A Probabilistic Analysis of Two University Parking Issues[1]

by

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We thank the Editor Euijune Kim and five anonymous reviewers for their helpful comments on two previous versions of this note. In addition, Batabyal acknowledges financial support from the Gosnell endowment at RIT. Finally, the usual absolution applies.

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Abstract

The uncertain nature of the demand for university parking has now created a major problem for university planners. From a facilities planning perspective, it is important to comprehend the *probabilistic* nature of the demand for parking and parking rules violations. Given this background, in our note, we shed light on two insufficiently studied issues concerning university parking. First, we focus on short-term and long-term parkers. We determine the mean parking time of an arriving car and then we compute the probability distribution function of the number of occupied parking spots at any particular time. Second, we concentrate on parking rules violators and we calculate the probability distribution function of the number of violators who are fined at an inspection.

Keywords: Long-Term Parking, Short-Term Parking, University, Violator

JEL Codes: R410, R480, D810

1. Introduction

The problem of insufficient parking facilities is a problem that plagues many urban areas in North America, Europe, and Asia. In contemporary times, this problem is also being felt in university campuses. Whether the university is located in the middle of a big city—as Boston University is in Boston, Massachusetts—or in a small town—as Cornell University is in Ithaca, New York—or in a mid-sized city—as the Rochester Institute of Technology is in Rochester, New York—the problem of not being able to find a suitable parking spot remains. The former President of the University of California, Clark Kerr once said "I find that the three major administrative problems on a campus are sex for the students, athletics for the alumni, and parking for the faculty." Parking is now a big problem in universities because in addition to faculty, parking is demanded by staff and, increasingly, by large numbers of students. This tripartite demand for parking has created not only an acute shortage of parking spots but also a major worry for university planners who must decide how best to allocate scarce parking spots.[4]

From a facilities planning perspective, it is clearly not possible *a la* Joni Mitchell in the *Big Yellow Taxi* to pave paradise and put up a parking lot! As noted by Anderson (1996) with regard to Stanford University in Palo Alto, California, parking lots in particular and parking structures in general are very costly to construct and maintain. Nor is it generally possible or desirable to provide parking spots at no cost.[5] The state of affairs that we have just described in this and in the previous

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We are not suggesting that university planners are the only ones who ought to be worrying about parking related problems. In principle, the task of constructing and managing parking facilities can be contracted out to private firms but this would create some well known "agency problems" for university planners. In addition, it may be difficult for a private firm to find land close enough to a university campus to make the private provision of parking facilities economically worthwhile. Because of these sorts of problems, it is university planners who typically end up worrying about parking problems in universities.

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Shoup (2005) has provided a painstaking account of why it is in general not a good idea to provide free parking.

paragraph and, in particular, the uncertainty surrounding the demand for parking together call for university planners to comprehend the *probabilistic* nature of the demand for parking and parking rules violations.

We now briefly discuss studies that have focused on the problems associated with parking in universities. Focusing on the Free University of Amsterdam, Verhoef *et al.* (1996) have used an empirical survey to study the social feasibility of regulatory parking policies. Brown *et al.* (2001) have used a survey study of thirty-five universities to show that programs like "Unlimited Access"—that provide fare free transit service to students—can reduce the demand for parking, increase student access to the campus, and reduce the cost of attending college. Michael *et al.* (2005) have used a classroom game to demonstrate that parking problems on campus can be alleviated by practicing price discrimination. Sandland (2006) has noted that the use of car share schemes is likely to be one solution to the acute parking problems faced by the University of East London at its Docklands campus. Clayton *et al.* (2006) have studied the usefulness of active prompting procedures in getting drivers leaving a university parking lot to increase seat belt use and decrease cell phone use. Finally, Clayton and Myers (2007) have analyzed the pros and cons of alternate ways of increasing the use of turn signals by drivers exiting university parking garages.

The papers discussed in the previous paragraph have certainly advanced our understanding of aspects of the parking problems encountered in universities. Even so, to reiterate something that we have already noted, parking problems in universities arise because the demand for parking exceeds the available supply and because the uncertainty surrounding the demand for parking makes planning difficult. This *stochastic* nature of the demand for parking and the inevitable parking rules violations that result have received inadequate attention in the extant literature. Therefore, in this note, we shed light on two insufficiently studied issues concerning university parking. First, we focus on short-term and long-term parkers. We determine the mean parking time of an arriving car and then we compute the probability distribution function of the number of occupied parking spots at any particular time. Second, we concentrate on parking rules violators and we calculate the probability distribution function of the number of violators who are fined at an inspection.

In this note, a university planner would like to provide parking to all university employees who are willing and able to pay for it. We do not distinguish between academic and non-academic staff in our subsequent analysis because we find no compelling reason to do so. Finally, whether an individual is considered to be a short-term or a long-term parker depends on the *length* of his or her parking time. Therefore, a student, in principle, could be either a short-term or a long-term parker. Similarly, an employee who works part-time at the university under study may be a short-term parker unlike an employee such as a Dean who would typically be a long-term parker.

The rest of this note is organized as follows. Section 2 analyzes a stochastic model of short-term and long-term parking in a university campus and sheds light on the first of the two issues mentioned above. Section 3 uses a model of parking rules infractions to discuss the second of the two issues stated above. Finally, section 4 concludes and offers suggestions for future research on the subject of this note.

2. Short-Term and Long-Term Parking

2.1. Preliminaries

Consider a representative parking lot in a university campus. Potential parkers on this lot are either short-term parkers or long-term parkers. As far as parking on this lot is concerned, one problem faced by a university planner[6] concerns how many spots on this lot to designate as either short-term or long-term. As we have already noted in section 1, this problem arises in part because our university planner does not definitively know the demand for either short-term or long-term parking in the lot under study. Put differently, our university planner is confronted with *stochastic* demand for short-term and long-term parking on this lot.

To model this key aspect of the problem, let us suppose that short-term and long-term parkers arrive at the lot under study in accordance with independent Poisson processes with rates λ_S and λ_L respectively.[7] The parking times of the short-term and the long-term parkers are independent of each other. Further, for tractability, we suppose that the parking time of a short-term parker is uniformly distributed on the interval $[a_S, b_S]$ and that of a long-term parker is also distributed uniformly but on the interval $[a_L, b_L]$.[8] Given this description of events, our university planner would like to know the mean or expected parking time of an arriving car in the lot under study.

2.2. Mean parking time of an arriving car

It is clear that the parking time of an arriving car is a *random* variable. Let us denote this random variable by P. We now wish to compute the expected value of P or $E[P]$. The reader will

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The entire analysis in this note is conducted from the perspective of a university planner. For this planner, the arrival processes of the short-term and the long-term parkers are *exogenous* and so are the distributions that govern the probabilistic parking times of these two categories of parkers.

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The focus of this paper is on modeling stochastic phenomena in the context of parking in universities. As Tijms (2003, p. 1) has noted, the "Poisson process is a natural modelling tool in numerous applied probability problems. It not only models many real-world phenomena, but the process allows for tractable mathematical analysis as well." This is why we are using the Poisson process in this note to study the two issues discussed in section 1. See Kulkarni (1995, chapter 5) or Tijms (2003, chapter 1) for textbook treatments of the Poisson process.

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The uniform distribution has been used previously to model parking times. See, for instance, Arnott (2005). We have modeled the arrivals and the parking times of short-term and long-term parkers as explicitly stochastic phenomena. However, there may be a deterministic component to either one or to both these phenomena.

note that we are working with two independent Poisson processes. Therefore, to compute $E[P]$, we will need to, in a sense, merge these two independent Poisson processes. A standard result for such merged processes is given in theorem 1.1.3 in Tijms (2003, p. 6) and we shall now use this standard result. Using this result, the expectation we seek is given by

$$E[P] = \sum_{i=S}^{L} \frac{\lambda_i}{\lambda_S + \lambda_L} \times \frac{a_i + b_i}{2}. \tag{1}$$

Equation (1) tells us that the mean parking time of an arriving car is the weighted sum of the two short-term and long-term parking time means $(a_i+b_i)/2,\ i=S,L$. The weights or $\lambda_i/(\lambda_S+\lambda_L)$ are the relative arrival rates of either the short-term or the long-term parkers. Our next task is to compute the probability distribution function of the number of occupied parking spots at any particular time.

2.3. The number of occupied parking spots

For definiteness, we shall compute the relevant probability distribution function at any time $t > b_L$. In other words, we are computing the probability distribution function of the number of occupied parking spots at any time t that exceeds the upper limit b_L of the interval over which the parking times of the long-term parkers is uniformly distributed. The problem before us has certain similarities with the well know $M/G/\infty$ model in queuing theory.[9] In the $M/G/\infty$ queuing model, the arrivals occur in accordance with a Poisson process, the service times are generally distributed, and there is an arbitrarily large number of servers. Now, computing the probability distribution function of the number of occupied parking spots is similar to computing the probability distribution function of the number of busy servers. The reader will note that in making this comparison, we are thinking

[9] See Kulkarni (1995, pp. 387-388) or Tijms (2003, pp. 9-15) for textbook accounts of the $M/G/\infty$ queuing model.

of occupied parking spots and busy servers equivalently.

Equation 1.1.6 in Tijms (2003, p. 9) tells us that the limiting distribution of the number of busy servers in the $M/G/\infty$ model is Poisson distributed. Using this result in our problem we reason that the number of occupied parking spots at any particular time $t > b_L$ is also Poisson distributed with mean

$$(\lambda_S + \lambda_L)[\sum_{i=S}^{L} \frac{\lambda_i}{\lambda_S + \lambda_L} \times \frac{a_i + b_i}{2}]. \tag{2}$$

The reader will note that the expression in the square brackets in equation (2) is the expectation $E[P]$ that we just computed in equation (1). Equation (2) and the discussion preceding it give us the answer to the question we posed at the beginning of this section 2.3.

2.4. Discussion

Our analysis of the two questions we posed and answered in sections 2.2 and 2.3 can be used for planning purposes in two ways. First, note that as stated in the first paragraph of section 2.1, one problem faced by a university planner concerns how many spots—in the parking lot under study—ought to be designated as either short-term or long-term. Now, this university planner will typically be working with existing stipulations for what precisely constitutes a short-term stay. So, for instance, short-term may mean one hour or less. If the mean parking time given by equation (1) is much less than one hour then this would suggest that, relative to the existing stipulation, most parkers in the lot under study are short-term parkers. Hence, it may make sense to designate a larger number of the spots in the lot under study as short-term spots.

Second, the fact that the number of occupied parking spots is Poisson distributed with mean given by equation (2) can be used to determine whether the capacity of the parking lot ought to be expanded. To see this more clearly, note the following. Given the findings in section 2.3 and conditional on the capacity of the parking lot, we can compute the probability that a car that enters the parking lot under study does not find a parking spot. When this happens, the car in question can exit the lot and this will involve a cost. Alternately, this car can simply "cruise around" until a spot opens up. From the standpoint of the university planner, the marginal benefit of expanding capacity on the lot would be the reduction in the loss stemming from not having fulfilled the extant demand for parking. In turn, the marginal cost of expanding capacity is the opportunity cost of the extra parking spot. Therefore, equations (1) and/or (2) and the discussion in this paragraph can be used to construct an objective function—such as an expected net benefit from parking function—that our university planner would maximize.

In our analysis thus far, we have not said anything explicitly about the "price" of parking in the university parking lot under study. This does not mean that price is an irrelevant factor. What it does mean is that this price issue is in the background of our current analysis. In this regard, note that our analysis has focused on a *physical* or *non-monetary* aspect of the university parking problem. As noted in sections 2.2 and 2.3, once we have an answer to our essentially physical question, we can then go about the task of determining the price of parking. One way to do this would be to solve an expected net benefit from parking maximization problem and then set the price or charge for parking to equal the marginal benefit from parking. A second way to determine a price for parking would be to set a base fee where the magnitude of this base fee would correspond to the mean parking time $E[P]$ in equation (1). Then, the actual fee paid by a parking individual would be higher or lower than this base fee depending on whether this individual's actual parking time is higher or lower than $E[P]$. As an illustration, suppose $E[P]$ equals 60 minutes in a specific instance. Then, we might set the base fee at $10. In this case, the actual fee paid by a specific parking individual would be higher (lower) than $10 depending on whether his or her actual parking time is higher (lower) than 60 minutes. This brings us to the second and final task of this note and this task involves focusing on parking rules violators and then calculating the probability distribution function of the number of violators who are fined at an inspection.

3. Parking Rules Violations

3.1. Preliminaries

Consider the representative parking lot of section 2.1. Our focus now is on individuals who park their cars in violation of one or more university parking lot rules. There can be a variety of parking lot rules but, for concreteness, let us consider two parking rules that exist in the campus of the Rochester Institute of Technology (RIT), in Rochester, New York. The first rule is that arriving vehicles must typically be registered in the parking office. The second rule is that whereas one may park in the more desirable reserved spots by paying a fee and then displaying the apposite reserved decal, individuals who have not paid the pertinent fee and hence do not possess the reserved decal may not park their cars in the reserved spots for any length of time.

These two parking rules are routinely broken at RIT but they are broken in an uncertain or stochastic manner. Now, to prevent the parking of unregistered cars and to ensure that people who have paid for the right to park in a reserved spot do, in fact, have a spot available to them, the parking office typically uses student inspectors to monitor the cars that are parked in the relevant parking lot. Unregistered cars and improperly parked cars are ticketed (fined) by the inspectors and hence inspections clearly generate revenue for the university.[10] Therefore, a university planner would be very interested in determining the probability distribution function of the number of violators who are ticketed at an inspection.

To compute this probability distribution function, it will be necessary to impose some more structure on the problem. To this end, suppose that potential parking rules violators enter the parking lot under study in accordance with a Poisson process with rate λ. Each potential rules violator parks his or her car in the lot for a time period that is Erlang $(2,\mu)$ distributed.[11] The university's policy is to inspect the parking lot under study every T time periods where T is fixed. Finally, each newly arrived rules violator is ticketed. We now proceed to our probabilistic characterization of the number of violators who are ticketed at an inspection.[12]

3.2. The probability distribution function

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We are, of course, assuming that the incentives within the university under study are such that violators "react" to tickets by paying the stipulated fines. If this were not the case then it would not make any sense to talk about the generation of revenues for the university. Finally, in this note, we are not interested in analyzing potential fine induced behavioral adjustments by parkers although the modeling framework employed here can be extended to account for this kind of behavior.

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The exponential distribution has been used previously—see Andersen and Rudemo (1982)—to model parking times. The Erlang distribution is a more general distribution than the exponential distribution in the sense that a *k*-Erlang distribution is a *k*-fold convolution of an exponential distribution. That is, it is the distribution of the sum of *k* independent and identically distributed (i.i.d) exponential random variables. In addition, the Erlang distribution is a large or two parameter family of phase-type distributions allowing only nonnegative values. In our case, $k=2$. This is because the parking times of rules violators can be thought of as being made up of *two* phases. There is a phase in which a rules violator's parked car is not ticketed and a phase in which this same rules violator's parked car is ticketed. Further, each phase is—following the work of Anderson and Rudemo (1982)—assumed to be exponentially distributed with the same parameter μ. These are the reasons for using the Erlang $(2,\mu)$ distribution to model the parking times of potential rules violators. See Tijms (2003, pp. 442-443) or Hillier and Lieberman (2005, pp. 798-800) for more on the Erlang distribution.

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The parking times of rules violators is assumed to be Erlang $(2,\mu)$ distributed. Therefore, the mean parking time of rules violators is $2/\mu$. This mean is *not* related to the mean computed in equation (1). In addition, in this third section of the note, we are not interested in the $2/\mu$ mean *per se*. Instead and as explained in section 3.1, we are interested in computing the probability distribution function of the number of violators who are ticketed at an inspection. The mean of this last probability distribution function is given below in equation (3). This mean is different from the equation (1) mean because the underlying probability distributions are different. In sections 2.1 and 2.2, we worked with the stationary Poisson process and uniform distributions. In contrast, in sections 3.1 and 3.2, we are working with a non-stationary Poisson process and an Erlang distribution. Finally, we have different answers for the means—equations (1) and (3)—and these means have to be considered separately because the questions posed in sections 2 and 3 are *different*.

Let X denote the random variable from the previous paragraph that is Erlang $(2,\mu)$ distributed. Now note that even though potential violators arrive at our parking lot in accordance with a Poisson process with rate λ, the arrival process of *violators* who will be ticketed is a non-homogeneous (also called non-stationary) Poisson process with intensity function $\lambda(t)$ where $\lambda(t)=\lambda\times Prob\{X>T-t\}$ for $t\in[0,T]$. This specification of the intensity function and the properties of Erlang distributed random variables together tell us that $Prob\{2>x\}=1-e^{-\mu x}-\mu xe^{-\mu x}$.

Using the last expression in the previous paragraph, we reason that the probability distribution of the number of potential rules violators who are ticketed at an inspection is Poisson distributed. Further, the mean of this Poisson distributed random variable is given by

$$\int_{t=0}^{T}\lambda(t)dt=\lambda\int_{t=0}^{T}[1-e^{-\mu(T-t)}-\mu(T-t)e^{-\mu(T-t)}]dt=[\lambda T-\frac{2\lambda}{\mu}](1-e^{-\mu T}). \qquad (3)$$

Equation (3) gives us the expression for the mean number of violators who are ticketed.

3.3. Discussion

Our analysis thus far in sections 3.1 and 3.2 can be used to facilitate planning in three ways. First, note that T is typically a control variable for our university planner. Therefore, this planner would like to know the relationship between T and the mean number of violators who are ticketed because knowing this relationship will allow the planner to know the nature of the relationship between T and the revenue from the issuance of parking tickets. Differentiating the right-hand-side (RHS) of equation (3) with respect to T tells us that $\partial[\{\lambda T-(2\lambda/\mu)\}\{1-e^{-\mu T}\}]/\partial T=\lambda\{1+\mu Te^{-\mu T}-3e^{-\mu T}\}$. This derivative is positive when $e^{\mu T}+\mu T>3$. Therefore, when this last inequality holds, raising T will increase ticket revenues. However, it is clear from our analysis thus far that the inequality specified above need not always hold and hence, in general, there is no necessary monotonic relationship between the university planner's control variable T and revenue from the issuance of parking tickets. Put differently, the nature of this relationship between T and ticket revenues depends on the *specifics of a given situation* and a university planner can gain additional insight into the nature of this relationship by conducting numerical analysis with specific values for the parameter μ. From footnote 11 we know that $(1/\mu)$ is the *mean* length of each of the two phases that make up the total parking time of a rules violator. Therefore, for the purpose of the numerical analysis that we have just referred to, the selection of reasonable values of the parameter μ will depend on an examination of extant data on parking enforcement for the university parking lot under study.

Second, suppose that our university planner would like to maximize the revenue from the issuance of parking tickets. In this case, this planner could set up and solve a "revenue from parking tickets" function maximization problem in which the optimal value of T or the time between successive inspections is determined endogenously. One way to approach the question of maximizing the revenue from parking tickets function by selecting T optimally would be to use renewal-reward theoretic methods of the sort employed in Batabyal and Beladi (2002). Specifically, when a renewal-reward theoretic framework[13] is used, the so called renewal-reward theorem[14] can be used to set up, for instance, the revenue from parking tickets function in which T is an argument. Then, the optimal T, T^*, will be that value of T which maximizes this renewal-reward theoretic

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Renewal-reward processes are a particular kind of stochastic process. See Kulkarni (1995, pp. 452-459) or Tijms (2003, pp. 33-80) for more on such processes.

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A succinct statement of the renewal-reward theorem can be found in Tijms (2003, p. 41).

revenue from parking tickets function that we have just talked about.

Finally, keeping the political fallout from the issuance of excessive parking tickets in mind, the RHS of equation (3) can play the role of a constraint in either a constrained expected total cost of parking inspections minimization problem or a constrained expected net benefit from the allocation of parking facilities maximization problem. Here, the idea would be to choose one or more control variables to ensure that, probabilistically speaking, the number of violators who are ticketed at an inspection is no larger than some maximal acceptable number. [15]

4. Conclusions

In this note, we conducted a preliminary investigation of parking issues on university campuses. Specifically, we analyzed two hitherto unstudied issues that arise from the excess demand for parking facilities and from the probabilistic nature of this demand. We first focused on short-term and long-term parkers. We determined the mean parking time of an arriving car and then we computed the probability distribution function of the number of occupied parking spots at any particular time. Second, we concentrated on parking rules violators and we calculated the probability distribution function of the number of violators who are fined at an inspection.

Our goal in this note was not to conduct an exhaustive analysis of parking issues in university campuses. Therefore, it is certainly the case that our analysis in this note can be extended in a number of directions. Here are two specific suggestions for extending the research described in this note. First, it would be useful to generalize our analysis by examining scenarios in which parkers arrive at a university parking lot in accordance not with Poisson processes but with more

[15] Section 2.4 contains a brief discussion of the optimal capacity of a parking lot. However, a full discussion of this issue is beyond the scope of this note and such a discussion will need to be part of a more elaborate study of parking issues in university campuses.

general renewal processes. Second, following the discussion in sections 2.4 and 3.3, it would be useful to set up and solve optimization problems in which, *inter alia*, the optimal value of T is determined endogenously. Studies of parking policies in universities that incorporate these features of the problem into the analysis will provide additional insights into a contemporary problem that has salient economic and social ramifications.

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