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THE ABNEY EFFECT: CHROMATICITY COORDINATES OF UNIQUE AND OTHER CONSTANT HUES

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Abstract—We compared unique and other constant hue loci measured at a fixed retinal illuminance for the same observers. When expressed in Judd chromaticity coordinates, unique hue and constant hue data agreed. Unique blue loci were curved, and unique red and green loci were noncollinear. These data imply that unique hues are not a linear transformation of color matching functions. Linear models are only an approximation, even at a single luminance level.

Color Vision Abney effect Unique hues

INTRODUCTION

Constant hues are colors equal in hue, but different with respect to another attribute, such as saturation. For instance, two lights may appear the same hue, e.g. blue, when one is a spectral light and the other is a mixture of two or more lights. Since mixtures of a chromatic light and an achromatic light often differ in hue as well as saturation (Abney, 1910), constant hue loci generally do not form straight lines in the CIE chromaticity diagram (Newhall, 1940; Newhall *et al.*, 1943; Wyszecki and Stiles, 1967, pp. 564–565). While there is disagreement about which constant hue loci are straight, most plots of constant hue loci show fairly straight lines for yellows, but curved lines for greens, reds and blues. These curved lines represent the Abney effect.

Unique hues are colors which have unitary or psychologically unique percepts (Dimmick and Hubbard, 1939a). Four unique hues are recognized: red, green, yellow and blue. For instance, unique yellows are those colors which are yellow, but neither reddish nor greenish. Since unique hues match in hue regardless of their saturations, they are also constant hues.

Color vision models based upon perceptual aspects of color use the four unique hues as parameters (e.g. Jameson and Hurvich, 1955; Ingling and Tsou, 1977). Unique yellow and blue are neither red nor green and have been considered as balance or null points for a red/green color opponent mechanism, i.e. there is no response to a unique yellow or blue. Likewise, unique red and green are considered the balance or null points for a yellow/blue mechanism. Many investigators find or predict balance points of the red/green opponent mechanism consistent with a linear combination of cone fundamentals (e.g. Schrödinger, 1925; Judd, 1949; Hurvich and Jameson, 1955; Judd and Yonemura, 1970; Larimer *et al.*, 1974; Raaijamakers and de Weert, 1975; Ingling *et al.*, 1978a; Romeskie, 1978). However, for the yellow/blue mechanism some investigators conclude that a nonlinear combination of cone fundamentals is necessary (Koenderink *et al.*, 1972; Larimer *et al.*, 1975; Ingling *et al.*, 1978a; Nagy, 1979; Werner and Wooten, 1979; Ikeda and Ayama, 1980).

Linear models predict unique hue loci which are straight lines in the chromaticity diagram. In addition, both the unique red and unique green loci and the unique yellow and unique blue loci should be collinear, resulting in two intersecting straight lines. Nonlinear models with linear balance points make the same predictions. (Linear balance points do not imply linear mechanism response; Krantz, 1975.) The predictions of linear unique hue loci disagree with constant hue loci data (Newhall *et al.*, 1943; MacAdam, 1950; Yonemura, 1970), which are curved near unique hue loci. The linear models also predict that a mixture of unique hues that are the balance points of one mechanism should appear unique (cf. Krantz, 1975), disagreeing with the unique hue data of Dimmick and Hubbard (1939a, b) and Valberg (1971) which show nonlinear unique hue loci.†

We compared unique and other constant hue loci under similar conditions for two observers. Constant hues were determined by matching a variable light to a standard light, and unique hues were determined by matching a variable light to an internal representation. Since our stimuli included desaturated lights, we used a color mixture technique. With two or more suitably chosen primaries, we could generate and

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†Constant hue loci were presented at the ARVO annual meeting (Elsner *et al.*, 1979) and additional unique hue loci and modeling at the OSA annual meeting (Burns *et al.*, 1979).

specify any chromaticity required by the observer. Our data were displayed in the Judd (1951a) chromaticity diagram for four reasons. First, this diagram facilitates the comparison of data with the linear model predictions, since it is a projective transformation of color matching functions (cf. Vos, 1978). Second, constant hue loci are usually plotted in chromaticity diagrams (cf. Fry, 1945; MacAdam, 1950; Judd and Wyszecki, 1956; MacAdam, 1969; Yonemura, 1970; etc.) Third, using a mixture diagram prevents treating metamers as separate data points. Fourth, using chromaticity coordinates allows us to express our results in a way which does not depend on the exact desaturant, as long as we know the chromaticity coordinates of the desaturant.

GENERAL METHODS

Subjects

Two of the authors served as observers, both with normal color vision, as ascertained by Rayleigh matches, the Farnsworth Munsell 100 Hue Test, and plate tests. Observer S.B. was corrected to normal acuity with lenses placed before the exit pupil, while A.E. wore contact lenses. Both observers were experienced and knowledgeable about the experimental hypotheses, but naive about their own results during a session.

Apparatus

A four channel, Maxwellian view optical apparatus (Burns *et al.*, 1982) presented stimuli and recorded responses under computer control. Each channel had a neutral density wedge and shutter located at a secondary image of the source. Lightly ground glass was placed just after this image, resulting in a highly uniform field. Light from channel 1 was adjustable in wavelength by means of a monochromator. The spectral composition of lights from channels 2, 3 and 4 was controlled by use of interference (Ditric Optics) or color compensating filters (Kodak). The channels were recombined by either a 100% contrast mirror grating or a beamsplitter. The mirror grating was produced by engine ruling a grating onto the silvered hypotenuse of a right angle prism. A second right angle prism was cemented to the first prism to form a cube with alternating reflecting and transmitting stripes. The grating and the beamsplitter were mounted on separate carriers to facilitate rapid and reproducible changes between field configurations. An adjustable bite bar allowed the eye to be placed at the 0.8 mm artificial pupil. An achromatizing lens was placed in the beam before this pupil.

A computer controlled and monitored the digital and analog apparatus components. Neutral density wedges, as well as the monochromator drive, were controlled by d.c. servo systems. All adjustments of either wavelength or wedge position were made via a feedback loop with the computer. Shutters could be controlled by either the computer, the experimenter, or an observer-operated function generator.

To obtain high purity throughout the spectrum, we minimized stray light. Stray light from the monochromator was at least 4.5 log units less than the peak transmission and was further attenuated at the spectral extremes by the use of blocking filters. Inconel neutral density filters were calibrated for relative density at the exit pupil by means of a photomultiplier tube (RCA 1P28) and microammeter (Farrand) operated within their linear ranges. Spectral density curves for these filters were obtained with a Beckman DU spectrophotometer. (For further instrumentation and calibration details, see Burns *et al.*, 1982.)

Stimuli

The circular test field subtended either 2 deg or 40 min of visual angle. The rest of the visual field was black. All stimuli were 20 td. The chromaticity coordinates of the white were $x = 0.362$, $y = 0.394$ for Experiment 1 and $x = 0.322$, $y = 0.396$ for the other experiments. To avoid biasing the judgments (Jameson and Hurvich, 1967), we used no fixation point.

Procedure for obtaining constant illuminance

Stimuli were equated in retinal illuminance by heterochromatic flicker photometry (HFP). For one HFP measurement, a channel flickered in counterphase against a standard channel of 20 td, while the observer adjusted a neutral density wedge to minimize flicker. The flicker rate was under observer control. At least three measurements were made per wavelength per channel. For the monochromator channel, each set of measurements covered about 100 nm in roughly 3 nm steps. The computer interpolated wedge settings for all intermediate wavelengths. No test wavelength was near the end of this calibration. These calibrations were stored and accessed by the computer.

The experimenter set the colorimetric purity (either 1.0, 0.79, 0.63 or 0.50), while maintaining constant retinal illuminance, by inserting calibrated neutral density filter combinations into two or more channels. Colorimetric purity is

$$C_p = \frac{L_\lambda}{L_\lambda + L_w}.$$

Where L_λ is the luminance of the dominant wavelength and L_w is the luminance of the white. Chromaticity could also be varied at constant retinal illuminance under computer control by mixing light from two channels. As either the wavelength or the proportion of two channels was adjusted, whether by the observer or the computer, the appropriate neutral density wedges were turned by the computer so that retinal illuminance remained constant. All HFP or hue matching judgments were made only after the wedge had stopped.

Procedure for determining unique and constant hues

The method of adjustment was used for hue matching a variable hue to a standard and for setting unique hues. The observer made his setting quickly, then signaled the experimenter or computer that he was finished. The experimenter could offset the observer's setting between trials. There was no equipment endpoint at either end of the adjustment, nor was there any tactile cues from the adjustment knob and drive shaft. We required that the observer have a precise visual criterion on each side of the setting to participate in the condition. For instance, when setting a unique blue, an observer had to see a variable light too red when adjusting in one direction and too green in the other direction. These visual endpoints allowed the observer to make successive approximation adjustments quickly. For desaturated stimuli the method of adjustment allowed the observer to determine whether there were clear visual criteria or not. The method of adjustment produced results similar to a double random staircase technique (Experiment 2), but was much faster. Since unique hues drift over time (Osaka *et al.*, 1978), it was important to gather a complete set of conditions in a single session.

A session of about 45 min was devoted typically to either (a) hue matching lights which varied in colorimetric purity to a single standard light or (b) determining unique reds and greens or yellows and blues. The observer was dark adapted 5 min before making any adjustments. In each session there were 15 adjustments per unique hue condition or 20 per other constant hue condition, usually gathered in blocks of five adjustments.

Except where noted, a block of each condition was performed in random order before any block was repeated. While adjusting the hue, the observer looked at the center of the circular test field, but could look away or blink between adjustments. Between blocks, shutters obscured the test field. The initial dark adaptation period and the randomization of blocks were adopted to minimize adaptation effects. The data showed no consistent effect of either time elapsed within a session or the previous block type, indicating that these measures were successful.

EXPERIMENT 1: CONSTANT HUE LOCI

We determined constant hue loci by hue matching a variable light to a standard light at constant retinal illuminance. The variable light changed in colorimetric purity across trials.

Methods

Standard lights were spectral lights of 440, 470, 500, 520, 530, 560, 590 or 610 nm. A square-wave grating of 0.54 c/deg visual angle divided the 2 deg test field into three parts. The variable light was the center stripe, flanked by two stripes of the standard light. (This stimulus is similar to a bipartite field with a phase shift of 1/4 cycle.) We chose this configuration to decrease color changes across the field due to asymmetric chromatic induction.

The experimenter set the purity of the variable light by mixing the instrument white with spectral light from the monochromator at constant retinal illuminance. Then the observer adjusted the wavelength of the variable light to match the standard in hue. As the wavelength changed, the wedge in the monochromator channel was adjusted by the computer to maintain constant retinal illuminance. Thus, the colorimetric purity was constant throughout the adjustment.

Results

Our constant hue loci, presented in the Judd (1951a) chromaticity diagram (Fig. 1), agreed with previous measurements of the Abney effect (Judd, 1972). That is, generally, lines of constant hue are curved. Our data showed that desaturating a "red" or "orange" bar caused it to appear redder than the standard bar. Desaturating a "blue" bar caused it to

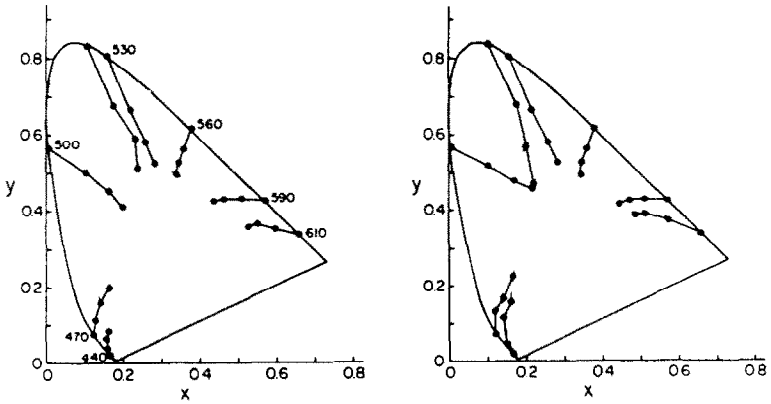

Fig. 1. Constant hue loci plotted in the Judd (1951a) chromaticity diagram: observer S.B. left, A.E. right. Error bars indicate confidence intervals of 2 SEM around the mean. For data with no error bars, either the confidence interval was less than 1 nm, or too small to be seen on the figures.

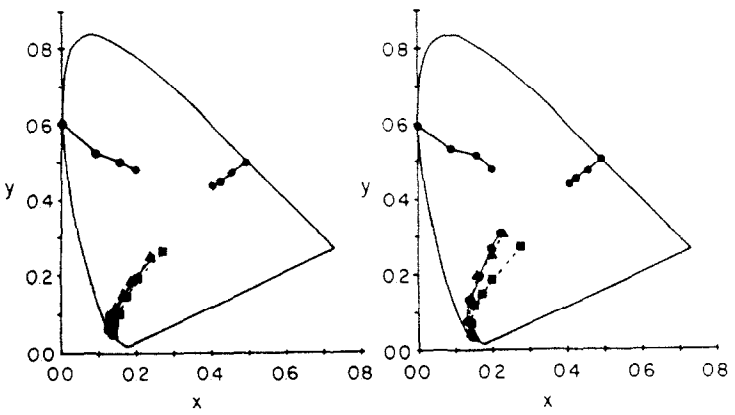

Fig. 2. Unique blue, green and yellow loci plotted in the Judd (1951a) chromaticity diagram: observers as above. Unique hues were obtained as mixtures of white and monochromatic lights (circles and solid lines), as mixtures of unique yellow and monochromatic lights (squares and dashed lines), or as mixtures of a 571 nm light and monochromatic lights (triangles and dashed lines).

appear redder than the standard bar*. To maintain a hue match the desaturated variable lights had to be adjusted to shorter wavelengths for the long wavelength standards and to longer wavelengths for the short wavelength standards.

EXPERIMENT 2: UNIQUE HUE LOCI—WAVELENGTH ADJUSTMENT TECHNIQUE

The existence of an Abney effect for most hues places a constraint on color opponent models. If these models predict linear unique hue loci, then the loci must fall in a region of chromaticity space where constant hues do not curve. Since our constant hue loci for 560 nm were nearly straight, they are consistent with a unique yellow around 575 nm with little or no curvature. However, all our blue constant hue loci were curved in the same direction, and, thus, are inconsistent with linear red/green opponent models. To compare the constant hue data with unique hue data we measured the chromaticity coordinates of (a) unique blues (lights which appear blue but neither red nor green), (b) unique yellows (lights which appear yellow but neither red nor green), and (c) unique greens (lights which appear green but neither yellow nor blue). All unique hues in this experiment were generated by proportionally mixing a light adjustable in wavelength with a white or spectral desaturant. No measure of unique red could be obtained with this method, since all long wavelength test fields appeared yellowish (e.g. Dimmick and Hubbard, 1939b). This method was virtually identical to that used for measuring constant hues, except that there was no physical standard. It is unlikely that the presence of an adjacent physical standard is critical in obtaining curved constant hue loci, since published constant hue loci gathered with different stimulus configurations and procedures show a similar Abney effect. For instance, similar Abney effects can be obtained with a ratio technique, in which the similarity of spatially separated colors is judged (Newhall *et al.*, 1949), and a matching technique (MacAdam, 1950).

Methods

The purity of the variable light was set using the fixed neutral density filter combinations. The observer adjusted the wavelength of the variable light to obtain a given unique hue. Unique blue loci were also obtained with a unique yellow and a 571 nm desaturant. The wavelength of the unique yellow desaturant was determined immediately before the session. Typically, there were 2 sessions per unique hue for each desaturant.

Results and Discussion

The chromaticity coordinates of the unique hue loci are shown in the Judd (1951a) chromaticity diagram (Fig. 2). For an observer with normal color matching functions, if linear color opponent mechanisms are assumed, then unique hue loci should plot as straight lines. The unique yellow and green data were roughly linear. For unique blue, however, this was not the case. Desaturating a spectral unique blue made it look reddish. This finding held for both observers, whether the desaturant was the instrument white, 571 nm (a slightly greenish yellow), or their own unique yellows. For the unique yellow desaturant, not only the change in chromaticity coordinates but also the change in wavelength, was a measure of the Abney effect. The wavelength change for mixtures of unique blue and unique yellow was 7 nm for S.B. and 8 nm for A.E. This result implies that the red/green opponent mechanism is nonlinear,

*Huff and Guth (1968). The effect on apparent hue of adding white to monochromatic lights. Personal communication of a paper presented at *Midwest Psychol. Assoc.*, Chicago, IL.

most obviously for short wavelength, high purity lights.

Comparison of unique and other constant hue loci. Our unique hue and other constant hue loci agreed. In the blue region of the chromaticity chart, constant hue loci bracketed the unique blue loci. The constant hue loci originating at 470 nm appeared slightly green, yet curved in the same direction as the unique hue loci. The unique yellow loci were fairly straight. They fell in a region where there is little curvature of constant hue loci. The unique green loci and 500 nm constant hue loci also agreed.

We found that the red/green color opponent mechanism is nonlinear, whereas most previous investigators conclude that it is linear. A potential reason for this disagreement is the use of different psychophysical techniques. To examine this we replicated our unique blue results using a forced choice, double random staircase, technique.

Confirmation with a brief presentation time using a double random staircase procedure. One of the authors (S.B.) was the subject. Equiluminant stimuli were presented for 250 msec, once every 12 sec. The desaturant was a 571 nm, "greenish-yellow" stimulus. On each trial the observer used a key press to signal "too red" or "too green". Between trials the computer adjusted the wavelength according to the previous wavelength and response in that staircase. The step size was decreased by octave steps during the first three reversals. The experiment continued until each staircase had at least five more reversals. Results are shown in Fig. 3, together with results using the method of adjustment and the 571 nm desaturant. The locus of unique blue was curved for both sets of data.

EXPERIMENT 3: UNIQUE RED AND UNIQUE GREEN LOCI— PRIMARY MIXTURE TECHNIQUE

The yellow/blue mechanism has a nonlinear term in several color opponent models (Larimer *et al.*, 1975; Werner and Wooten, 1979). Is this nonlinearity reflected in unique red and green loci? While published constant hue loci differ somewhat in the middle wavelength region of the chromaticity chart, generally there is little curvature near unique green. Our unique green loci did not curve. However, constant hue loci may curve near unique red (Newhall *et al.*, 1943), although the literature is inconsistent on this point (cf. Judd, 1972). A second possibility is that unique green and unique red are each linear, but they may not be collinear. Consistent with these possibilities, unique green and unique red are not complements (Dimmick and Hubbard, 1939b).

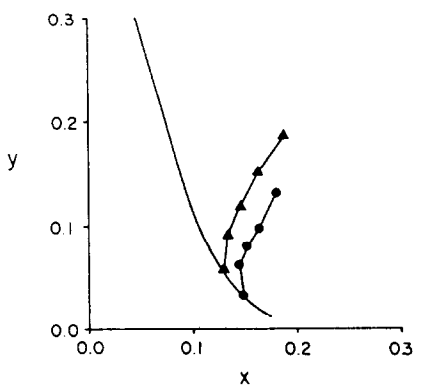

Fig. 3. Comparison of unique blue loci for observer S.B. using a double random staircase technique (circles) and the method of adjustment (triangles). The desaturant was 571 nm for both experiments. Data were collected about 3 months apart.

We determined unique red and unique green loci for lights matched in retinal illuminance, but varying in chromaticity. We mixed a given short wavelength light with a series of long wavelength lights. Our method differed from cancellation paradigms in that our observers adjusted the proportion of the short wavelength light in the mixture, instead of its wavelength or illuminance. Our paradigm was designed to show (a) whether the unique red loci are curved, and (b) whether unique red and unique green are collinear.

Our paradigm also tested a specific model of the yellow/blue mechanism in which yellow is proportional to luminosity (or $\bar{y}$) and blue to the short wavelength sensitive (SWS) cone fundamental, (e.g. Judd, 1951b; Jameson and Hurvich, 1955; Boynton, 1979). A mechanism of this type has constant yellow output for spectral lights of constant retinal illuminance at wavelengths longer than 540 nm (for which there is minimal SWS cone sensitivity). When any of these long wavelength lights is mixed with a short wavelength light to obtain unique red or green, the model prediction is that the proportion of short wavelength light should be constant.

Methods

To measure unique reds and greens, we mixed a short wavelength light (either 450 or 470 nm) with a series of long wavelength lights (530, 550, 570, 580, 590, 610 and 640 nm). Two degree targets were tested.

The experimenter set each long wavelength light on the monochromator. The observer matched the long and short wavelength lights with HFP, then made 15 settings of unique red or green. The proportion of short wavelength light in the mixture was adjusted at constant retinal illuminance by the computer. There were four sessions, two sessions with each short wavelength light.

Results and Discussion

Constant unique red and unique green loci are plotted in the Judd (1951a) chromaticity diagram in Fig. 4. Chromaticity coordinates were the average of two sessions. There was no striking curvature of the unique red loci. Unique red loci were not collinear with unique green loci. An equiluminance mixture of unique green and unique red would appear yellowish.

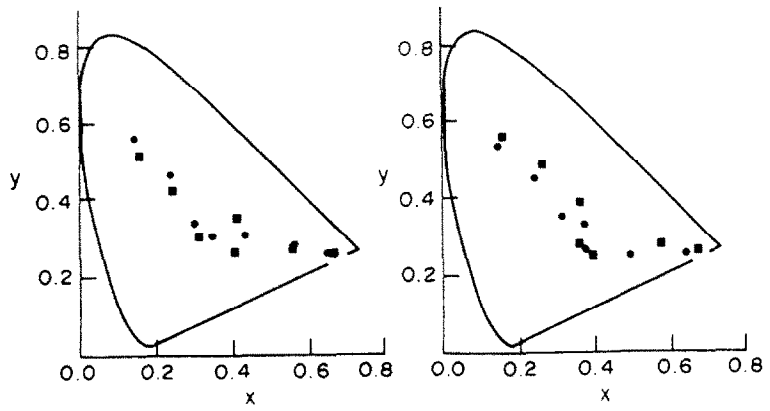

Fig. 4. Unique red and green loci plotted in the Judd (1951a) chromaticity diagram: observers as above. Two primaries, a long and short wavelength, were combined at constant luminance. Squares—450 nm short wavelength primary; circles—470 nm short wavelength primary.

The proportion of short wavelength light in the mixture of short plus long wavelength light was neither constant nor monotonic. For our primaries, the proportion of short wavelength light was maximum for mixtures with 570, 580 or 590 nm lights. Mixtures with longer or shorter wavelength lights required smaller proportions.

The results showed that the yellow/blue opponent mechanism is not a linear transformation of color matching functions. This conclusion depends upon only the following assumptions: first, SWS cone absorptions at wavelengths longer than 540 nm are insignificant (Judd, 1945; Smith and Pokorny, 1975); second, the ratio of long wavelength sensitive (LWS) to middle wavelength sensitive (MWS) cone absorptions is a monotonic function of wavelength for the 540 to 670 nm primaries (Walraven, 1974; Smith and Pokorny, 1975); third, luminosity is roughly a linear combination of cone fundamentals for each observer.

Confirmation with a 40 min field and with three primaries. To test the generality of our 2 deg findings, we also examined unique hue loci for a 40 min target. For the unique blue-unique yellow loci we used the two-primary technique, with the subject's own unique yellow as the desaturant. These data were collected at about the same time as the 2 deg data.

The 40 min and 2 deg results are compared in Fig. 5. For the higher purity unique blues, most 40 min settings were shorter in wavelength than those of the 2 deg fields ($P < 0.01$*), but the nonlinearity of the red/green mechanism remained. For unique yellow there was no significant difference between the 2 deg and the 40 min settings. These data were consistent with the decreased redness of small, short wavelength fields (Ingling *et al.*, 1969).

We also redetermined unique red and green loci by

*Confidence intervals were calculated around the mean of the 2 deg data, using a one-tailed t distribution, $\alpha = 0.01$, and d.f. = 29. Differences were expected in the direction of requiring more SWS cone input or shorter wavelength adjustments for the 40 min field size.

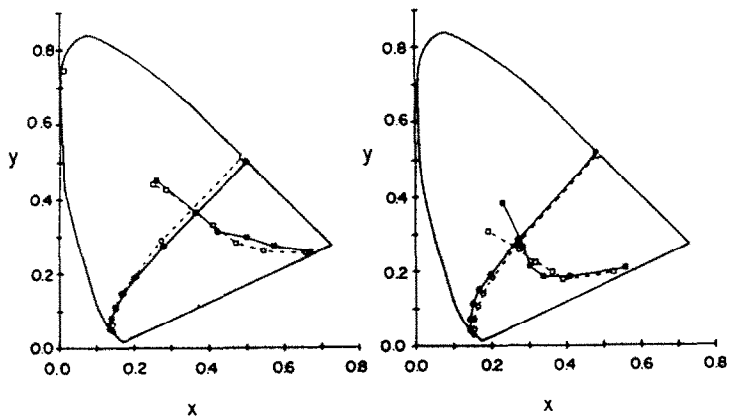

Fig. 5. Comparison of unique hues generated for the 2 deg (solid lines) and 40 min (dashed lines) conditions: observers as above. Unique blues and yellows were determined by mixing various proportions of each observer's unique yellow and a variable short wavelength light. Unique red and green loci were determined using a three primary method.

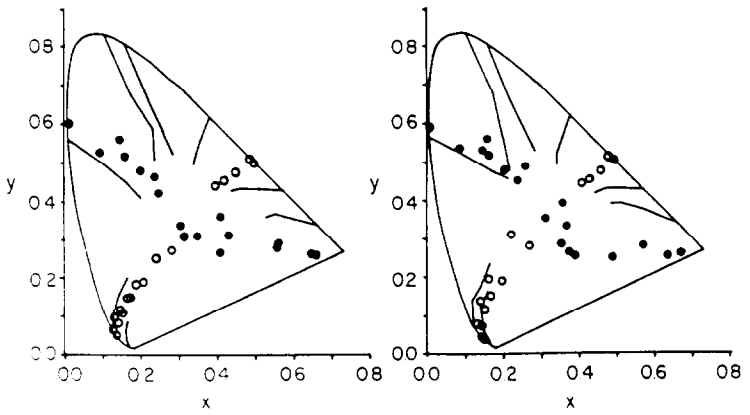

Fig. 6. Summary of constant hue data plotted in the Judd (1951a) chromaticity diagram: observers as above. Solid lines-data from hue matching at 0.54 c/deg; open symbols—unique yellows (white desaturant) or unique blues (white and unique yellow desaturants); closed symbols—unique reds and greens (spectral and two-primary data).

proportionally mixing three primaries, 510, 460 and 650 nm, matched by HFP. This allowed us to present the various stimuli in a random block design, and thus minimize possible adaptational affects in the measurement of the unique red and green loci. Both 2 deg and 40 min stimuli were tested. For both field sizes, the unique red and unique green loci were noncollinear (Fig. 5). The unique red and green data shown in Fig. 5 were gathered several months after the data shown in Fig. 4.

Variability of the data. The variability of the unique and the other constant hue data was low enough for meaningful comparisons between tasks. Constant hue data collected over a period of 3 months (Fig. 6) are in good agreement.

The variability of unique hue data within a session was smaller than across sessions. For instance, in a given condition with unique red and green, the variability of the proportion blue set within a session was much less than across sessions. Unique red and green loci were never collinear within a session. Likewise, the equiluminous mixtures of unique yellow and unique blue always looked reddish. Unique red and blue varied more than unique green or yellow both across sessions (Osaka *et al.*, 1978) and within sessions.

The variability of our spectral constant hue loci compared favorably with wavelength discrimination studies. In our spectral task, as in the chromaticity matching task of MacAdam (1942), the criterion was equality of the variable and standard fields. We formed confidence intervals from 3 times the standard deviation (20 settings). Discrimination as a function of wavelength agreed well with previous wavelength discrimination results. Discrimination was best at 560 nm (1.07 and 1.42 nm for S.B. and A.E., respectively) and at 440 nm (1.26 and 1.49 nm). Discrimination was poorest at 470 nm (1.80 and 2.87 nm), 520 nm (2.13 and 3.03 nm), and 610 nm (1.80 and 2.02 nm). The data of 500 nm (1.77 and 1.50 nm) and 590 nm (1.74 and 1.65 nm) were intermediate. Medians agreed with means. The variability was higher for nonspectral constant hues, since the criterion was equality only of hue, i.e. there was a saturation difference between variable and standard. However, in most cases the variability was too small to be seen in Fig. 1.

GENERAL DISCUSSION

Constant hue loci are typically curved (the Abney effect) when plotted in the Judd (1951a) chromaticity diagram. Yet this effect is rarely considered with regard to unique hues, which are constant hues as well as the balance points for many models of chromatic mechanisms. Measurements of constant and unique hues were consistent for our observers (Fig. 6). The unique blue loci curved in a manner similar to the blue constant hue loci. The unique yellow loci were fairly straight, and fell in a region of the chromaticity diagram where there is little curvature. The unique green loci were straight, as were the green constant hue loci. The unique red loci were fairly straight, and were bracketed by constant hue loci curving in opposite directions. Unique reds were not collinear with unique greens. The noncollinearity is consistent with the chromaticities of typical unique reds, greens and whites specified in the literature (cf. Dimmick and Hubbard 1939b; Valberg 1971).

If color opponent mechanisms are modeled with the unique hues of our two observers, then neither color opponent mechanism can be a linear transformation of color matching functions. Whether our observers have color matching functions which differ from the Judd observer is immaterial to this conclusion. Since a mixture of an observer's unique blue and unique yellow appeared reddish (Experiment 2), his red/green mechanism must be nonlinear regardless of his color matching functions. Also, for unique reds and greens, (Experiment 3) the amount of blue

Table 1. Unique blue determinations for a variety of luminances, field sizes, and methodologies

Study	No. of observers	Range of unique blue (nm)
Purdy (1931)	1	474–478
Dimmick and Hubbard (1939)	10	467–482
Hurvich and Jameson (1955)	2	467–475
Rubin (1961)	262	468.3
Boynton and Gordon (1965)	3	462–474
Akita *et al.* (1964)	4	455–473
Larimer *et al.* (1974)	5	474–484
Eichengren (1976)	4	458–470
Gordon and Abramov (1977)	2	460–470
Osaka *et al.* (1978)	3	463–481
Nagy (1979)	3	468–490
Werner and Wooten (1979)	3	468–469
Larimer (1981)	3	465–475
Present study	2	462, 468

Values were obtained either from published tables or by interpolation. The entry for Rubin (1962) represents a mean for 262 observers. The values for the present study are the averages for the spectral unique blues in the unique yellow desaturant condition of Experiment 2.

required in a mixture with long wavelength primaries was a nonmonotonic function of wavelength. This finding implies that the yellow/blue mechanism is nonlinear given reasonable color matching functions. That our observers have reasonable color matching functions is supported by two findings. First, our radiometric and colorimetric methods for specifying white agreed well (Burns *et al.*, 1982). Second, departures from linearity were similar for our two observers.

Although our constant hue loci are consistent with previously published loci, our conclusions are not consistent with many studies of unique hues. In addition to individual differences, there are several possible causes for this disagreement. First, consider the possibility that our unique blues are atypical, and this leads to a spurious finding of nonlinearity. The spectral unique blues of our observers are shorter than some in the literature. Examination of Table 1 shows that measured spectral unique blues have ranged between 455 and 484 nm. This range is not segregated according to methodology. For instance, Boynton and Gordon (1964), using a color naming technique obtain 100 td unique blues ranging from 465 to 472 nm, and Nagy (1979) obtains values ranging from 469 to 484 nm using a staircase procedure and 1 sec flashes. For our observers and the 2 deg target, A.E. typically had unique blues in the low 460's, while S. B. had values in the high 460's, and occasionally as long as 471 nm. These values are not due to the use of the method of adjustment since the staircase procedure resulted in a shorter unique blue and the nonlinearity persisted (Experiment 2).

A second possibility is that some methodologies do not test a full range of chromaticities. In a cancellation paradigm, many measurements are made on a very limited range of chromaticities, with only a few data points in the crucial regions of chromaticity space. Consider a subject with a unique blue unique yellow locus similar to ours, and a unique green of 520 nm (a representative value). The "green" valence curve would be obtained by cancelling the greenness in wavelengths between 480 and 560 nm with a 670 nm light (a representative value for the long wavelength primary). The "red" valence curve would be obtained by cancelling wavelengths from 400 to 460 nm and from 570 to 700 nm. Figure 7 shows the chromaticities which result from this procedure. Note that the entire long wavelength redness valence curve plots in a very restricted region of chromaticity space. A linear combination of photopigments must fit these points since they have nearly identical chromaticities. Thus if a cancellation procedure tests a limited range of chromaticities and the statistical analysis does not take this into account, linearity may be concluded.

A third possibility is that stray light affects some of the measurements of unique blue reported in the literature. Given the sharp curvature of the blue loci which we measured, small amounts of stray light could shift unique blue to longer wavelengths. We estimated that for our observers a stray light of 0.1% of the input energy would have caused a shift in the spectral unique blue of about 2 nm*. Higher levels of stray light or the use of sources weighted towards the long wavelengths (e.g. tungsten) would result in greater shifts. Since the nonlinearity is most pronounced at high purities this could also lead to spurious findings of linearity. For our experiment the

*The possible effect of stray light can be computed as follows. Suppose that a monochromator is 100% efficient at its peak wavelength, has a stray light rejection of three log units, a half-bandwidth of 10 nm, and that the light source is an equal energy spectrum. The light within the passband can be treated as a triangle function with half of the energy that would be present in a 10 nm band of the source spectrum. Between 400 and 700 nm there are thirty 10 nm bands of stray light, each with 0.1% of the input energy. The result is a stray light equivalent of a 6% desaturation by an equal energy white.

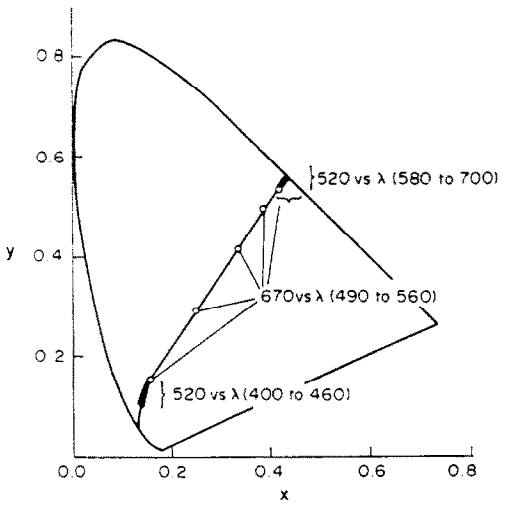

Fig. 7. Example of the chromaticities tested in a hypothetical cancellation experiment. The primaries used are 520 and 670 nm. The thin solid line represents an actual unique blue unique yellow locus. The heavy dark areas represent areas of chromaticity space where the cancellation paradigm would make at least six determinations. The circles represent points where the cancellation paradigm would make a single determination. If all determinations are treated independently there will be a strong bias towards the conclusion of linearity based on the laws of color mixture.

stray light was attenuated to 0.003% or less by the monochromator and was further attenuated by blocking filters (Burns *et al.*, 1982).

Constraints on modeling

As discussed above, no linear model will fit our data. Models which assume that (a) mechanism response is at baseline for unique hues and (b) the cone outputs are summed and differenced prior to a nonlinearity (e.g. an exponent) predict our data no better than do linear models. Our data may be explained by models (a) which apply a nonlinearity to one or more cone outputs before a summation occurs or (b) which have two or more summation terms and apply a nonlinearity to one or more of them before combining terms for a mechanism output.

If the nonlinearity is modeled at the cone level, the class of nonlinearities is limited by the locus of white. For instance if exponential nonlinearities with different powers on different terms are used (e.g. Larimer *et al.*, 1975; Werner and Wooten, 1979) then the locus of white will change with illuminance (Pokorny *et al.*, 1981). Such a change in color appearance may occur, but it is not large. Thus, acceptable models should use a different type of nonlinearity (e.g. one which is self-normalizing). A type of nonlinearity which meets this criterion is the Naka–Rushton equation (Naka and Rushton, 1965). Seim and Valberg (1980) have proposed such a model to explain the effects of chromatic induction. Our unique yellow and blue data may be modeled by putting such a nonlinearity on the output of the cones.

$$R(\mathrm{SWS}) = \frac{S(\mathrm{SWS})^n}{S(\mathrm{SWS})^n + \sigma}$$

where $R(\mathrm{SWS})$ is the response of the SWS cone mechanism, $S(\mathrm{SWS})$ is the quantal catch of the SWS cone, n is a real number, and σ is the semi-saturation constant. Mechanism response can then be expressed as

$$r/g = K_1[R(\mathrm{SWS})] - K_2[R(\mathrm{MWS})] + K_3[R(\mathrm{LWS})]$$

where r/g is the response of the red/green color opponent mechanism, and the k_i's are constants.

For the red/green mechanism such a model adequately fit our data for a wide range of parameters. The role of the nonlinearity on the output of the LWS and MWS cones is small, since the change in their relative sensitivity for the chromaticity range of unique yellow and blue is relatively small.

Attempts to model the yellow/blue opponent mechanism using this approach were less successful. Although a qualitative fit was obtained, a quantitative fit required additional arbitrary parameters. For the yellow/blue opponent mechanism if the nonlinearity is modeled as acting on a difference term, then a number of approaches are possible. One approach is to use an exponential nonlinearity (Larimer *et al.*, 1975; Werner and Wooten, 1979). Both of these models are inconsistent with the present as well as other data. (See Pokorny *et al.*, 1981.) A more successful approach is to apply a nonlinearity to a difference term which approaches zero near white. This allows the locus of white to remain constant as illuminance is varied. With this approach a wide range of nonlinearities can be applied to the data, e.g. exponents or absolute values (Richter, 1979). A third approach is to posit a nonlinear interaction between opponent mechanisms. This again serves to leave the white point stable, since both opponent mechanisms have zero output at white, but allows for considerable interaction in other regions of the chromaticity diagram (Pokorny *et al.*, 1981). We have not attempted to model explicitly these types of nonlinearities, since in most cases our data do not provide the necessary constraints to rule out one class of model as opposed to another.

CONCLUSIONS

Comparison of unique and other constant hue loci shows that constant hue and unique hue loci are different ways of characterizing the same underlying visual mechanisms. For our observers, there are no striking discrepancies between these two types of data. Therefore, conflicting interpretations in the literature are probably due to methodological and analytical technique or individual differeces. Our data rule out general linear models which have unique hues as balance points for either of the two opponent mechanisms.

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