Congruence & Similarity with Proofs: Notes

Introductory Theorems for Proofs!

Triangle sum theorem: all 3 interior angles of a triangle equal 180°

$a + b + c = 180$

Exterior Angles theorem: the sum of 2 remote interior angles equal the exterior angle

$50 + 70 = 120$

Congruent Figures: figures that have the same shape and same size

Congruence statements: 2 statements that have a one-to-one correspondance w/ each other

→ $\triangle ABC \cong \triangle DEF$

Base Angles theorem: If the base angles are congruent, then the legs are congruent
(the 2 congruent angles in an isosceles triangle)

Converse of the Base Angles theorem: If the legs are congruent, then the base angles are congruent

Equilateral Triangle Extension: If a triangle is equilateral, then it is equiangular

Converse of the Equilateral Triangle Extension: If a triangle is equiangular then it is equilateral

All 3 sides and angles are congruent

Reflexive Property: any side or angle is congruent to itself

$\overline{ab} \cong \overline{ab}$

$\angle c \cong \angle c$

Included side: the side between 2 angles

Included angle: the angle between 2 sides

Proving Triangles Congruent:

Side-Side-Side (SSS) Postulate (Proof)

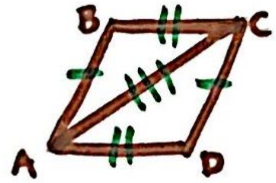
If 3 sides of 1 triangle are congruent to 3 sides of another triangle, then the triangles are congruent.

If $\overline{JK} \cong \overline{NK}$
$\overline{AJ} \cong \overline{AN}$
$\overline{AK} \cong \overline{AK}$
Then
$\triangle JKA \cong \triangle NKA$

Example: Complete the Proof.

Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$

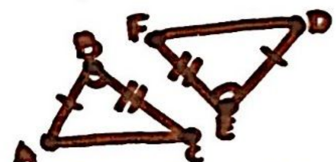
Prove: $\triangle ABC \cong \triangle CDA$



Statements:	Reasons:
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{BC} \cong \overline{AD}$	2. Given
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive Property
$\triangle ABC \cong \triangle CDA$	4. SSS

Side-Angle-Side (SAS) Postulate (Proof)

If 2 sides and its included angle of 1 triangle is congruent to 2 sides and its included angle of another triangle, then the triangles are congruent.

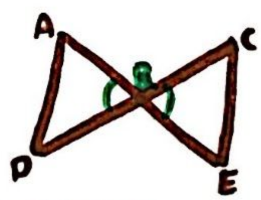


If $\overline{AB} \cong \overline{EG}$
$\angle B \cong \angle E$
$\overline{BC} \cong \overline{EF}$
Then
$\triangle ABC \cong \triangle DEF$

Example: Complete the Proof

Given: B is the midpoint of $\overline{AE}$
B is the midpoint of $\overline{CD}$

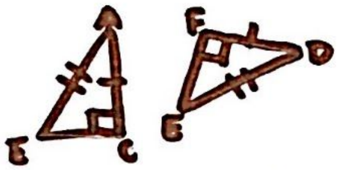
Prove: $\triangle ABD \cong \triangle EBC$



Statements:	Reasons:
1. B is the midpoint of $\overline{AE}$	1. Given
2. $\overline{AB} \cong \overline{BE}$	2. Definition of midpoint
3. B is the midpoint of $\overline{CD}$	3. Given
4. $\overline{CB} \cong \overline{BD}$	4. Definition of midpoint
5. $\angle ABD \cong \angle EBC$	5. vertical angles
6. $\triangle ABD \cong \triangle EBC$	6. SAS

Hypotenuse - Leg (HL) Theorem

If a leg and a hypotenuse of one right triangle is congruent to a leg and a hypotenuse of a second right triangle, then the triangles are congruent.



If $\angle C$ & $\angle F = 90°$ then $\triangle ACB \cong \triangle DFE$
$\overline{AB} \cong \overline{DE}$
$\overline{AC} \cong \overline{DF}$

Example: complete the Proof

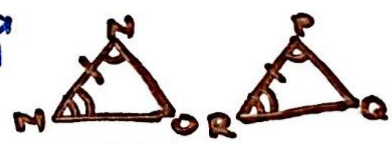
Given: $OM \perp LN$, $\overline{OL} \cong \overline{ON}$

Prove: $\triangle OML \cong \triangle OMN$

Statements:	Reasons:
1. $OM \perp LN$	1. Given
2. $\angle OML$ & $\angle OMN$ are right triangles	2. Definition of Perpendicular
3. $\overline{OL} \cong \overline{ON}$	3. Given
4. $\overline{OM} \cong \overline{OM}$	4. Reflexive Property
5. $\triangle OML \cong \triangle OMN$	5. HL

Angle-side-Angle (ASA) Postulate (Proof)

If 2 angles and 1 included side of a triangle is congruent to 2 angles and 1 included side of another triangle, then the triangles are congruent.

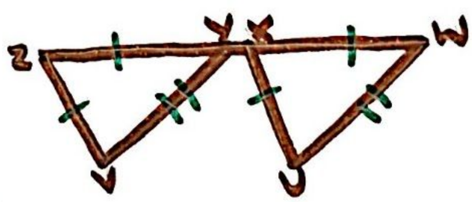


$\angle N \cong \angle P$
$\overline{MN} \cong \overline{RP}$
$\angle M \cong \angle R$

Example: complete the Proof

Given: $WU \parallel YV$, $XU \parallel ZV$, $WX \cong YZ$

Prove: $\triangle WXU \cong \triangle YZV$



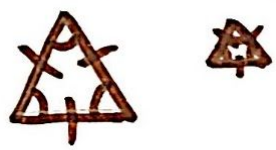
Statements:	Reasons:
1. $WU \parallel YV$	1. Given
2. $\angle UWX \cong \angle VYZ$	2. Corresponding Angles
3. $XU \parallel ZV$	3. Given
4. $\angle UXW \cong \angle VZY$	4. Corresponding Angles
5. $\overline{WX} \cong \overline{YZ}$	5. Given
6. $\triangle WXU \cong \triangle YZV$	6. ASA

Angle - Angle - Side (AAS) theorem

If 2 angles and 1 side next to them in triangle A are congruent to triangle B, then triangle A is congruent to triangle B.

Anti-Theorems: DON'T USE THESE

Angle - Angle - Angle (AAA)



Angle-Side-Side (SSA)

Remember: Order matters in congruence statements

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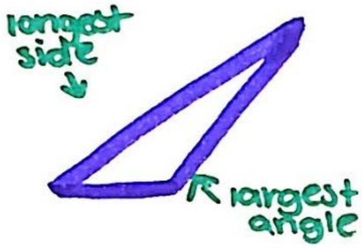
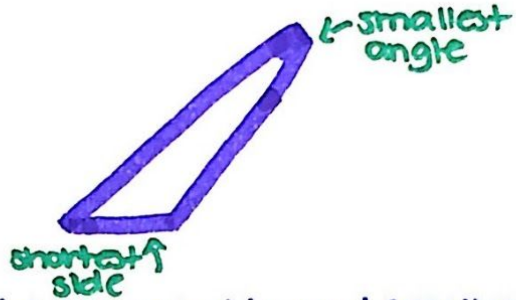
Triangle Inequalities:



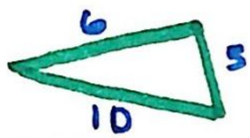
They (longest side & largest angle) are opposite each other always.



The shortest side and smallest angle are opposite each other always.

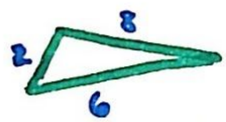
Triangle Inequality Theorem: The sum of the lengths of the 2 shortest sides must be greater than the length of the 3rd side in order to be a triangle

Examples:



$6+5 > 10$ ✓

$11 > 10$



$6+2=8$ ✗

$8=8$

* the triangles are just models, they are not exact

Finding Possible Side Lengths: The unknown side of a triangle is greater than the difference and less than the sum of the 2 known sides

Example:

$$(12-8) < x < (12+8)$$

$$4 < x < 20$$

x is between 4 and 20

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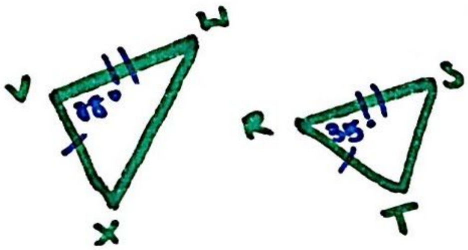
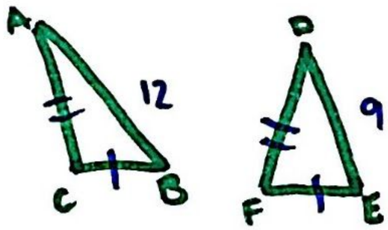
Hinge Theorem:

Hinge Theorem: If 2 sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle.



$WX > ST$

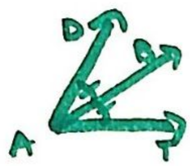
Converse of the Hinge Theorem: If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second triangle.



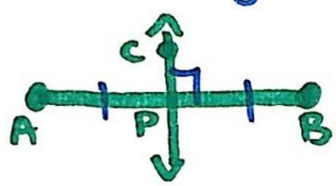
$m\angle C > m\angle F$

Angle Bisectors, Medians, Altitudes:

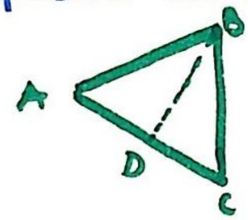
Angle Bisector: cuts an angle in half



Perpendicular Bisector: cuts a segment in half at 90°



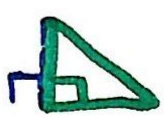
Median: starts at a vertex and cuts the opposite side in half



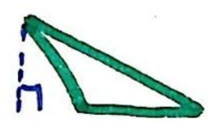
$\overline{BD}$ is a median

Altitude (height): perpendicular segment that connects a vertex to the opposite side or the line that contains the opposite side

Acute triangle



Right triangle

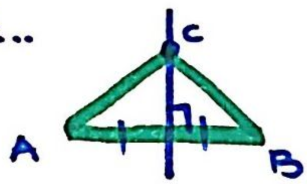


obtuse triangle

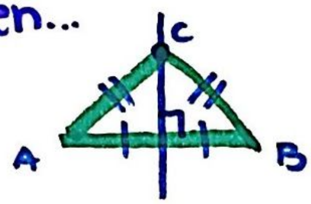
Bisector Theorems:

Perpendicular Bisector Theorem: If a point lies on the perpendicular bisector of a segment, then that point is equidistant from the endpoints of the segment

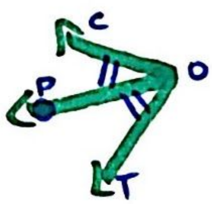
If...

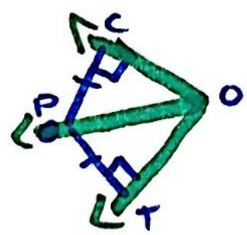


then...



Angle Bisector Theorem: If a point lies on the angle bisector of an angle, then that point is equidistant from the rays of the angle at a 90° angle





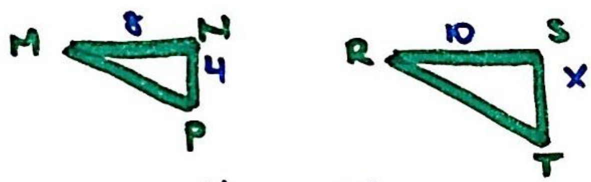
Proportions in Geometry:

Properties of Proportions: $\frac{a}{b} = \frac{c}{d}$

1. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$
2. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$
3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$
4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$

Example in Geometry:

In the diagram, $\frac{MN}{RS} = \frac{NP}{ST}$



so: $\frac{4}{8} = \frac{x}{10}$

$8x = 40$

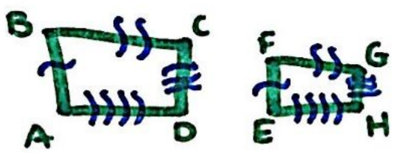
$x = 5$

* you can use this knowledge as a foundation for complex problems involving proportions in geometry.

Similar Polygons:

Similar Polygons: polygons that have congruent angles and proportional sides

If $ABCD \sim EFGH$



Then...

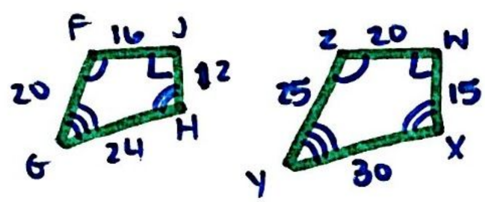
$\angle A = \angle E$
$\angle B = \angle F$
$\angle C = \angle G$
$\angle D = \angle H$

Scale Factor: If 2 polygons are similar, then the ratio of 2 of the corresponding sides will be equal

* To determine if 2 polygons are similar, check...
- angles are congruent
- sides are similar/proportional

Perimeters of Similar Polygons: equal the scale or side factor

Example: Determine if polygons are similar.



$\frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30}$

$\frac{4}{5} = \frac{4}{5} = \frac{4}{5} = \frac{4}{5}$

$\boxed{\text{Yes}}$

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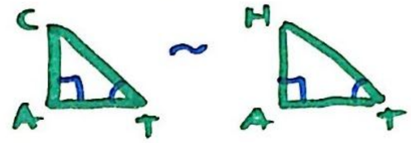
Proving Triangles Similar:

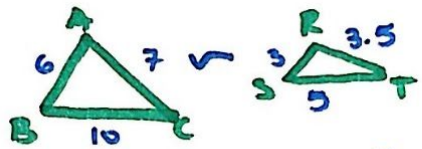
Angle Angle Similarity Postulate (AA~)

If at least 2 angles are congruent to 2 other angles, then the triangles are similar.



Side-Side-Side Similarity Theorem (SSS~)

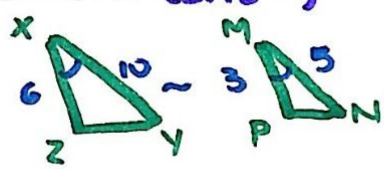
If the 3 sides of Δ A are proportional to the 3 sides of Δ B, then Δ A is similar to Δ B.



$\frac{6}{3} = \frac{7}{3.5} = \frac{10}{5} \rightarrow \frac{2}{1}$

Side-Angle-Side Similarity Theorem (SAS~)

If 2 sides of Δ A are similar and the angle between them is congruent to ΔB, then ΔA is similar to ΔB.



$\Delta XYZ \sim \Delta MNP$

$\frac{10}{5} = \frac{6}{3} \rightarrow \frac{2}{1}$