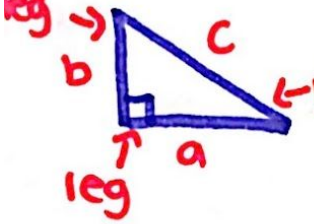
Radical Expressions and Equations: Notes

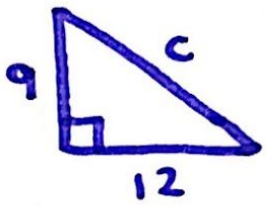
Pythagorean Theorem:

* applies only to right triangles



$a^2 + b^2 = c^2$

SO, ...



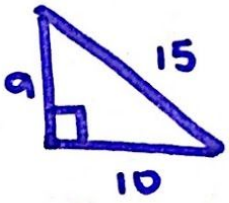
$9^2 + 12^2 = c^2$

$81 + 144 = c^2$

$225 = c^2$

$c = 15$

and



$a^2 + 10^2 = 15^2$

$a^2 + 100 = 225$

$a^2 = 125$

$a \approx 11.18$

Identifying a right triangle:

Time 2 smaller sides squared (a, b) and added together should equal the longer side (c) squared $a^2 + b^2 = c^2$

Sides: 6m, 24m, 25m

$6^2 + 24^2 = 25^2$

$36 + 576 = 625$

$612 = 625$

Not a right triangle

Sides: 24cm, 10cm, 26cm

$24^2 + 10^2 = 26^2$

$576 + 100 = 676$

$676 = 676$

Right triangle

Simplifying Radicals:

Simplifying radicals by removing perfect square factors:

A radical expression is simplified when:

- if the radicand has no perfect square factors other than 1
- the radicand has no fractions
- no radicals appear in the denominator of a fraction

$\sqrt{18} = \sqrt{9}\sqrt{2} = \underline{3\sqrt{2}}$ $\quad \sqrt{450} = \sqrt{25}\sqrt{9}\sqrt{2} = 5 \cdot 3\sqrt{2} = \underline{15\sqrt{2}}$

Simplifying with a variable:

* any variable with an even exponent has a square root

$\sqrt{48a^5} = \sqrt{16}\sqrt{3}\sqrt{a^4}\sqrt{a} = \underline{4a^2\sqrt{3a}}$

$\sqrt{x^4y^5} = \sqrt{x^4}\sqrt{y^4}\sqrt{y} = \underline{x^2y\sqrt{y}}$

Multiplying two radical expressions:

* multiply the coefficients in front of the radicals and the radicands together

$2\sqrt{7t} \cdot 3\sqrt{14t^2} = 6\sqrt{98t^3} = 6 \cdot \sqrt{49}\sqrt{2}\sqrt{t^2}\sqrt{t} = 6 \cdot 7 \cdot t\sqrt{2t} = \underline{42t\sqrt{2t}}$

Simplifying fractions within radicals:

* reduce the fraction if possible then break apart the numerator and denominator and simplify them seperately

$\sqrt{\frac{2}{72}} = \sqrt{\frac{1}{36}} = \underline{\frac{1}{6}}$ $\quad \sqrt{\frac{80}{25}} = \frac{\sqrt{80}}{\sqrt{25}} = \frac{\sqrt{16}\sqrt{5}}{\sqrt{25}} = \underline{\frac{4\sqrt{5}}{5}}$

Rationalizing the denominator: making the denominator without radicals

* multiply the fraction by a value of one that contains the radical in the denominator

$\frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{\sqrt{4}} = \underline{\frac{\sqrt{14}}{2}}$ $\quad \frac{\sqrt{7}}{\sqrt{8n}} \cdot \frac{\sqrt{8n}}{\sqrt{8n}} = \frac{\sqrt{56n}}{\sqrt{64n^2}} = \frac{2\sqrt{14n}}{8n} = \underline{\frac{\sqrt{14n}}{4n}}$

Operations with Radical Expressions:

Combining like radicals:

↳ To add/subtract radicals, you need to do the operation to like terms

$3\sqrt{7} + 8\sqrt{7} = 11\sqrt{7}$ $\quad 9\sqrt{y} - \sqrt{y} = 8\sqrt{y}$

Simplifying before adding/subtracting:

$\sqrt{12} + \sqrt{27}$

$2\sqrt{3} + 3\sqrt{3}$

$5\sqrt{3}$

$\sqrt{1256} + 3\sqrt{206} - \sqrt{456}$

$5\sqrt{56} + 6\sqrt{56} - 3\sqrt{56}$

$8\sqrt{56}$

Multiplying Radical Expressions:

↳ you can distribute

$\sqrt{2}(5 + \sqrt{12})$

$5\sqrt{2} + \sqrt{24}$

$5\sqrt{2} + 2\sqrt{6}$

$5\sqrt{5}(-4 + 6\sqrt{5})$

$-20\sqrt{5} + 30\sqrt{25}$

$-20\sqrt{5} + 150$

Multiplying Sums and Differences of Radicals:

↳ FOIL or multi-distribute

$(\sqrt{2} + 2\sqrt{5})(6 - \sqrt{5})$

$\sqrt{36} - \sqrt{30} + 2\sqrt{30} - 2\sqrt{25}$

$6 - \sqrt{30} + 2\sqrt{30} - 10$

$-4 + 1\sqrt{30}$

$(4 - \sqrt{3})(\sqrt{3} + 5)$

$4\sqrt{3} + 20 - \sqrt{9} - 5\sqrt{3}$

$-1\sqrt{3} + 17$

Rationalizing a denominator using conjugates

↳ Conjugates- the sum and difference of the same 2 terms

$\frac{10}{\sqrt{7} - \sqrt{2}} \cdot \frac{\sqrt{7} + \sqrt{2}}{\sqrt{7} + \sqrt{2}} = \frac{10(\sqrt{7} + \sqrt{2})}{7 - 2} = \frac{10(\sqrt{7} + \sqrt{2})}{5} = 2(\sqrt{7} + \sqrt{2})$

$\downarrow$

$2\sqrt{7} + 2\sqrt{2}$

Solving Radical Equations:

Solving by Isolating the Radical:

- Squaring and taking the square root are inverse operations, so when a square root has a variable in it, square both sides to isolate what is underneath the radical
- Before you square both sides, you must isolate the radical sign

$\sqrt{x} = 8$

$(\sqrt{x})^2 = 8^2$

$x = 64$

$\sqrt{x} + 3 = 10$

$\sqrt{x} = 7$

$(\sqrt{x})^2 = 7^2$

$x = 49$

$\sqrt{2x-1} + 4 = 7$

$\sqrt{2x-1} = 3$

$(\sqrt{2x-1})^2 = 3^2$

$2x - 1 = 9$

$2x = 10$

$x = 5$

Solving Equations with Square Roots on Both Sides:

- try to place one radical on each side of the equal sign

$\sqrt{x+1} = \sqrt{3}$

$(\sqrt{x+1})^2 = (\sqrt{3})^2$

$x + 1 = 3$

$x = 2$

$\sqrt{2x-5} - \sqrt{6} = 0$

$\sqrt{2x-5} = \sqrt{6}$

$(\sqrt{2x-5})^2 = (\sqrt{6})^2$

$2x - 5 = 6$

$2x = 11$

$x = \frac{11}{2}$

* you need to check each solution in these problems b/c sometimes you can have extraneous solutions (a number is not a solution of the original equation)