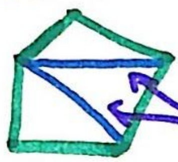
Quadrilaterals & Circles: Notes

Angles of Polygons:

- In a polygon, a diagonal is a segment that joins 2 nonconsecutive verticies



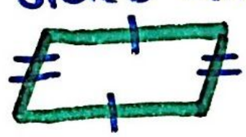
diagonals

# of sides	3	4	5	6	7	8	9	10	12	n
# of triangles	1	2	3	4	5	6	7	8	10	$n-2$
sum of interior angles	180°	360°	540°	720°	900°	1080°	1260°	1440°	1800°	$(n-2)(180)$
sum of exterior angles	360°	360°	360°	360°	360°	360°	360°	360°	360°	360°
interior angle measure	60°	90°	108°	120°	128.6°	135°	140°	144°	150°	$\frac{(n-2)(180)}{n}$
exterior angle measure	120°	90°	72°	60°	52.4°	45°	40°	36°	30°	$\frac{360}{n}$

Parallelogram:

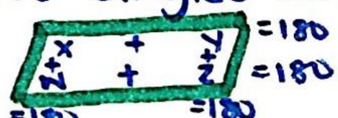
Parallelogram: Quadrilateral with 2 pairs of parallel sides

- opposite sides are congruent

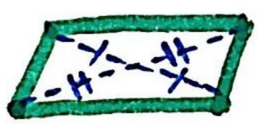


- opposite angles are congruent

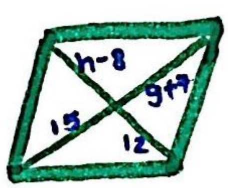
- consecutive angles are supplementary



- diagonals bisect each other



Example:



* opposites are congruent

$h - 8 = 12$ $g + 7 = 15$

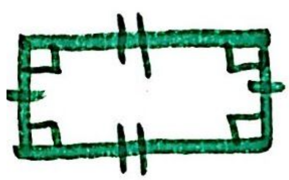
$\boxed{h = 20}$ $\boxed{g = 8}$

Square, Rectangle, Rhombus:

Rhombus: parallelogram with 4 congruent sides

↳ opposite sides are congruent
↳ opposite angles are congruent
↳ consecutive angles equal 180°
↳ Diagonals form 4 right angles
↳ Bisect opposite angle

Rectangle: parallelogram with 4 right angles



↳ opposite sides are congruent
↳ opposite angles are congruent
↳ consecutive angles equal 180°
↳ Diagonals are congruent

Square: parallelogram with 4 congruent sides and 4 right angles

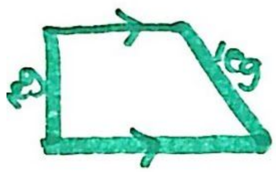
↳ Both a rectangle and a rhombus

* You can use these properties to solve geometric problems (unknown angles / sides)

Trapezoids & Kites:

Trapezoid: quadrilateral with EXACTLY one pair of parallel sides

*NOT A PARALLELOGRAM



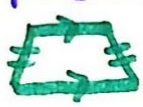
↳ non parallel sides are legs

↳ 2 pairs of base angles

↳ diagonals are non congruent and do not bisect each other

Isosceles Trapezoid: a trapezoid with congruent legs

↳ legs, base angles, and diagonals are congruent



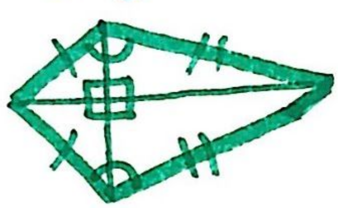
Midsegment of a Trapezoid: average of 2 bases of trapezoid

$$\frac{\text{base} + \text{base}}{2}$$

Kites: a quadrilateral with 2 pairs of consecutive congruent sides

↳ 1 pair of opposite angles are congruent

↳ Diagonals are perpendicular



Angles in Circles:

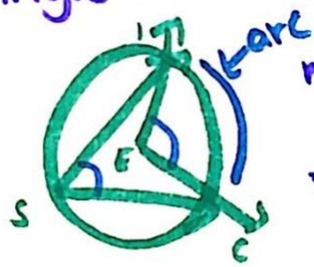
Central Angle: an angle whose vertex lies at the center of the circle
- central angle creates a major arc and minor arc

$\angle CEN$

Inscribed Angle: an angle whose vertex lies <u>ON</u> the circle

Intercepted Arc: an arc on the circle where angles go through

Measure of the Inscribed Angle: the arc degrees



$$\frac{m\angle IEC}{2} = m\angle ISC$$

* use this rule for circles with this kind of arc

Inscribed Angles Theorem:
If 2 inscribed angles of a circle intercept the same arc, then the inscribed angles are congruent

$\angle M \cong \angle N$

Inscribed Polygon:
A polygon whose vertices touch the circle

- inscribed quadrilateral
* opposite angles = 180°

AMA
LEARNING

Finding the Measure of Angles on the Circle:

on the circle:

$m\angle x = \frac{arc}{2}$

$m\angle y = \frac{arc}{2}$

inside the circle:

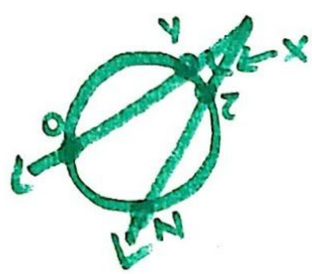
$m\angle 1 = \frac{\overset{\frown}{CB} + \overset{\frown}{AD}}{2}$

$\angle 1 \cong \angle 3$ b/c they are opposites

$m\angle 2 = \frac{\overset{\frown}{AC} + \overset{\frown}{DB}}{2}$

$\angle 2 \cong \angle 4$ b/c they are opposites

outside the circle:



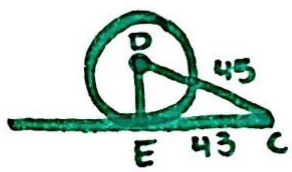
$m\angle X = \frac{\overset{\frown}{ON} - \overset{\frown}{YZ}}{2}$

* this is a kind of formula sheet to help you solve problems
* in actual problems the arcs mentioned will be given to you

Properties of Tangents to a circle:

Tangent Theorem: a line outside the circle forms a right angle with the radius at the point of tangency

Example: EC is tangent to circle D at point E. Find the radius.



$43^2 + x^2 = 45^2$

$1849 + x^2 = 2025$

$x^2 = 176$

$x = 13.3$

*use the tangent theorem to use the Pytagorean theorem for this problem

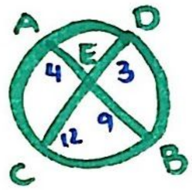
DE = 13.3

radius = 13.3

segments:

2 chords intersect INSIDE the circle:

the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord



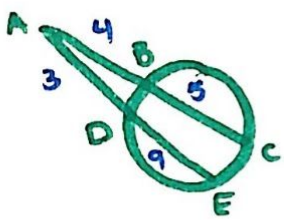
$AE \times EB = CE \times ED$

$4 \times 9 = 12 \times 3$

$36 = 36$

2 secants intersect OUTSIDE the circle:

the product of the lengths of one secant and its external segment equals the product of the lengths of the other secant and its external segment



$AC \times AB = AE \times AD$

$9 \times 4 = (9+3) \times 3$

$36 = 36$

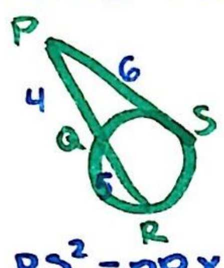
whole segment x outside

=

whole x outside

A secant and a tangent intersect OUTSIDE the circle:

the product of the secant and external segment equals the square of the tangent segment



$PS^2 = PR \times PQ$

$6^2 = (5+4) \times 4$

$36 = 36$

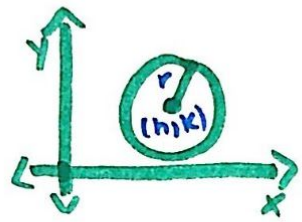
tangent ^{2} = whole x outside

Equations for Circles:

$$(x-h)^2 + (y-k)^2 = r^2$$

center: (h,k)

Radius: $\sqrt{r^2} = r$



Examples: $(x-1)^2 + (y+2)^2 = 9$

center: $(1,-2)$ $(-1,+2 \rightarrow 1,-2)$

Radius: 3 $(\sqrt{9})$

Write the equation of a circle with center $(0,-3)$ and a radius of 5.

$$(x-h)^2 + (y-k)^2 = r^2$$

$h = 0$

$k = -3$

$r = 5$

$$(x-0)^2 + (y+3)^2 = 5^2$$

$$\boxed{x^2 + (y+3)^2 = 25}$$