July 27, 2004

Rick Norland
Norland Research
8475 Lisa Lane
Las Vegas, Nevada 89113

Hi Rick;

Attached is the write-up for an activity I did with a small group of secondary math teachers this summer. The differences between the real-world of friction, gravity, inexpensive servos, and the pure, abstract world of mathematics made such a simple problem a real challenge for these teachers. I gave them a short course in programming with Terrapin Logo as the turtle in Logo is a sophisticated Mathbot. This helped them solve the math problems before encountering the more time-consuming engineering problems.

You're welcome to do whatever you like with these pages.

Jim Wilson

AIMS Education Foundation
P. O. Box 8120
Fresno, CA 93747-8120
jawilson@fresno.edu

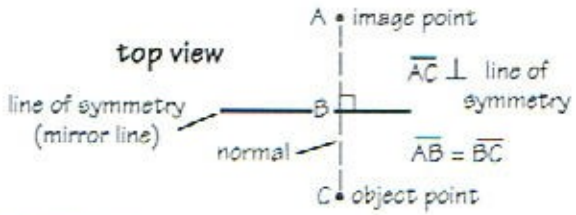
Teaching the Mathbot the Law of Plane Reflection

Review:

Mathematics - Line Symmetry

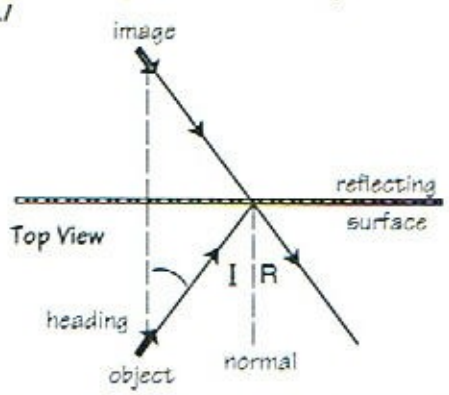
The mathematical definition of line symmetry says that two points are symmetric with respect to a line, if and only if, that line perpendicularly bisects the line segment joining the two points.

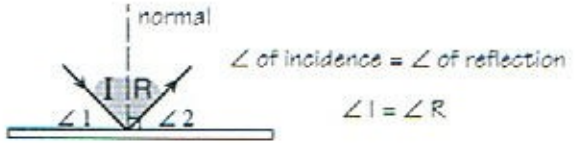
For a plane mirror, the line connecting an object and its mirror image is perpendicular (normal) to the mirror and, the object and image are equidistant from the mirror. Line symmetry explains the relationship between an object placed in front of a vertical, plane mirror, and the image of the object as seen in the mirror.



Science - Law of Plane Reflection

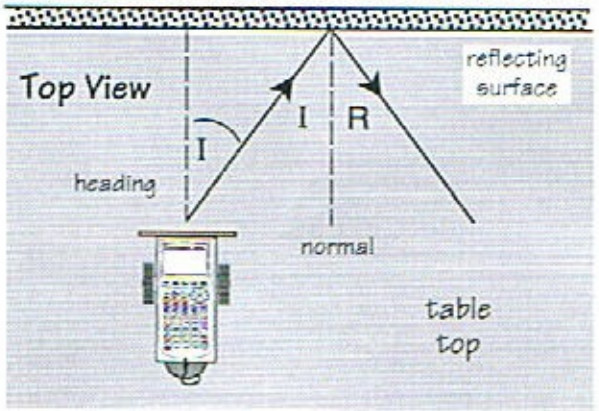
For a plane mirror, the angle made by the incident ray and the line perpendicular (normal) to the mirror is equal to the angle made by the reflected ray and the normal. This result can be reached from the *line of symmetry* definition./





The Problem:

Position the Mathbot so that its "heading" (the direction in which the Mathbot is pointing) is perpendicular to a flat, vertical surface. (A table positioned close to a wall works well.)



Program the Mathbot so that:

- it accepts as input the angle of incidence,
- turns right through the appropriate angle.
- moves forward in a straight line until it strikes the reflecting surface,
- turns through the appropriate angle so that the angle of reflection equals the angle of incidence, and
- moves forward.

Additional Test Specifications:

- if $\angle I = 0°$, the Mathbot reflects off the barrier back along the perpendicular,
- if $\angle I = 90°$, the Mathbot moves parallel to the reflecting surface,
- if $\angle I = 45°$, the Mathbot reflects along the 45° line.

CLR Commands

Command	Left Servo Direction	Right Servo Direction
1 = timed movement only 2 = move until switch is pressed 3 = time **OR** until switch is pressed	0 = backward 1 = no rotation 2 = forward	0 = backward 1 = no rotation 2 = forward

The Program:

Added Challenge:

Modify your program so that the Mathbot continuously reflects ($\angle I = \angle R$) whenever it strikes a vertical surface.

For the Teacher

The program prompts the user to input the angle of incidence.

```
PROGRAM:IEQRBOT
ClrHome
Prompt I
```

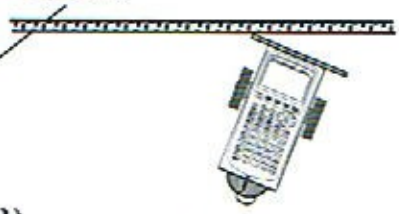
The Mathbot, placed perpendicular to the reflecting surface, turns to the right through this angle. The command for this turn was calculated as follows:

For the surface on which the Mathbot was placed, the command **Send({120,65}: Get(R)** turned the Mathbot to the right through a 90° angle. The ratio 65/90 is equivalent to 0.72/1. These commands converts the input angle and turns the Mathbot to the right. [The ratio 65/90 is specific to the Mathbot I was using and the surface material of the table top.]

```
I*0.72→I
Send({120,I})
Get(R)
```


```
Send({222,100})
Get(R)
```

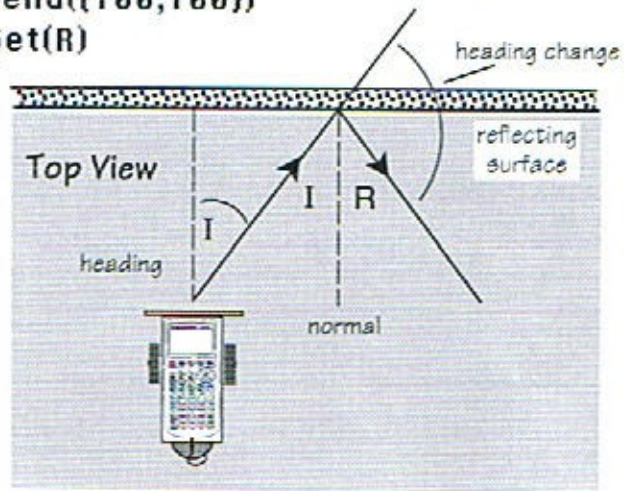
The Mathbot then moves forward until it strikes the reflecting surface at ∠I (measured from the normal). [If two parallel lines are cut by a transversal, the opposite interior angles along the transversal are equal.]



The Mathbot backups a short distance.

```
Send({100,100})
Get(R)
```

As shown in the diagram, If the Mathbot is to reflect off the surface so that ∠I = ∠R, it must turn right to change its heading by (180° – 2I)degrees.




```
(180*0.72) - (2*I)→I
Send({120,I})
Get(R)
```

The Mathbot again moves forward along a straight line, having obeyed the *angle of incidence* equals the *angle of reflection* law for plane reflection.

```
Send({222,200})
Get(R)
```