Exponents and Exponential Functions: Notes

Integer Exponents:

Review:

$3^2 = 3 \times 3 = 9$ | $-3^2 = -3 \times 3 = -9$ | $(-3)^2 = (-3)(-3) = 9$

Zero and Negative Exponents:

* any nonzero # raised to the zero power is 1
* any nonzero # raised to a negative exponent is equal to 1 divided by that # raised to the opposite (positive) exponent

$3^0 = 1$ | $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ | $a^0 b^{-3}$ $a = 8$ $b = -2$; $(8)^0(-2)^{-3}$; $1\left(\frac{1}{(-2)^3}\right)$; $1\left(\frac{1}{-8}\right) = -\frac{1}{8}$ | $3y^{-2}$; $3 \cdot y^{-2}$; $3 \cdot \frac{1}{y^2}$; $\frac{3}{y^2}$

Multiplying Powers with the same base

* Keep base same, add the exponents (Property of Exponents)

$3^4 \cdot 3^2 = 3^{4+2} = 3^6$ | $2a \cdot 9b^4 \cdot 3a^2$; $(2 \cdot 9 \cdot 3)(a \cdot a^2)(b^4)$ ← combine only those bases that are alike; $54a^3b^4$

Simplifying a Power raised to a Power

* Keep base same, multiply the exponents

$(n^7)^4 = n^{7 \cdot 4} = n^{28}$ | $x^2(x^6)^4$; $x^2(x^{24})$; x^{26} | $(4m^2)^3 = 4^3 \cdot (m^2)^3$ ← distribute the exponent; $64m^6$

* solve these step by step

Division Properties of Exponents

* Keep base same, divide the exponents

$\frac{6^7}{6^4} = 6^{7-4} = 6^3$ | $\frac{a^5b^9}{(ab)^4} = \frac{a^5b^9}{a^4b^4}$; ab^5 | $\frac{2^3 \cdot 3^2 \cdot 5^7}{2 \cdot 3^4 \cdot 5^5} = 2^2 3^{-2} 5^2$; $\frac{2^2 \cdot 5^2}{3^2}$; $\frac{100}{9}$

Rational Exponents & Radicals:

Finding roots:

Index: with a radical sign, the number that indicates the degree of the root

The index tells you what root to look for.
↳ To find that root, look for what number you can take to the power of the index and that's the root!

$\sqrt{36}=6$	$\sqrt[3]{64}$ $x^3=64$ $x=4$	$\sqrt[4]{81}$ $x^4=81$ $x=3$	$\sqrt[5]{100{,}000}$ $x^5=100{,}000$ $x=10$

Exponents can also be fractions
↳ The numerator tells you to raise the base to that power and the denominator tells you to take that root of the answer

$125^{\frac{1}{3}}$ $(\sqrt[3]{125})^1$ 5^1 5	$16^{\frac{3}{4}}$ $(\sqrt[4]{16})^3$ 2^3 8	$27^{\frac{4}{3}}$ $(\sqrt[3]{27})^4$ 3^4 81

Converting to exponential form:

↳ use exponent on radicand as numerator
↳ use index as denominator

$\sqrt[5]{b^3}=b^{\frac{3}{5}}$

$\sqrt[3]{27d^5}=(27d^5)^{\frac{1}{3}}$

$27^{\frac{1}{3}}d^{\frac{5}{3}}$

$3d^{\frac{5}{3}}$

Exponential Functions:

* A function that repeatedly multiplies an initial amount by the same positive number

Form: $y = ab^x$ $b > 0$, $b \neq 1$

Evaluating an Exponential Function:

↳ substitute in a value for the variable exponent and solve the equation

An initial population of 20 rabbits triples every half year. $f(x) = 20 \cdot 3^x$ gives the population after x half year periods. How many rabbits will be there after 1.5 years?

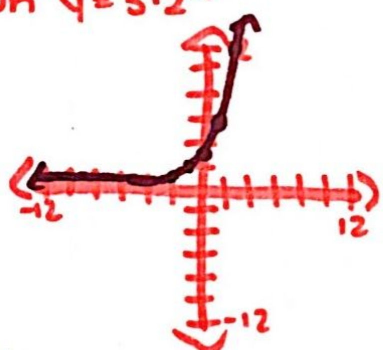
$f(3) = 20 \cdot 3^3$ $f(3) = 20 \cdot 27$ $f(3) = 540$ rabbits

Graphing an Exponential Function:

↳ make a table values that includes negatives, zero and positive values

Graph $y = 3 \cdot 2^x$

x	y
-2	3/4
-1	1½
0	3
1	6
2	12

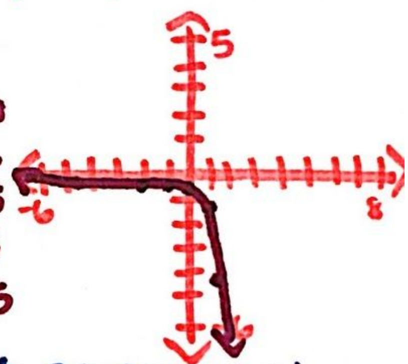


* this is exponential growth

↳ b/c 3 is positive in the equation

Graph $y = -0.5 \cdot 3^x$

x	y
-2	-1/18
-1	-1/6
0	-0.5
1	-1.5
2	-4.5



* this is exponential decay

↳ b/c -0.5 is negative in the equation

Solving one Variable Equations:

↳ isolate the power and then figure out what would the exponent have to be to make the equation equal

$3 \cdot 2^x = 24$
$2^x = 8$
$x = 3$

$5 \cdot 2^x - 152 = 8$
$5 \cdot 2^x = 160$
$2^x = 32$
$x = 5$