Linear Functions: Notes

Slope:

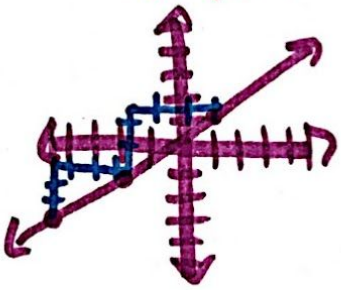
Slope: a measure of the steepness of a line

Rise: the difference in the y values of 2 points on a line

Run: the difference in the x values of 2 points on a line

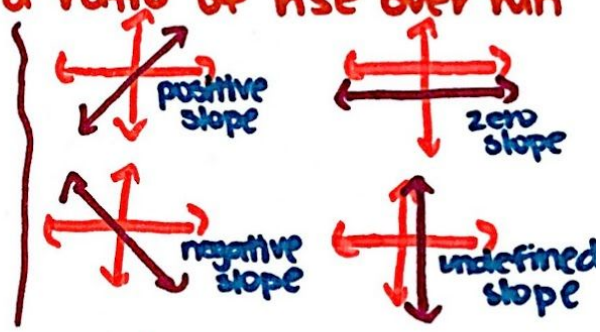
To find the slope of a line that has been graphed:

- begin at one point and count vertically to the level of the 2nd point to find the rise
- then count horizontally to the level of the 2nd point to find the run
- write your answers as a ratio of rise over run



slope: $\frac{2}{3}$

$\frac{\text{rise}}{\text{run}}$



Finding the slope using points:

- When you have 2 points that are on the line you use the formula...

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

↑ slope

so, when you have only points given, you can find the slope

(4, -2) and (-1, 2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2-2}{4--1} = \frac{-2-2}{4+1}$$

$$\boxed{m = -\frac{4}{5}}$$

Slope-Intercept Form:

$$y = mx + b$$

- y: y value
- m: slope
- x: x value
- b: y-intercept - y coordinate of the point where a line crosses the y-axis

Identifying slope and y-intercept:

To find the slope and y-intercept from a linear equation, you need to get it into the proper form by solving for y.

$y = 3x + 1$

$m = 3$

$b = 1$

$2y - 6 = 3x$

$2y = 3x + 6$

$y = \frac{3}{2}x + \frac{6}{2}$

$y = \frac{3}{2}x + 3$

$m = \frac{3}{2}$

$b = 3$

Writing an equation:

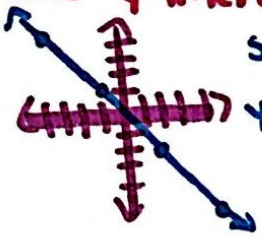
substitute the values into the equation.

slope: 3 y-int: 2

$y = 3x + 2$

Writing an equation for a graph:

- Find the rise and run (slope)
- Find the y-intercept



slope: $\frac{-3}{3} = -1$

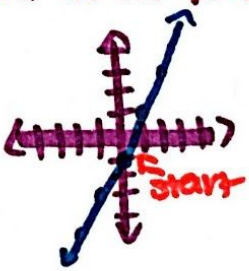
y-int: 0

$y = -x + 0$

$\boxed{y = -x}$

Graphing a linear equation from slope-intercept form.

- use rise and run to plot points
- repeat until you form a line



$y = 2x - 1$

$m = \frac{2}{1}$

$b = -1$

Writing an equation from 2 points,

- use slope formula to find m
- plug in one point to find b
- write equation w/ m and b

(2, 1) and (5, -8)

$m = \frac{1 - -8}{2 - 5} = \frac{9}{-3} = -3$

$y = mx + b$

$1 = -3(2) + b$

$1 = -6 + b$

$7 = b$

$\rightarrow \boxed{y = -3x + 7}$

Point-Slope Form:

$y - y_1 = m(x - x_1)$ m = slope (x_1, y_1) = point on the line

- same as slope-intercept except you are given a point on the line instead of the y-intercept

Writing linear equations in point-slope form:

- have the basic form down
- substitute the values into the equation

slope = $\frac{5}{2}$; $(-3, 0)$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{5}{2}(x - -3)$

$y = \frac{5}{2}(x + 3)$

Slope-point form → Slope intercept form:

You should always graph in slope intercept form

↳ To convert the equation, solve for y

slope = -4 ; $(-1, -2)$

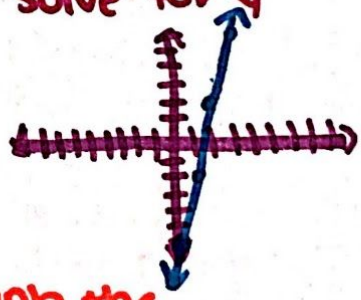
$y - y_1 = m(x - x_1)$

$y - -2 = -4(x - -1)$

$y + 2 = -4(x + 1)$

$y + 2 = -4x - 4$

$y = -4x - 6$ then you can graph the line too!



Using two points to write an equation:

- Find the slope
- Pick one point and use it in the point-slope formula
- simplify the answer

$(1, -4)$ and $(3, 2)$

$m = \frac{-4-2}{1-3} = \frac{-6}{-2} = 3$

$y - 2 = 3(x - 3)$

$y - 2 = 3x - 9$

$\boxed{y = 3x - 7}$

Standard Form of a Line:

$ax + by = c$

a, b, c are real #s

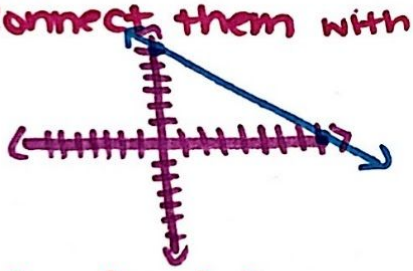
a and b $\neq 0$

Graphing in standard form:

- find the x and y intercepts
  - insert 0 for one variable to find the intercept for the other
- plot them and connect them with a line

$3x + 4y = 24$

x	y
0	6
8	0



Transferring to standard form:

- make sure that there are no fractions in the equation
- to get rid of them, multiply by a common denominator
- get the x and y term on the same side

$y = -\frac{3}{7}x + 5$

$7y = -3x + 35$

$3x + 7y = 35$

$y - 2 = \frac{1}{3}(x + 6)$

$y - 2 = \frac{1}{3}x + 2$

$y = \frac{1}{3}x + 4$

$3y = x + 12$

$-x + 3y = 12$

Writing an equation from 2 points in standard form:

- write it in slope-intercept form
- get the x and y term on the same side

(-6, 4) (3, -5)

$m = \frac{4 - -5}{-6 - 3} = \frac{9}{-9} = -1$

$y - 4 = -1(x + 6)$

$y - 4 = -1x - 6$

$y = -1x - 2$

$\boxed{x + y = -2}$

Slopes of Parallel and Perpendicular Lines:

Parallel Lines:
- Slope is the exact same
- y-intercepts / points are different

Original Line: $y = 4x + 3$ $\quad m = 4$ ✓, $b = 3$

Parallel Line: $y = 4x - 1$ $\quad m = 4$ ✓, $b = -1$

Perpendicular Lines:
- slope is the negative reciprocal of original slope
- rise and run are switched and positive → negative, negative → positive
- y-intercepts / points are different

Original Line: $y = 4x + 3$ $\quad m = 4$ ✓, $b = 3$

Perpendicular Line: $y = -\frac{1}{4}x - 1$ $\quad m = -\frac{1}{4}$ ✓, $b = -1$