Solving Equations: Notes

One Step Equations:

- To find the answer to an equation, isolate the variable
  - a variable is isolated when it is by itself on one side of the equation
  - isolate the variable by using the inverse operation, which will "undo" operations on the variable
    - Addition ↔ Subtraction
    - Multiplication ↔ Division
- An equation is like a balance scale
  - to keep it balanced, perform the same operation to both sides

Addition & Subtraction:

- We use the Addition Property of Equality which says we can add or subtract to both sides of an equation equally and the equation is equal

$x - 10 = 4$
$+10 \quad +10$
$x - 0 = 14$
$\boxed{x = 14}$

$x + 7 = 9$
$-7 \quad -7$
$x + 0 = 2$
$\boxed{x = 2}$

Multiplication & Division:

- We use the Multiplication Property of Equality which says we can multiply or divide to both sides of an equation equally and the equation is equal

$\frac{k}{-5} = -4$
$(-5)\frac{k}{-5} = -4(-5)$
$\boxed{k = 20}$

$7x = 56$
$\frac{7x}{7} = \frac{56}{7}$
$\boxed{x = 8}$

Two Step Equations:

Steps to solve equations: Inverses

- Inverse of Addition & Subtraction (First)
- Inverse of Multiplication & Division (Later)

$2x + 3 = 15$
$-3 \quad -3$
$2x = 12$
$\frac{2x}{2} = \frac{12}{2}$
$\boxed{x = 6}$

$\frac{n}{7} - 2 = 3$
$+2 \quad +2$
$\frac{n}{7} = 5$
$(7)\frac{n}{7} = 5(7)$
$\boxed{n = 35}$

- Two Step Equations w/ Fractions:

$\frac{x}{15} - \frac{1}{5} = \frac{3}{5}$
$+\frac{1}{5} \quad +\frac{1}{5}$
$\frac{x}{15} = \frac{4}{5}$
$(15)\frac{x}{15} = \frac{4}{5}(15)^3$
$\boxed{x = 12}$

OR
Get rid of fractions using LCD

$\frac{x}{15} - \frac{1}{5} = \frac{3}{5}$
$15\left(\frac{x}{15} - \frac{1}{5} = \frac{3}{5}\right)$
$x - 3 = 9$
$+3 \quad +3$
$\boxed{x = 12}$

- Equations w/ numerators: * multiply both sides by denominator

$\frac{x-7}{3} = -12$
$(3)\frac{x-7}{3} = -12(3)$
$x - 7 = -36$
$+7 \quad +7$
$\boxed{x = -29}$

$\frac{a+10}{2} = 4$
$(2)\frac{a+10}{2} = 4(2)$
$a + 10 = 8$
$-10 \quad -10$
$\boxed{a = -2}$

Multi Step Equations:

Steps to solving Equations:
- Distribute to get rid of parentheses
- combine like terms
- Inverse of Addition & Subtraction
- Inverse of Multiplication & Division

*If you have a set of grouping symbols, you need to distribute whatever term is immediately in front of them to get rid of them. (Remember to keep the sign w/ the term)

$-8(2x-1)=36$

$-16x+8=36$

$-8 \quad -8$

$\frac{-16x}{-16}=\frac{28}{-16}$

$x=-\frac{28}{16}$ or $-\frac{7}{4}$

$3(3x-6)=18$

$9x-18=18$

$+18 \quad +18$

$\frac{9x}{9}=\frac{36}{9}$

$x=4$

* After you distribute, you sometimes have like terms on the same side of the Equal sign. You should combine those like terms to simplify the equation.

$5m-23+2m=5$

$7m-23=5$

$+23 \quad +23$

$\frac{7m}{7}=\frac{28}{7}$

$m=4$

$n+5(n-1)=7$

$n+5n-5=7$

$6n-5=7$

$+5 \quad +5$

$\frac{6n}{6}=\frac{12}{6}$

$n=2$

* To get rid of multiple fractions in an equation, multiply by the LCD.

$\frac{b}{3}+\frac{1}{8}=19$

$24\left(\frac{b}{3}+\frac{1}{8}=19\right)$

$8b+3=456$

$-3 \quad -3$

$\frac{8b}{8}=\frac{453}{8}$

$b=\frac{453}{8}$ or $56\frac{5}{8}$

Equations with Variables on Both Sides:

Steps to solve Equations:

- Distribute to get rid of parentheses
- Combine like terms
- Get variables to one side * add or subtract the variables to make one side 0
- Inverse of Addition & Subtraction
- Inverse of Multiplication & Division

$7k = 4k + 15$
$-4k \quad -4k$
$\frac{3k}{3} = \frac{15}{3}$
$\boxed{k = 5}$

$5x - 2 = 3x + 4$
$-3x \quad -3x$
$2x - 2 = 4$
$+2 \quad +2$
$\frac{2x}{2} = \frac{6}{2}$
$\boxed{x = 3}$

*Simplifying both sides:

$3 - 5x + 2x = -2 - 2(1 - x)$
$3 - 5x + 2x = -2 - 2 + 2x$
$3 - 3x = -4 + 2x$
$+3x \quad +3x$
$3 = -4 + 5x$
$+4 \quad +4$
$\frac{7}{5} = \frac{5x}{5}$
$\boxed{\frac{7}{5} = x}$

*Equations with Infinite or No Solutions

$x + 4 - 6x = 6 - 5x - 2$
$-5x + 4 = 4 - 5x$
$+5x \quad +5x$
$4 = 4$
Infinite # of solutions

$-8x + 6 + 9x = -17 + x$
$x + 6 = -17 + x$
$-x \quad -x$
$6 = -17$
No solutions

Literal Equations:

Literal Equations: an equation with 2 or more variables

You can rearrange literal equations using the rules for equations to isolate any of the variables. This is called solving for a variable.

Solve for y:

$$10x + 5y = 80$$

$$\frac{5y}{5} = \frac{80 - 10x}{5}$$

$$\boxed{y = 16 - 2x}$$

Ratios, Rates, and Conversions:

Ratio: compares 2 numbers by using division
- the ratio of a and b can be written as
  - $\frac{a}{b}$
  - a : b
  - a to b

Unit Rates: we use rates to compare similar situations

to find a unit rate we need to get our second unit equal to one

If John can eat 53.5 hot dogs in 12 minutes, how many can he eat in 1 minute?

$\frac{53.5}{12} = 4.46$ hot dogs per minute

* You can use this to compare items as well.

Converting Units: To convert from one unit of measure to another, we use a conversion factor. This is a ratio with equal units so that it is equal to 1 that we multiply the original unit by to change it

convert 330 minutes to hours:

$$\frac{330 \text{ min}}{1} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{330 \text{ hr}}{60} = \boxed{5.5 \text{ hrs}}$$

Converting Rates: same as ↑

A student ran the 50 yd dash in 5.8 sec. At what speed did the runner run in miles per hour?

$$\frac{50 \text{ yd}}{5.8 \text{ sec}} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = \frac{180000 \text{ mi}}{10208 \text{ hr}} \approx \boxed{17.63 \text{ mih}}$$

Solving Proportions:

Proportion: An equation that states 2 ratios are equal

*We can solve a proportion by using the cross multiplying method.

1) You take the numerator of one ratio and multiply it by the denominator of the other. those 2 products should be equal.

$\frac{7}{8} \times \frac{m}{12}$

$84 = 8m$

$\boxed{m = 10\frac{1}{2}}$

$\frac{b-8}{5} \times \frac{b+3}{4}$

$4(b-8) = 5(b+3)$

$4b - 32 = 5b + 15$

$-32 = b + 15$

$\boxed{b = -47}$

You can also use proportions to solve problems.

An 8 oz. can of orange juice contains 97 mg of vitamin C. About how many mg of vitamin C would be in a 12 oz. can?

$\frac{8}{97} = \frac{12}{x}$

$8x = 1164$

$x = 145.5$

$\boxed{145.5 \text{ mg}}$

* You can also use this for percent/percentage!

find 50% of 20

$\frac{50}{100} \cdot 20 = \frac{1000}{100} = 10$

what percent of 60 is 15?

$p \cdot 60 = 15$

$p = \frac{15}{60} = \frac{1}{4} = 25\%$