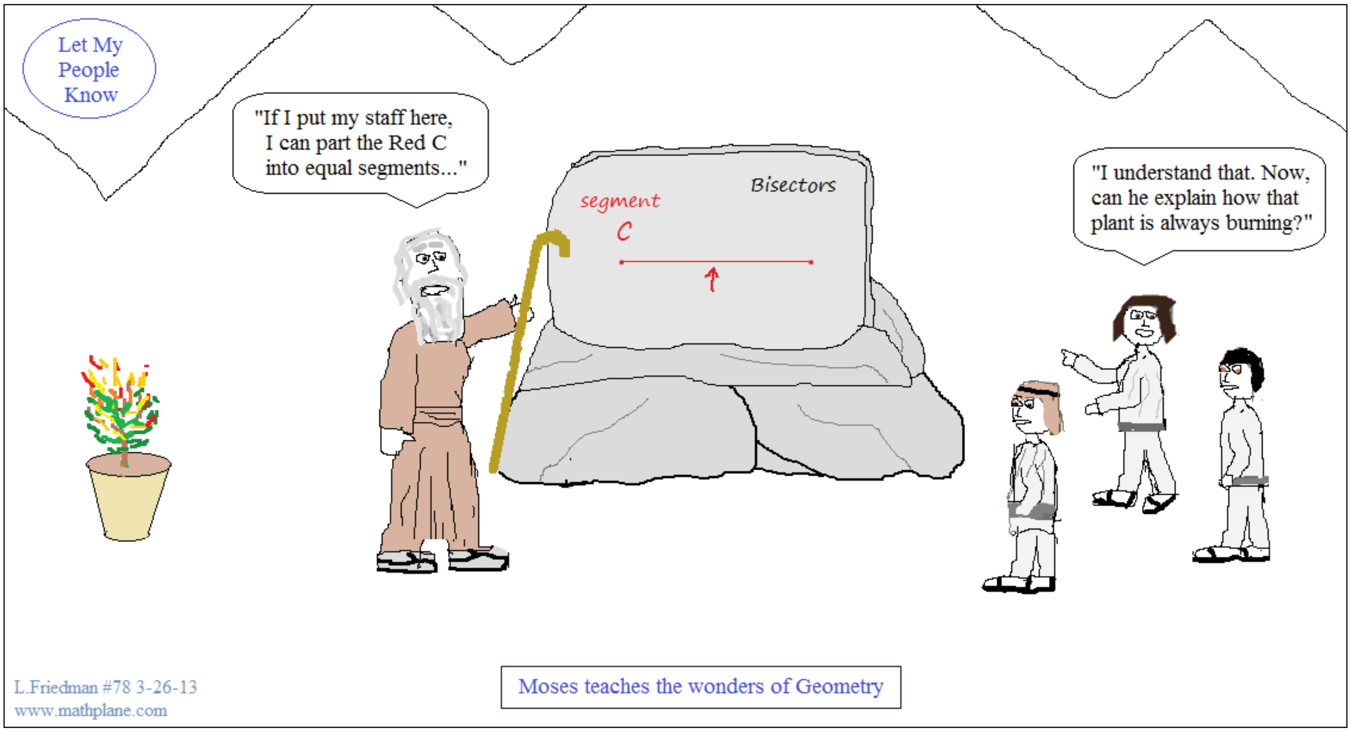
$(4/3)x = (-8/3)x + 12$

$(12/3)x = 12$

$x = 3 \quad y = 4$

centroid is at (3, 4)





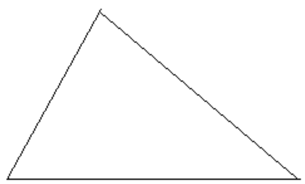
PRACTICE EXERCISES

Triangle Test: Median, Altitude, Perpendicular Bisector, and Angle Bisector

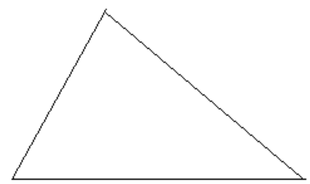
I. Identify the parts of the triangle

Draw the following:

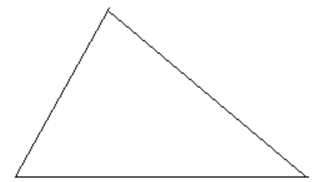
A) 3 medians



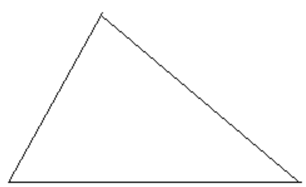
B) 3 angle bisectors



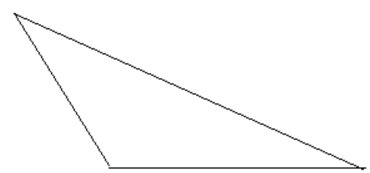
C) 3 perpendicular bisectors



D) 3 Altitudes



E) 3 Altitudes



II. Definitions and Concepts

Match each of the following geometry terms with the appropriate triangle points:

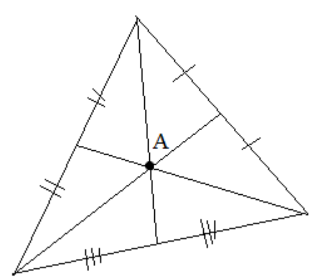
Incenter
Centroid
Orthocenter
Circumcenter

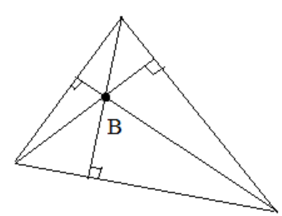
A)

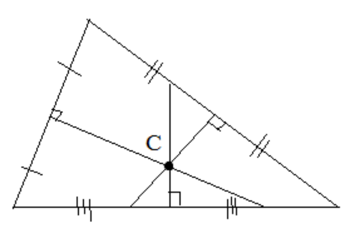
B)

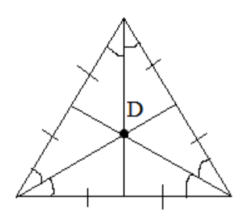
C)

D)





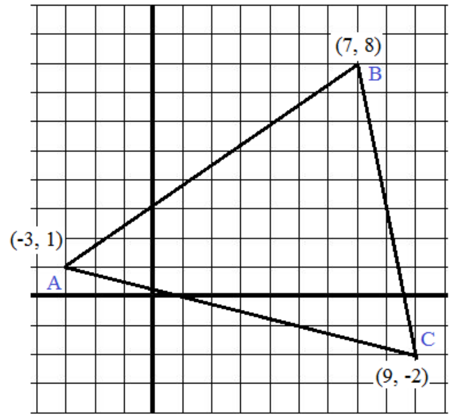




III. Geometry Applications

Find lines that include the following (from triangle ABC):

1) The median from A to $\overline{BC}$
(write as a linear equation in *point-slope form*)



2) The Altitude from B to $\overline{AC}$
(write as a linear equation in *standard form*)

3) The Perpendicular Bisector of $\overline{BC}$
(write the linear equation *slope intercept form*)

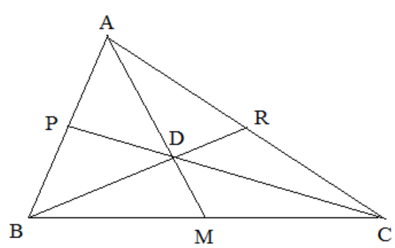
IV: Miscellaneous

1) Given: $\triangle ABC$ with medians $\overline{AM}$ $\overline{BR}$ $\overline{PC}$
$\overline{DM} = 4$ cm
area of $\triangle PBC = 52$ sq. cm

What is the area of triangle ABC?

What is the area of triangle ABR?

What is the length of $\overline{AM}$?



2) What type of triangle can have an identical median, perpendicular bisector, and altitude?

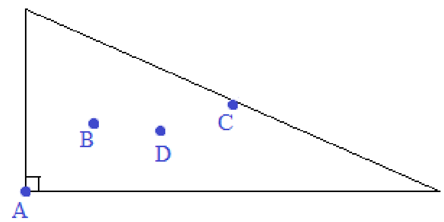
3) Draw a triangle where all 3 altitudes have identical lengths.

More Triangle Parts Questions....

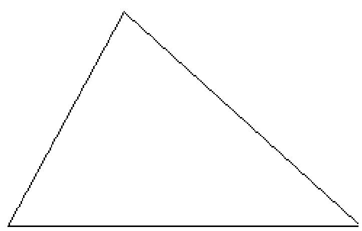
1) Where is the point of concurrency?

(Determine the point where the 3 lines intersect)

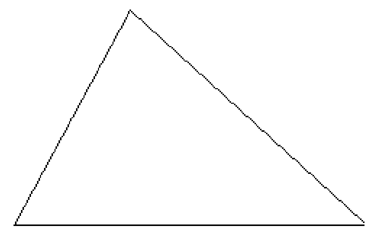
1. Perpendicular Bisectors _____
2. Medians _____
3. Altitudes _____
4. Angle Bisectors _____



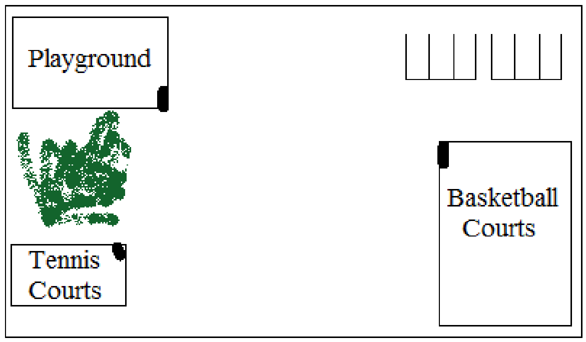
2) a) Inscribe a circle:



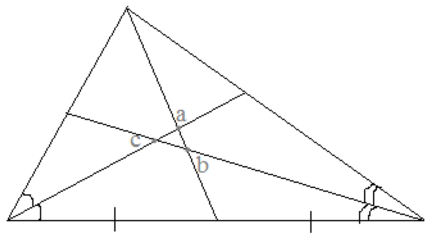
b) Circumscribe a circle:



3) The sketch is a diagram of a local park. (the entrances are marked). Where should they place a drinking fountain that is equal distance from the playground, tennis courts, and basketball courts?

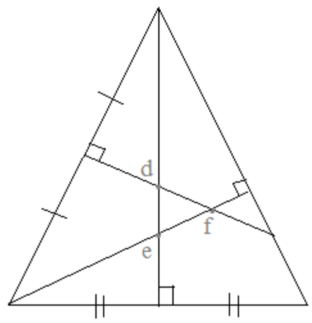


4A)



Which letter is the point of concurrency of the angle bisectors? (the incenter)

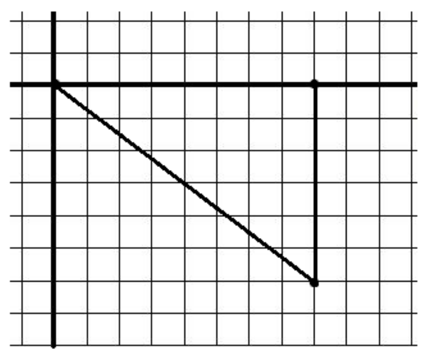
4B)



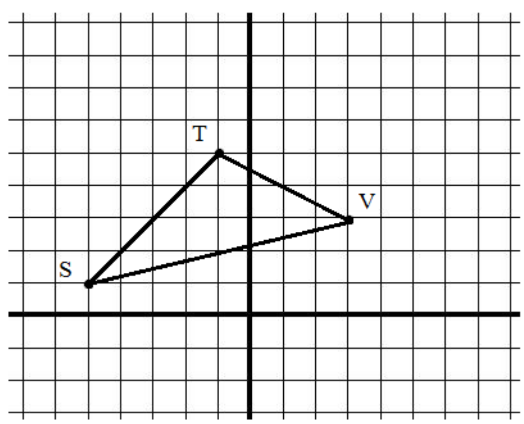
Which letter is the point of concurrency of the altitudes? (the orthocenter)

Which letter is the point of concurrency of the perpendicular bisectors? (the circumcenter)

5) Find the *center of a circle* that circumscribes a triangle with vertices (0, 0) (8, 0) and (8, -6)



6) Find the coordinates of the *centroid* C in △ STV where S (-5, 1) T (-1, 5) V (3, 3)



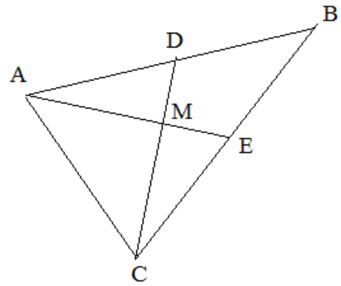
Triangle Test: Median, Altitude,
Perpendicular Bisector and Angle Bisector

7) $\overline{AE}$ and $\overline{CD}$ are medians.

$\overline{AE} = 12$

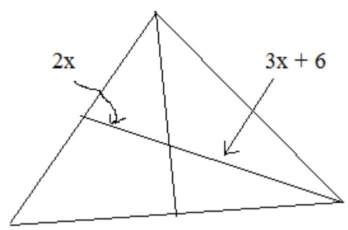
What is $\overline{ME}$?

$\overline{AM}$?



8) The diagram shows a triangle and its 2 medians.

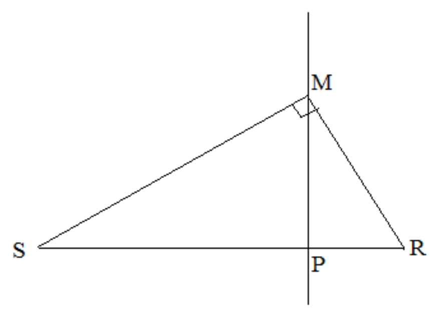
What is the length of the labeled median?



9) Given: Right triangle SMR with altitude $\overline{MP}$ and horizontal hypotenuse $\overline{SR}$

M: (3, 4) S: (-5, -1)

Find: Coordinate R



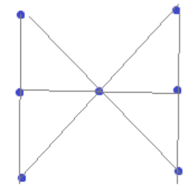
Applying properties of triangle parts and concurrency

Here are a few questions. Note the application of triangle parts!

Question #1

Arrange the 7 dots so that 6 (three dot) lines can be drawn...

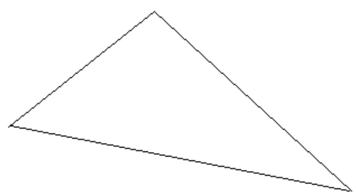
example of 5 lines...



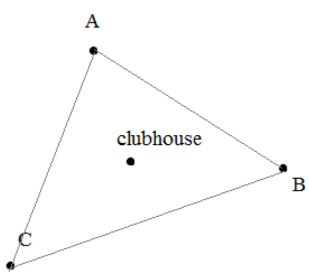
can you figure
out the arrangement
to get 6 straight lines?

Question #2

My birthday cake is a strange triangle shape...
If I have 5 friends at my party, how do I cut the cake so that everyone gets an equal amount?



Question #3



Which item would determine if the clubhouse is equidistant to locations A, B, and C?

a) incenter

b) circumcenter

c) centroid

d) orthocenter

Question #4

True or False? The point where a triangular table could balance on one leg is always the same distance from each side of the triangle...

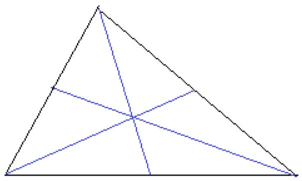
Triangle Test: Median, Altitude, Perpendicular Bisector, and Angle Bisector

SOLUTIONS

I. Identify the parts of the triangle

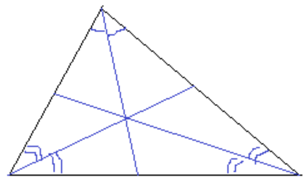
Draw the following:

A) 3 medians

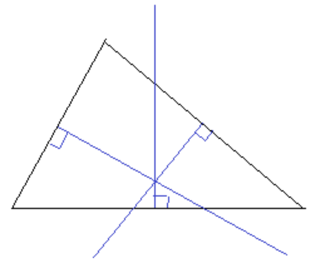


(vertex to midpoint of opposite side)

B) 3 angle bisectors

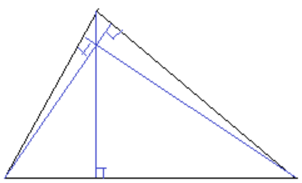


C) 3 perpendicular bisectors



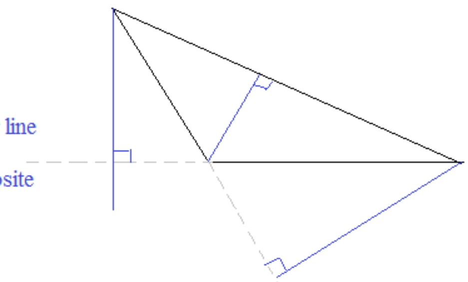
(extend from midpoint of each side)

D) 3 Altitudes



E) 3 Altitudes

(perpendicular line segment from vertex to opposite side)



II. Definitions and Concepts

Match each of the following geometry terms with the appropriate triangle points:

Incenter
Centroid
Orthocenter
Circumcenter

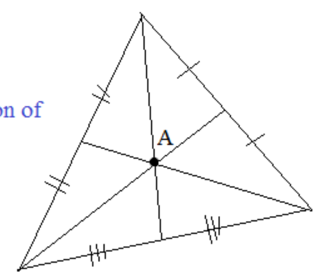
A) Centroid

B) Orthocenter

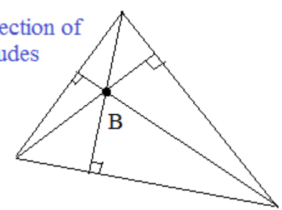
C) Circumcenter

D) Incenter, Centroid, Orthocenter, or Circumcenter

Intersection of 3 medians

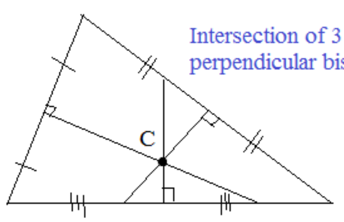


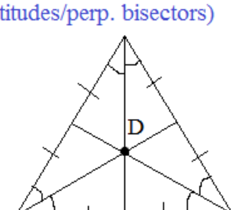
Intersection of 3 altitudes



intersection of medians AND angle bisectors (and altitudes/perp. bisectors)

Intersection of 3 perpendicular bisectors





(equilateral triangle)

Triangle Test: Median, Altitude, Perpendicular Bisector, and Angle Bisector

III. Geometry Applications

SOLUTIONS

Find lines that include the following (from triangle ABC):

1) The median from A to $\overline{BC}$
(write as a linear equation in *point-slope form*)

To express the equation of a line, we need the slope and a point:

Point: A -- (-3, 1)
Slope: the slope going through A and the midpoint of $\overline{BC}$

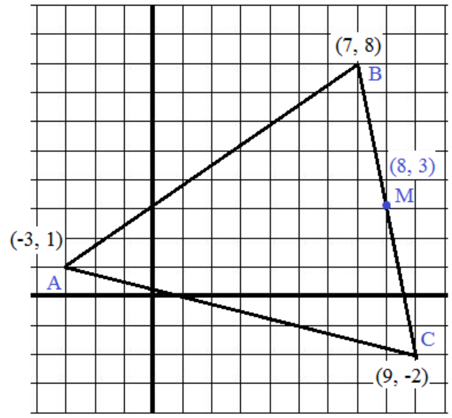
Midpoint M $= \left(\frac{7+9}{2}, \frac{8+(-2)}{2}\right) = (8, 3)$

Slope of line going through A and M: $\frac{3-1}{8-(-3)} = \frac{2}{11}$

$y - 1 = \frac{2}{11}(x + 3)$

or

$y - 3 = \frac{2}{11}(x - 8)$



2) The Altitude from B to $\overline{AC}$
(write as a linear equation in *standard form*)

We need a point and the slope...

Point: B (7, 8)
Slope: *perpendicular* to $\overline{AC}$

slope of $\overline{AC}$ is $\frac{1-(-2)}{-3-9} = \frac{3}{-12}$

slope of line perpendicular to $\overline{AC}$ is 4 (opposite reciprocal)

Altitude line:

$y - 8 = 4(x - 7)$

$y - 8 = 4x - 28$

$4x - y = 20$

3) The Perpendicular Bisector of $\overline{BC}$
(write the linear equation *slope intercept form*)

Need the midpoint of $\overline{BC}$
and the slope of a line perpendicular to $\overline{BC}$

Midpoint of $\overline{BC}$ = (8, 3) (found in question 1))

slope of $\overline{BC} = \frac{8-(-2)}{7-9} = -5$

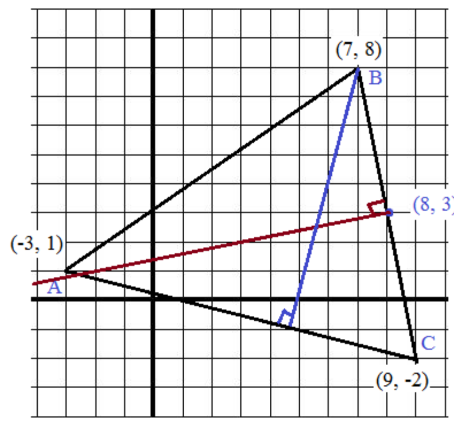
slope of line perpendicular to $\overline{BC}$ = 1/5

linear equation of perpendicular bisector:

y - 3 = 1/5(x - 8)

y - 3 = 1/5x - 8/5

y = 1/5x + 7/5



IV: Miscellaneous

1) Given: $\triangle$ABC with medians $\overline{AM}$ $\overline{BR}$ $\overline{PC}$
$\overline{DM}$ = 4 cm
area of $\triangle$PBC = 52 sq. cm

What is the area of triangle ABC?

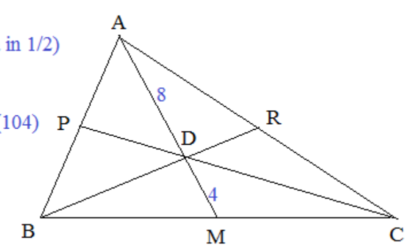
104 sq. cm (a median cuts a triangle's area in 1/2)

What is the area of triangle ABR?

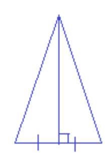
52 sq. cm (since ABC is 104, ABR is 1/2(104))

What is the length of $\overline{AM}$?

$\overline{DM}$ = 4 $\overline{AD}$ = 8
AD:DM is 2:1 12 cm
(AD is 2/3 of $\overline{AM}$)



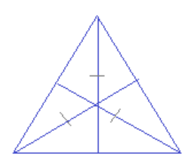
2) What type of triangle can have an identical median, perpendicular bisector, and altitude?



Isosceles triangle

3) Draw a triangle where all 3 altitudes have identical lengths.

equilateral triangle



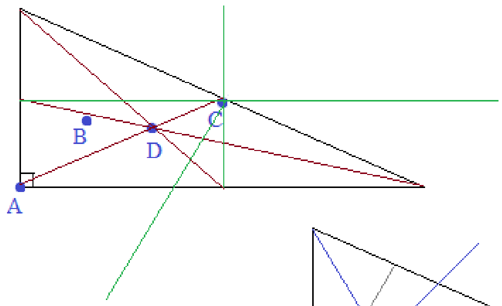
More Triangle Parts Questions....

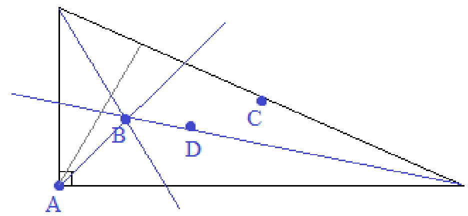
SOLUTIONS

1) Where is the point of concurrency?

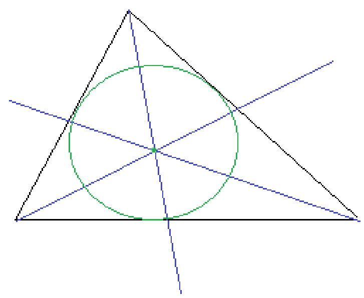
(Determine the point where the 3 lines intersect)

1. Perpendicular Bisectors __C__
2. Medians __D__
3. Altitudes __A__
4. Angle Bisectors __B__





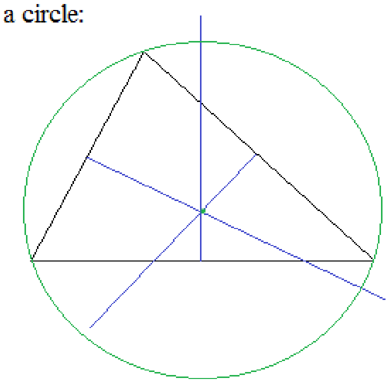
2) a) Inscribe a circle:



Draw angle bisectors.
Then, inscribe the circle...

(the intersection is equidistant from each side of the triangle)

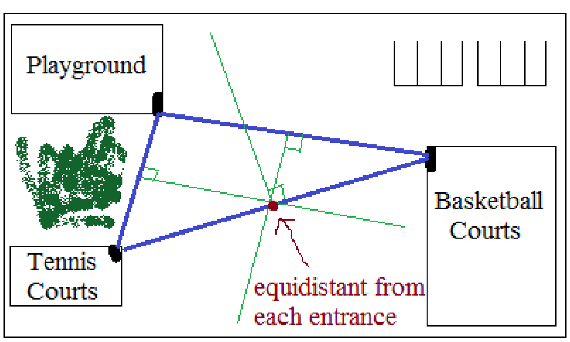
b) Circumscribe a circle:



Draw perpendicular bisectors.
Then, circumscribe the circle...

(the point of concurrency/intersection is the center of the circle. and the distance to each vertex is the radius)

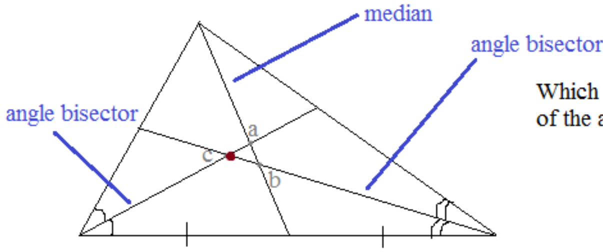
3) The sketch is a diagram of a local park. (the entrances are marked). Where should they place a drinking fountain that is equal distance from the playground, tennis courts, and basketball courts?



Draw a triangle connecting the entrances.
Then, construct the perpendicular bisectors.
The intersection of the 3 ⊥ bisectors are equidistant from the 'vertices'.

SOLUTIONS

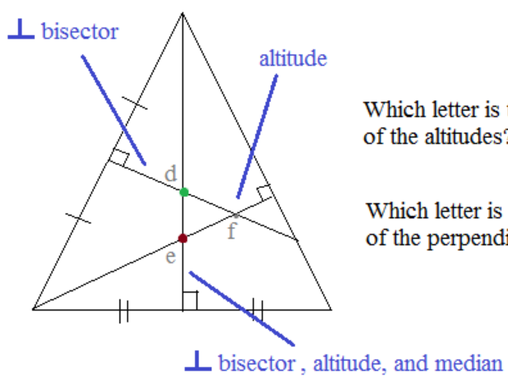
4A)



Which letter is the point of concurrency of the angle bisectors? (the incenter)

c is where the angle bisectors intersect

4B)



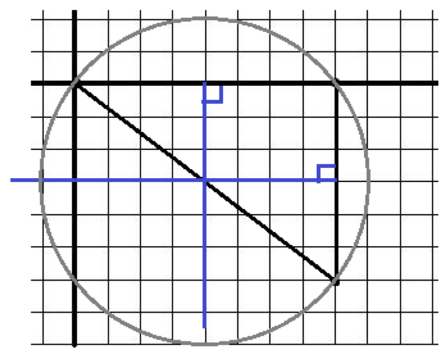
Which letter is the point of concurrency of the altitudes? (the orthocenter)

e is the orthocenter

Which letter is the point of concurrency of the perpendicular bisectors? (the circumcenter)

d is the circumcenter

5) Find the *center of a circle* that circumscribes a triangle with vertices (0, 0) (8, 0) and (8, -6)



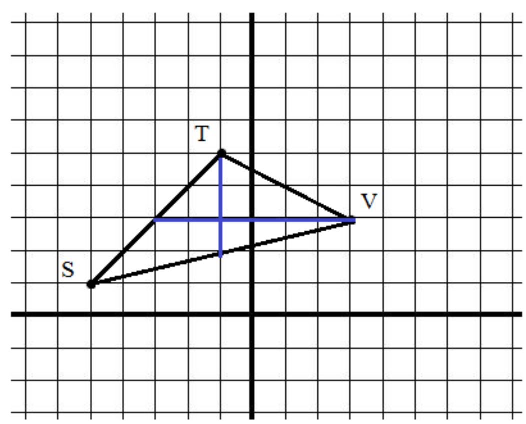
To find the circumcenter, identify where the perpendicular bisectors meet...

-- midpoint of (0, 0) and (8, 0) is (4, 0) and, since it is perpendicular to the side of the triangle, the segment is vertical....

-- midpoint of (8, 0) and (8, -6) is (8, -3) and, this segment is horizontal...

their intersection is at (4,-3)....

6) Find the coordinates of the *centroid* C in △ STV where S (-5, 1) T (-1, 5) V (3, 3)



The centroid is where the medians intersect...

-- median from vertex V to $\overline{ST}$ is horizontal line

-- median from vertex T to SV is vertical line

the medians are concurrent at (-1, 3)

(the third median will pass through (-1, 3) also)

(**Note: each distance from vertex to centroid is 2/3 of the length of the median)

Triangle Test: Median, Altitude, Perpendicular Bisector and Angle Bisector

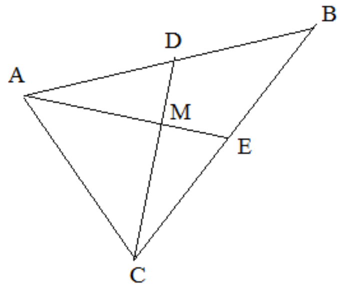
SOLUTIONS

7) $\overline{AE}$ and $\overline{CD}$ are medians.

$\overline{AE} = 12$

What is $\overline{ME}$? 4

$\overline{AM}$? 8



2/3 Centroid Theorem

The centroid is 2/3 along any median...

2/3 of AE = AM   2/3(12) = 8

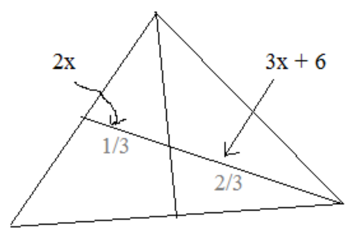
1/3 of AE = ME   1/3(12) = 4

8) The diagram shows a triangle and its 2 medians.

What is the length of the labeled median?

$3x + 6 = 2(2x)$ $x = 6$ length is 36...

(because the large portion is 2/3 the length of the median and the small portion is 1/3 the length of the median... i.e. the larger portion is twice as large and the small)



9) Given: Right triangle SMR with altitude $\overline{MP}$ and horizontal hypotenuse $\overline{SR}$

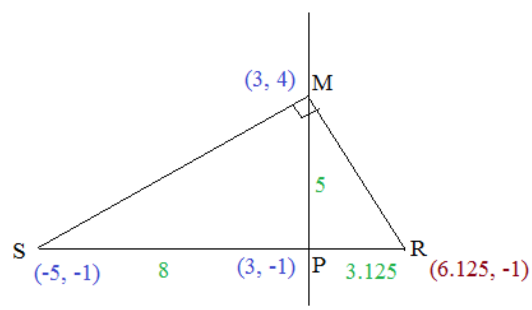
M: (3, 4)   S: (-5, -1)

Find: Coordinate R

Since MP is an altitude, it is perpendicular to SR...
If SR is horizontal, then MP is vertical and P is (3, -1)

Length of MP is 5 and SP is 8

"Altitude to Hypotenuse": MP is the geometric mean of PR and SP



$$\frac{8}{5} = \frac{5}{PR}$$ 8(PR) = 25   PR = 3.125

Therefore, R is (6.125, -1)

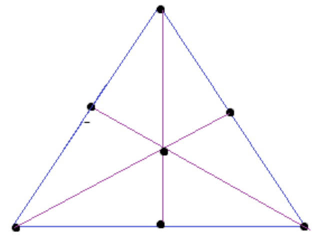
Here are a few questions. Note the application of triangle parts!

Applying properties of triangle parts and concurrency

Question #1

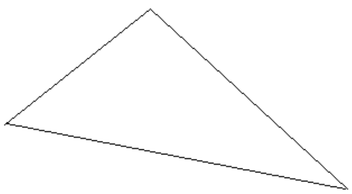
Arrange the 7 dots so that 6 (three dot) lines can be drawn...

Answer: apply the concept of medians and centroid...

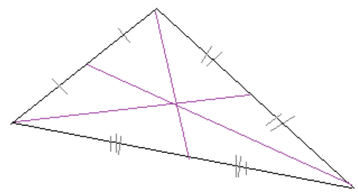


Question #2

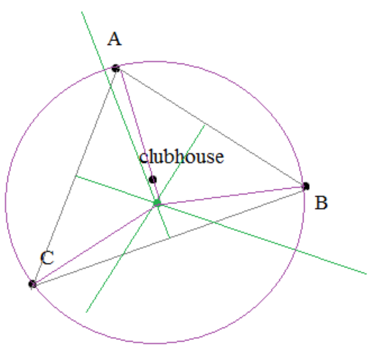
My birthday cake is a strange triangle shape...
If I have 5 friends at my party, how do I cut the cake so that everyone gets an equal amount?



answer: draw medians... each triangle is the same area!



Question #3



Which item would determine if the clubhouse is equidistant to locations A, B, and C?

a) incenter

b) circumcenter

c) centroid

d) orthocenter

Draw perpendicular bisectors...

If the clubhouse is equidistant to the vertices A, B, and C,
then it is located at the circumcenter...

(NOTE: the Green dot is where the clubhouse should be located!)

Question #4

True or False? The point where a triangular table could balance on one leg is always the same distance from each side of the triangle...

False....

The centroid "divides the area" of the triangle into equal weights...

And, centroid occurs where each median meets -- 2/3 from each vertex...

(the incenter is equidistant to each side, but it doesn't necessarily divide the area...)

Thanks for downloading this geometry packet. (Hope it was useful!)

If you have questions, suggestions, or feedback, let us know.

Cheers,

Lance@mathplane.com

Mathplane is also at pinterest, and teacherspayteachers

Also, Mathplane.ORG for mobile and tablets.