Quadratic Equations and Functions: Notes

Quadratic Graphs:

Quadratic Function - a function that can be written as $ax^2 + bx + c$, graph is a parabola

Parabola - shape of a quadratic function ; U shape

Axis of Symmetry - the line that divides a graph into 2 matching halves

Vertex - the highest or lowest point on a parabola

Minimum - the y-value of the lowest point on the graph of a function

Maximum - the y-value of the highest point on the graph of a function

* a quadratic function always has a degree of 2

Graphing a quadratic:

↳ find enough ordered pairs to see the shape of the parabola

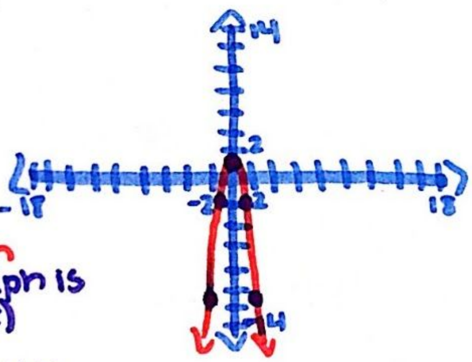
↳ choose x values and find their y values

↳ connect the points with a smooth curve

$y = -3x^2 + 1$

x	y
-2	-11
-1	-2
0	1
1	-2
2	-11

← vertex & Maximum (since graph is negative)



U shape - minimum

∩ shape - maximum

x values - domain

y values - range

Quadratic Functions:

Finding the axis of symmetry using the formula:

↳ for ax^2+bx+c, use $x=\frac{-b}{2a}$ to find the axis of symmetry

$y=x^2+3x+4$
$a=1 \quad b=3$
$x=-\frac{3}{2(1)}=\boxed{-\frac{3}{2}}$

$y=-3x^2+10x+9$
$a=-3 \quad b=10$
$x=\frac{-10}{2(-3)}=\frac{-10}{-6}=\frac{10}{6}=\boxed{\frac{5}{3}}$

Finding the vertex of a parabola:

↳ since the vertex lies on the axis of symmetry, you know the x-value of the vertex

↳ if you input the x-value into the quadratic, you can find the y-value and thus find the coordinate of the vertex

$y=5x^2-10x+3$
$a=5 \quad b=-10$
$x=-\frac{-10}{2(5)}=-\frac{-10}{10}=1$
so, $y=5(1)^2-10(1)+3$
$y=5-10+3$
$y=-2$
vertex: $(1,-2)$

$y=x^2-4x-10$
$a=1 \quad b=-4$
$x=-\frac{-4}{2(1)}=-\frac{-4}{2}=2$
so, $y=(2)^2-4(2)-10$
$y=4-8-10$
$y=-14$
vertex: $(2,-14)$

Graphing a quadratic function:

you need to...

↳ find the axis of symmetry
↳ find the vertex
↳ find the y-intercept
↳ create the symmetry
↳ draw the parabola

Solving Quadratic Equations:

Quadratic Equation: ax^2+bx+c

Root of an equation: solution to a quadratic equation

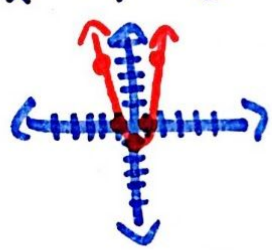
Zero of a function: x-intercept of a graph of a function

Solving by graphing:

- accurately graph the quadratic then find where the graph crosses the x-axis
  - those are the values that make y equal zero and since the quadratic equation must equal zero these are the solutions for the quadratic equation

$x^2-1=0$

zeros are 1 and -1 so those are the 2 solutions



Solving by using square roots:

- for a quadratic that has x^2 but no b term, you can isolate the x^2 then take the square root of both sides to find the value(s) of x

$x^2=16$

$\sqrt{x^2}=\sqrt{16}$

$x=\pm 4$

$x^2=-4$

$\sqrt{x^2}=\sqrt{-4}$

NO solution

$4x^2-25=0$

$4x^2=25$

$x^2=\frac{25}{4}$

$\sqrt{x^2}=\sqrt{\frac{25}{4}}$

$x=\pm\frac{5}{2}$

$36x^2=1$

$x^2=\frac{1}{36}$

$\sqrt{x^2}=\sqrt{\frac{1}{36}}$

$x=\pm\frac{1}{6}$

Factoring to Solve Quadratic Functions:

Using the zero Product Property:

↳ For all real numbers a and b, if $ab = 0$ then $a = 0$ or $b = 0$

* To use this property to solve a factored quadratic, find what could make each factor equal 0

$(x-3)(x+7)=0$

if $x-3=0$ then $x=3$

if $x+7=0$ then $x=-7$

solutions: 3 and -7

$(x)(x-5)=0$

if $x=0$

if $x-5=0$ then $x=5$

solutions: 0 and 5

Factoring:

↳ write the equations in $y=ax^2+bx+c$ form

↳ factor the quadratic to form 2 items that multiply to get zero

↳ then solve using the zero product property

$x^2+7x+10=0$

$(x+5)(x+2)=0$

$\boxed{x=-5 \quad x=-2}$

$-2x^2=18-12x$

$0=2x^2-12x+18$

$0=2(x^2-6x+9)$

$0=2(x-3)(x-3)$

$\boxed{x=3}$

$x^2-25=0$

$(x+5)(x-5)=0$

$\boxed{x=5 \quad x=-5}$

The Quadratic Formula:

- another way to solve for x

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

a, b, and c come from the quadratic

$2x^2+3x-5=0$

$a=2 \quad b=3 \quad c=-5$

$$x = \frac{-3 \pm \sqrt{3^2-4(2)(-5)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9+40}}{4}$$

$$x = \frac{-3 \pm \sqrt{49}}{4}$$

$$x = \frac{-3 \pm 7}{4}$$

$$x = \frac{-3+7}{4} \qquad x = \frac{-3-7}{4}$$

$$x = \frac{4}{4} \qquad x = \frac{-10}{4}$$

$$\boxed{x=1} \qquad \boxed{x=-\frac{5}{2}}$$

Using the Discriminant:

↳ b^2-4ac in the quadratic formula that tells the number of solutions

if $b^2-4ac > 0$ then there are 2 solutions

if $b^2-4ac = 0$ then there is 1 solution

if $b^2-4ac < 0$ then there are No solutions

$3x^2+10x+2=0$

$10^2-4(3)(2)$

$100-24$

76

2 solutions

$x^2-x+1=0$

$(-1)^2-4(1)(1)$

$1-4$

-3

No solutions