Polynomials and Factoring: Notes

Adding & Subtracting Polynomials:

Finding the degree:

of a monomial:

Monomial - a polynomial w/ 1 term

Degree of a Monomial - the sum of the exponents of the variables in the monomial

Polynomial - monomial / sum or difference of monomials

Degree of a Polynomial - largest degree of a term within a polynomial

Standard Form of a polynomial - a polynomial where the terms are in order from greatest to least

- $2a^2b^4$: degree = 6 b/c 2+4
- $4x - 18x^5$ = 1st term degree = 1, 2nd term degree = 5, degree = 5
- $y^3 + y^5 + 4y$ = degree = 3, 5, 1 so order is $y^5 + y^3 + 4y$

Adding Polynomials:

↳ Line polynomials vertically by like terms and add

↳ use associative, commutative properties to collect like terms and add horizontally

$(2x^2 - x) + (x^2 + 3x - 1)$

$$\begin{array}{r} 2x^2 - x \phantom{{}-1} \\ +\ \ x^2 + 3x - 1 \\ \hline 3x^2 + 2x - 1 \end{array}$$

OR

$(2x^2 + x^2) + (-x + 3x) + (-1)$

$3x^2 + 2x - 1$

$(4x^2 + 3x - 6) + (2x^2 - 4x + 5)$

$$\begin{array}{r} 4x^2 + 3x - 6 \\ +\ 2x^2 - 4x + 5 \\ \hline 6x^2 - x - 1 \end{array}$$

OR

$(4x^2 + 2x^2) + (3x - 4x) + (-6 + 5)$

$6x^2 - x - 1$

Subtracting Polynomials:

↳ Remember subtracting is like adding the opposite

↳ Write the opposite of each term in 2nd polynomial then add

$(a^4 - 2a) - (3a^4 - 3a + 1)$

$(a^4 - 2a) + (-3a^4 + 3a - 1)$

$$\begin{array}{r} a^4 - 2a \phantom{{}-1} \\ +\ -3a^4 + 3a - 1 \\ \hline -2a^4 + a - 1 \end{array}$$

$(2x^2 - 3x^2 + 1) - (x^2 + x + 1)$

$(2x^2 - 3x^2 + 1) + (-x^2 - x - 1)$

$(-x^2 + 1) + (-x^2 - x - 1)$

$$\begin{array}{r} -x^2 \phantom{{}-x} + 1 \\ +\ -x^2 - x - 1 \\ \hline -2x^2 - x \phantom{{}-1} \end{array}$$

Multiplying & Factoring Polynomials:

Multiplying a monomial & polynomial:

↳ distribute the monomial to each term in the polynomial
↳ combine like terms

$2x(3x+1)$ → $6x^2+2x$

$-x^3(9x^4-2x^3+7)$ → $-9x^7+2x^6-7x^3$

$5n(3n^3-n^2+8)$ → $15n^4-5n^3+40n$

Finding the Greatest Common Factor:

↳ find the prime factorization of each term
↳ take the smallest power of each common factor base

24 and 60

$24 = 2^3 \cdot 3$

$60 = 2^2 \cdot 3 \cdot 5$

$GCF = 2^2 \cdot 3$ or $\boxed{12}$

$5x^3+25x^2+45x$

$5x^3 = 5 \cdot x^3$

$25x^2 = 5^2 \cdot x^2$

$45x = 3^2 \cdot 5 \cdot x$

$GCF = 5 \cdot x$ or $\boxed{5x}$

Factoring out a GCF:

↳ divide each term by the GCF
↳ write the polynomial as 2 factors: GCF, what was left

Factor: $4x^5-24x^3+8x$ GCF: $4x$ Divide by $4x$ and you get: x^4-6x^2+2

Final answer: $4x(x^4-6x^2+2)$

Factor: $6x^3-15x^2+12x$ GCF: $3x$ Divide by $3x$ and you get: $2x^2-5x+4$

Final answer: $3x(2x^2-5x+4)$

Multiplying Binomials:

Using the distributive property:

↳ use the distributive property more than once
↳ distribute each term in one of the binomials to the other binomial

$(x+2)(x-5)$
$x(x-5)+2(x-5)$
$x^2-5x+2x-10$
$x^2-3x-10$

$(x+3)(x-3)$
$x^2-3x+3x-9$
x^2-9

← if you see a problem like this, go straight to the third step (the answer will always be the product of the first and last 2 terms)

FOIL Method:

↳ Multiply
- ↳ 1st 2 terms
- ↳ outer 2 terms
- ↳ inner 2 terms
- ↳ Last 2 terms

$(x+3)(x+2)$
$x^2+2x+3x+6$
x^2+5x+6

Multiplying Polynomials:

↳ use the distributive property several times

$(x+2)(x^2-5x+4)$
$x(x^2-5x+4)+2(x^2-5x+4)$
$x^3-5x^2+4x+2x^2-10x+8$
x^3-3x^2-6x+8

$(x^2-4x+6)(x^2-2x+5)$
$x^2(x^2-2x+5)-4x(x^2-2x+5)+6(x^2-2x+5)$
$x^4-2x^3+5x^2-4x^3+8x^2-20x+6x^2-12x+30$
$x^4-6x^3+13x^2-32x+30$

Factoring x^2+bx+c:

Factor by guess and check:

- look for 2 numbers that are factors of the constant (c)
- use those 2 binomials and multiply them using FOIL, if it doesn't work, try 2 other factors

$x^2+19x+60$

$(x+15)(x+4) \rightarrow x^2+4x+15x+60$

$x^2+19x+60$ ✓

Factoring x^2+bx+c when c is positive:

- factors of the constant will always add together to form the middle term
- find the 2 factors that have a sum of the middle coefficient (b) and multiply to equal the constant (c)

x^2+6x+8
$(x+4)(x+2)$

x^2+5x+6
$(x+3)(x+2)$

$x^2-10x+16$
$(x-2)(x-8)$

$x^2+13x+42$
$(x+6)(x+7)$

Factoring x^2+bx+c when c is negative:

- the 2 factors will be opposite signs (+, −) and equal the b term and multiply to equal c

$x^2+7x-18$
$(x+9)(x-2)$

$x^2-5x-24$
$(x-8)(x+3)$

$x^2+2x-15$
$(x+5)(x-3)$

x^2+2x-8
$(x+4)(x-2)$

Factoring a trinomial with 2 variables:

- can only do this if middle term has both variables and outer terms each contain one of the variables

$x^2+6xy-55y^2$
$(x+11y)(x-5y)$

$a^2+12ab+32b^2$
$(a+8b)(a+4b)$

Factoring $ax^2 + bx + c$:

- change the middle term into 2 terms using the factors found (product of ac and sum of b)
- Factor by grouping

$4x^2+16x+15$
AC = 60 B = 16
Factors: 10 and 6
$4x^2+10x+6x+15$
$2x(2x+5)+3(2x+5)$
$(2x+5)(2x+3)$

$6x^2+11x+3$
AC = 18 B = 11
Factors: 9 and 2
$6x^2+9x+2x+3$
$3x(2x+3)+1(2x+3)$
$(3x+1)(2x+3)$

When c is positive:
↳ both factors must have the same sign

$2x^2+11x+12$
$(2x+3)(x+4)$

When c is negative:
↳ factors will have opposite signs

$4y^2+7y-2$
$(4y-1)(y+2)$