**Basics of Algebra: Notes**

# Variables and Expressions:

**Quantity:** Anything that can be measured/counted

**Variable:** letter/symbol used to represent a value

**Constant:** value that does not change

**Numerical Expression:** contains constants and operations

**Algebraic Expression:** contains variables, constants, and operations

**Words Indicating Operation:**

| Addition | Subtraction | Multiplication | Division |
|---|---|---|---|
| plus | minus | times | divided by |
| sum | quotient | product | quotient |
| increased by | less than | equal groups of | goes into |

**Examples:**

| $x + 6$ | $h - 2$ | $4 \cdot x$ | $k \div 7$ |
|---|---|---|---|
| the sum of x and 6 | difference of h and 2 | 4 times x | quotient of k and 7 |

**or**

Tim reads 20 pages per hour. Write an algebraic expression for the number of pages he reads in x hours.

→ $20x$ or $20 \cdot x$

* you can also do this same thing with 2 or more operations!

# Order of Operations and Evaluating Expressions:

Order of operations: PEMDAS

Parentheses / other grouping symbols
Exponents
Multiplication } left to right
Division
Addition } left to right
Subtraction

ex.) $(6-2)^3 \div 2$

$(4)^3 \div 2$

$64 \div 2$

$\boxed{32}$

## Evaluating Algebraic Expressions:

To evaluate algebraic expressions, you substitute values for the variables (that are given to you). Then, you have a numerical expression to solve.

$x^2 + x - 12 \div y^2$ ; $x = 5$ $y = 2$

$5^2 + 5 - 12 \div 2^2$

$25 + 5 - 12 \div 4$

$25 + 5 - 3$

$30 - 3$

$\boxed{27}$

* Remember order of operations here!

# Real Numbers and The Number Line:

**Set:** a collection of numbers / variables
**Element of a Set:** a member of a set
**Subset:** a set that has elements from a given set
**Natural Numbers:** the counting numbers
**Whole Numbers:** the natural numbers and zero
**Integers:** whole numbers and their opposites
**Rational Numbers:** numbers that can be $\frac{a}{b}$
**Irrational Numbers:** numbers that can't be $\frac{a}{b}$
**Real Numbers:** the set of all rational and irrational numbers

## Classifying Real Numbers:

$\frac{8}{9}$ belongs to which sets of real numbers — rational

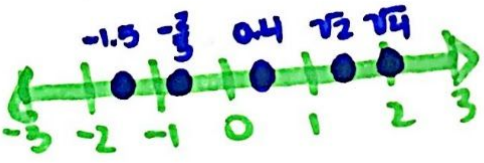
18 belongs to which sets of real numbers — natural; whole, integer; rational

## Comparing Real Numbers then Graphing Them:

**Inequality:** compares the values of 2 expressions

$>$ greater than; $<$ less than; $\geq$ greater than or equal to; $\leq$ less than or equal to

Order: $\sqrt{4}$, 0.4, $-\frac{2}{3}$, $\sqrt{2}$, and $-1.5$



# Properties of Real Numbers:

**Equivalent Expressions:** 2 algebraic expressions that have the same value for all values of the variable

**Commutative Property:** you can add numbers in any order and multiply numbers in any order

$2+7=7+2$ $\quad a+b=b+a$

$3 \cdot 9 = 9 \cdot 3$ $\quad ab=ba$

**Associative Property:** When you are only adding or multiplying you can group any of the numbers together

$(6+8)+2=6+(8+2)$ $\quad a+(b+c)=(a+b)+c$

$(7 \cdot 4) \cdot 5 = 7 \cdot (4 \cdot 5)$ $\quad (ab)c=a(bc)$

**Additive Property: (Identity)** Adding zero does not change the value of the number

$a+0=a$ $\quad 5+0=5$

**Multiplicative property: (Identity)** Multiplying by 1 doesn't change the value of the number

$a \cdot 1 = a$ $\quad 5 \cdot 1 = 5$

**Zero Property of Multiplication:** Any value multiplied by zero is zero

$a \cdot 0 = 0$ $\quad 5 \cdot 0 = 0$

**Multiplication Property of -1:** Any value multiplied by -1 is the opposite of the value

$-1 \cdot a = -a$ $\quad -1 \cdot 5 = -5$

$-1 \cdot -5 = 5$

# The Distributive Property:

**Distributive Property:** You can multiply a number by a sum or multiply by each number in the sum then add. the result is the same.

$3(4+8) = 3(4) + 3(8)$

$a(b+c) = ab + ac$

## Simplify each expression:

$3(x+8) = 3x + 24$

$-(3x-2) = -3x + 2$

$(5b-4)(-7) = -35b+28$

$y(2y-1) = 2y^2 - y$

## Combining Like Terms:
(values w/ the same exact variable)

$20x + 12x = 32x$

$4n + 11n^2 = 4n + 11n^2$ b/c $n$ and $n^2$ are not like terms

## Simplify:

$2(x+6)+3x$

$2x + 12 + 3x$

$5x + 12$

$4a - 2(a-1)$

$4a - 2a + 2$

$6a + 2$

$6(x-4) + 9$

$6x - 24 + 9$

$6x - 15$

You can also rewrite fractions using the distributive property:

$$\frac{6x+4}{3} = \frac{1}{3}(6x+4) = 2x + \frac{4}{3}$$

# Intro. to Equations:

**Equation:** a math sentence with an equal sign

**open sentence:** equation (one or more variable in it) that may be true or false depending on value of variable

| $3y+6=5y-8$ | $16-7=4+5$ | $32\div 8=2\cdot 3$ |
|---|---|---|
| open | true | false |

**Solution to an equation:** value of variable that makes the equation true

Is $x=6$ a solution to $32=2x+12$?

No $32 \neq 24$ (≠ = not equal to)

Is $x=\frac{1}{2}$ a solution to $6x-8=-5$?

Yes $-5=-5$

* the values given were substituted into the equations

**Writing an equation:**

The sum of $4x$ and 3 is 8: $4x+3=8$

**Finding a Solution:**

**Mental Math**

$x+8=12$

$x=4$

b/c you know by heart that $4+8=12$

**Table**

$5n-8$

| n | $5n+8$ | value |
|---|---|---|
| 5 | $5(5)+8$ | 33 |
| 6 | $5(6)+8$ | 38 |
| 7 | $5(7)+8$ | 43 |
| 8 | $5(8)+8$ | 48 |