Solving Inequalities: Notes

Inequalities and their Graphs:

Inequality: a statement that compares the values of 2 statements

↳ solutions include any value that makes the inequality true

$>$	$<$	$\geq$	$\leq$
greater than	less than	greater than or equal to	less than or equal to

All real numbers less than or equal to -7. → $x \leq -7$

6 is less than a number is greater than 13. → $x - 6 > 13$

The sum of x and 4 is at least 8. → $x + 4 \geq 8$

***Identifying solutions of inequalities:**

↳ we can find out if a number is a solution to an inequality by substituting the value in and seeing if it produces a true result.

Is -3 a solution to $2x + 1 > -3$?

$2(-3) + 1 > -3$

$-6 + 1 > -3$

$-5 > -3$

No

Is -1 a solution to $2x + 1 \geq -3$?

$2(-1) + 1 \geq -3$

$-2 + 1 \geq -3$

$-1 \geq -3$

yes

Consider the numbers -1, 0, 1, and 3. Which are solutions of $13 - 7y \leq 6$?

1 and 3

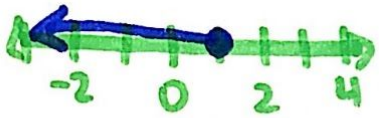
* Graphing Inequalities:

- In inequalities, there are too many possible solutions. So, we use a graph on a number line to show all the solutions.
- The solutions are shaded on the number line and an arrow shows that the solutions continue past those shown on the page.
- To show that an endpoint is a solution (≥, ≤) draw a solid circle at the number.
- To show that an endpoint isn't a solution (>, <) draw an empty circle.

$b < -1.5$

$r \geq 2$

* Writing an inequality from a graph:



$x \leq 1$

$x > 0$

Solving Inequalities: one step + multi step

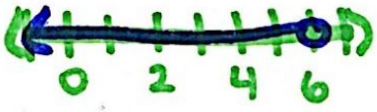
* solving inequalities is similar to solving equations. You NEED to isolate the variable using the properties of inequalities and inverse operations.

- Addition & Subtraction

$x + 9 < 15$

$-9 \quad -9$

$x < 6$



$d - 3 \geq -6$

$+3 \quad +3$

$d \geq -3$

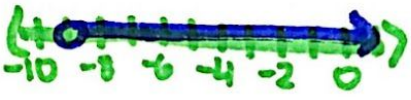
* same steps for multi step inequalities

- Multiplication & Division w/ Positive Numbers

$3x > -27$

$\frac{3x}{3} > \frac{-27}{3}$

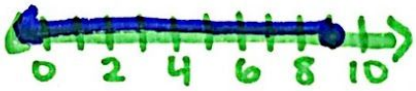
$x > -9$



$\frac{2}{3}r \leq 6$

$\left(\frac{3}{2}\right)\frac{2}{3}r \leq 6\left(\frac{3}{2}\right)$

$r \leq 9$



* same steps for multi step inequalities

- Multiplication & Division w/ Negative Numbers

* If you multiply or divide both sides by a negative number, the resulting inequality is not a true statement. You need to reverse the inequality symbol to make the statement true.

$-8x > 72$

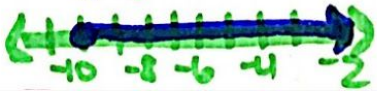
$\frac{-8x}{-8} > \frac{72}{-8}$

$x < -9$

$-x \leq 10$

$\frac{-x}{-1} \leq \frac{10}{-1}$

$x \geq -10$



* same steps for multi step inequalities

Writing Sets:

Roster Form: A notation for listing all of the elements in a set using braces and commas

Set-builder Notation: A notation used to describe the elements of a set

Write a set for the multiples of 2:

Roster Form: $\{2, 4, 6, 8 \ldots\}$

Set Builder Form: $\{x \mid x \text{ is a multiple of } 2\}$

* You can write the solutions to inequalities only in set-builder notation because we need to include every possible answer above or below a certain value.

$-5x + 7 \leq 17$

$-5x \geq 10$

$x \geq -2$

$\{x \mid x \geq -2\}$

$4n + 9 < 21$

$4n < 12$

$n < 3$

$\{n \mid n < 3\}$

* Finding subsets
  - ↳ a set of elements that are also elements in another set

List all the subsets of the set $\{3, 4, 5\}$

$\{\emptyset\}$ $\{3\}$ $\{4\}$ $\{5\}$ $\{3, 4\}$ $\{3, 5\}$ $\{4, 6\}$ $\{3, 4, 5\}$

If $A = \{x \mid x < -3\}$ and $B = \{x \mid x \leq 10\}$ is $A \subseteq B$?

$\subseteq$ means subset

Yes because all numbers less than -3 are less than 0.

Compound Inequalities:

Compound Inequality: 2 inequalities that are combines into one statement by the words AND or OR

*Solving Compound Inequalities involving AND

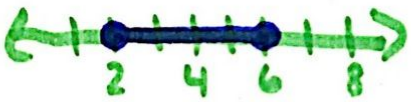
To solve a compound inequality involving AND, separate the 2 inequalities and solve both separately. then graph them together.

$4 \leq x+2 \leq 8$

$4 \leq x+2$ AND $x+2 \leq 8$

$2 \leq x$ AND $x \leq 6$

$2 \leq x \leq 6$



$-5 \leq 2x+3 < 9$

$-5 \leq 2x+3$ AND $2x+3 < 9$

$-4 \leq x$ AND $x < 3$

$-4 \leq x < 3$

*Solving Compound Inequalities involving OR

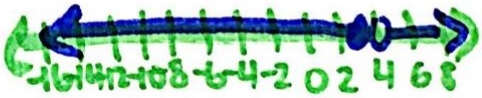
solve as above, but graph separately.

$-4+a > 1$ OR $-4+a < -3$

$a > 5$ OR $a < 1$

$2x \leq 6$ OR $3x > 12$

$x \leq 3$ OR $x > 4$



*Interval Notation

↳ used to show an inequality

$<$, $>$ → ( )

$\leq$, $\geq$ → [ ]

∞ = Infinity

Brackets shows that the value is included

Parentheses show that value isn't included

$x \geq 2 = [2, \infty)$

$x < 2 = (-\infty, 2)$