Chromaticity discrimination: effects of luminance contrast and spatial frequency

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We examined the effects of luminance contrast and spatial frequency on chromaticity discrimination of grating bars. Alternate bars of gratings were filled with light of a standard wavelength and light that could be varied in wavelength. The observer set the variable bars to match the standard bars in hue. Discrimination, as measured by the standard deviation of the matches, decreased as spatial frequency increased. Luminance contrast did not improve chromaticity discrimination but did lead to Bezold–Brücke hue shifts that were spatial-frequency dependent.

INTRODUCTION

"Color specifications reveal whether or not two sources radiating different spectral distributions appear to have the same color, for the average human observer."[1] Often it is desired to determine under what conditions the colors of two objects are reliably discriminated or what color a given object appears rather than merely whether a colored object is detected. Examples include discriminating the colors of signal lights and interpreting color-coded video graphics displays. In these cases, it is necessary to employ a task that requires discrimination based on color information and to manipulate experimentally or control other factors, such as luminance, luminance contrast, and spatial frequency. As the specification of real-world objects typically is stated with reference to a chromaticity diagram rather than to wavelength and luminance units, the term chromaticity discrimination has come to describe the discrimination of color.

Chromaticity discrimination depends on many factors, including the spatial structure of the viewing field and the chromaticity of the standard. There are comprehensive reviews of the data and theory for chromaticity discrimination,[1–7] but many of these describe discrimination for a narrow class of stimuli, such as bipartite fields.

For a wider class of stimuli, such as higher-spatial-frequency stimuli, chromaticity discrimination is not yet well characterized, particularly in the presence of luminance contrast. At higher spatial frequencies it is difficult to measure the visual-system response to chromatic contrasts unconfounded by luminance contrasts, especially when the contrast threshold is lower for luminance than chromatic contrast.[8–15] That is, although adding luminance contrast to a small object may make it easier to detect, there may be no advantage gained when discriminating the color of that object from the color of a second. Generally, detection of chromatic contrast or chromaticity discrimination is poorer for high-spatial-frequency stimuli[8–15] (or for small field sizes[16–18]) than lower spatial frequencies (or large field sizes). The wavelength dependence of chromaticity discrimination also varies as spatial frequency increases (or field size decreases).[8,15–20] Here, we investigate the role of luminance contrast in chromaticity discrimination for a range of spatial frequencies. Specifically, does the addition of luminance contrast improve the discrimination of hue *per se* of grating bars?

Luminance contrast, when added to chromatic contrast, improves the resolution of high-spatial-frequency grating bars and enhances the distinctness of borders between two color fields; i.e., it provides the necessary high-frequency information to make the border appear distinct.[13,14,21–23] Can this be taken to mean that there is an improvement in chromaticity discrimination or chromatic information processing? Previous reports of improving wavelength discrimination at medium to high spatial frequencies may be due to the higher threshold for chromatic contrast than for luminance contrast at these frequencies. It has long been known that performance of tasks that depend on high-spatial-frequency information, e.g., obtaining the clearest boundaries of two or more objects, is improved with the addition of luminance contrast (the Liebmann effect[24]). Recall that in many instances, although the boundaries of colored patches are indistinct, the colors *per se* are quite distinct in the regions away from the borders. Also, luminance contrast may be irrelevant to the judgment of other stimulus attributes of chromatic stimuli, provided that resolution is sufficient to perform the task. For instance, when luminance contrast is added to chromatic stimuli, there is no change in the orientation judgments of bipartite fields[25] or in the response time to the presence of chromatic stimuli,[26] even though the color appearance is significantly changed in both cases.

The Bezold–Brücke hue shift (a change in hue with a change in luminance or luminance contrast)[27–33] complicates interpretation of studies in which both hue and luminance contrast vary. Under appropriate stimulus conditions, a Bezold–Brücke hue shift occurs for small luminance differences (0.2–0.3 log unit) between the standard and variable,[31–32] with the luminance difference at which a hue difference is first perceived depending on wavelength.[32] The hue shift increases at higher spatial frequencies.[31] Thus, if there is luminance contrast between variable and standard grating bars, they may not match in hue when they are the same wavelength. This result might lead to artifacts in the usual measure of wavelength discrimination ($\Delta\lambda$). A more

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important consideration is that chromaticity discrimination does not remain constant when the standard or variable undergoes a hue shift; the observer must discriminate hue with respect to a new standard.[18,34] That is, the observer is now performing discriminations around a new operating point. If a psychometric function were determined for wavelength discrimination around both the old and the new operating points, there is no reason *a priori* to think that the slope of the function (indicating discrimination) would remain constant. As will be seen in the section headed Results, our findings agree with previous results[34,35] in that, as the standard changes, discrimination does not remain constant but depends on the standard and the underlying responses of visual mechanisms.

We measured chromaticity discrimination as a function of luminance contrast and spatial frequency. We matched the variable and standard in hue and, following MacAdam,[1] used the standard deviation of a series of matches as our index of discrimination. This method defines discrimination of chromaticity in an equal-luminance plane. A discrimination contour results, which may be elliptical in shape.[36] If luminance is also varied,[35] an ellipsoid results.

Previous results of chromaticity discrimination for higher spatial frequencies indicate a large, nonmonotonic change in performance when luminance contrast is added to chromatic contrast.[13,14] In contrast, recent results obtained for the detection of gratings show ellipses and ellipsoids, similar to those found for bipartite fields, for a wide range of spatial and temporal frequencies.[10,15] The discrimination in the chromatic- and luminance-contrast directions depends on spatial frequency. Above 1 cycle/degree (cpd), discrimination in the luminance-contrast direction improves, whereas chromatic-contrast discrimination worsens. However, these results do not indicate whether the hues of the grating bars were discriminated, since the criterion was "to detect *any* difference from a blank field, not the presence of hue modulation."[15] Additional detection results indicated a decrease in performance above 1 cpd if the criterion was not (1) the detection of grating bars versus a blank field but rather (2) the discrimination of their hues.[9] Accompanying this performance difference was a luminance contrast visible above 3 cpd, regardless of experimental manipulation. Above 20 cpd, only luminance contrast was visible.

Thus, given the greater sensitivity of achromatic visual mechanisms above 1 cpd, along with the numerous possible artifacts that can produce detectable luminance contrast at high spatial frequencies, a method is required that will tap color information, not merely form detection. Methodologies involving setting just-noticeable differences or obtaining two-alternative forced-choice judgments may allow responses to be partly or entirely based on luminance contrast. In contrast, the use of equality of hue allows a clear criterion, which may not be obtainable at high spatial frequencies with the other methods. At high spatial frequencies it is often difficult to identify which bars represent the standard, and a forced-choice judgment of longer wavelength depends on correct identification of the standard and variable bars. If the standard bars are brighter or darker than the variable ones, this stimulus uncertainty is eliminated. With a hue matching method, we can report the matching wavelength in addition to the standard deviations of hue matches for gratings with luminance contrast. The hypothesis to be tested is the following: Does chromaticity discrimination improve when there is luminance contrast between the variable and the standard? If so, then the standard deviation of hue matches will be largest for the equiluminant condition.

METHODS

Two of the authors served as observers. They had normal color vision and normal acuity with correction. They were knowledgeable about the experimental hypotheses but were naive about their results during a session.

The apparatus was a computer-controlled, four-channel Maxwellian-view stimulator[37,38] with a 2-mm artificial pupil. For the variable channel (channel 1), which the experimenter varied in illuminance and the observer in wavelength, the wavelength was determined by monochromator. The monochromator wavelength was monitored with a potentiometer and a digital voltmeter accurate to 0.1 nm. When additional precision was necessary, the wavelength was determined by attaching a laser optical lever to the monochromator. Channel 2 remained constant over the course of the experiment, serving as a luminance standard for the other channels. Channel 3, the standard channel, remained constant in wavelength and illuminance throughout a session. The wavelength was determined with a three-cavity interference filter (Ditric Optics) of 530 or 560 nm. Standard and variable bars were formed by reflection and transmission at one of a series of engine-ruled mirror grating beam-splitter cubes, including a bipartite-field beam splitter. Spatial frequency was altered by changing the beam-splitter cube between sessions. Channel 4 could provide either a desaturant or a surround.

The circular test field subtended 2-deg visual angle and had no fixation point. A square-wave grating of 1.07, 2.15, 4.30, or 8.60 cpd was presented, with standard bars alternating with variable bars. Standard stimuli were 100 Td. Luminance differences between the standard and variable were 0.2, 0.1, 0.03, or 0 log unit. As the observer could not possibly perform all conditions in a session without fatigue, subsets of the conditions were selected for each data set.

The method was similar to that of Burns *et al.*[38] As the observer adjusted the wavelength of the variable bars, retinal illuminance did not change. Before a session, each observer used heterochromatic flicker photometry (HFP) to match in luminance the variable channel to channel 2, the luminance standard, in 3-nm steps over a range of wavelengths greater than the narrow range required for discrimination of hue. HFP was done through the clear portion of the bipartite beam-splitter cube, placed to provide a uniform 2-deg field of view. Immediately before testing a given spatial frequency, the grating beam-splitter cube of the test frequency was inserted; then the observer obtained an isomeric match between the variable and standard bars to ensure a precise match of illuminance between the variable and standard channels. The computer was programmed to maintain constant retinal illuminance of the variable bars through dc servo control of a neutral-density wedge in the variable channel.

The two standard wavelengths were selected to be in spectral regions where the variable-channel wedge required minimal adjustment with change in wavelength. Discrimination for these two midspectral standards typically is good for normal observers, thus minimizing those artifacts (e.g., scattered light and chromatic aberration) that may be present when large wavelength differences are required for discrimination.

RESULTS

Main Conditions

For both standard wavelengths and all spatial frequencies, there was no case in which the chromaticity discrimination in the equal-luminance condition was significantly worse (or better) than in the conditions with low luminance contrasts. The apparent superiority of chromaticity discrimination in the equal-luminance condition over the other conditions was not statistically significant (Figs. 1–3).[39] At low spatial frequencies, discrimination was as good as that generally found in bipartite-field studies.[40] At higher spatial frequencies, the discrimination was worse, whether the variable and standard matched in luminance or not. However, our discrimination was better than that reported previously.[13,14] The matching hue changed with the amount of luminance contrast, with the largest hue shifts at the highest spatial frequencies (Figs. 1 and 3).

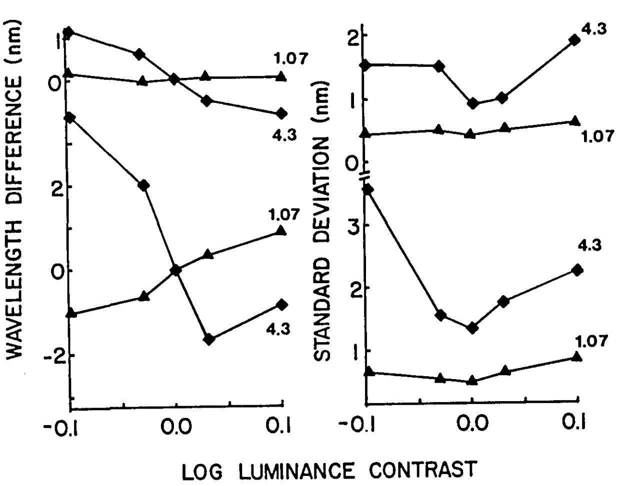

Fig. 1. Left panel: Bezold–Brücke hue shift as a function of luminance contrast between the variable and standard bars. Standard bars are 530 nm at 100 Td: triangles, 2.15 cpd; diamonds, 4.3 cpd. Top, data for observer SB; bottom, data for observer AE. Right panel: Hue discrimination, the median standard deviation for data in left panel, with five sessions for observer SB and four sessions for observer AE. $N = 10$ per session. Observers and symbols are as above.

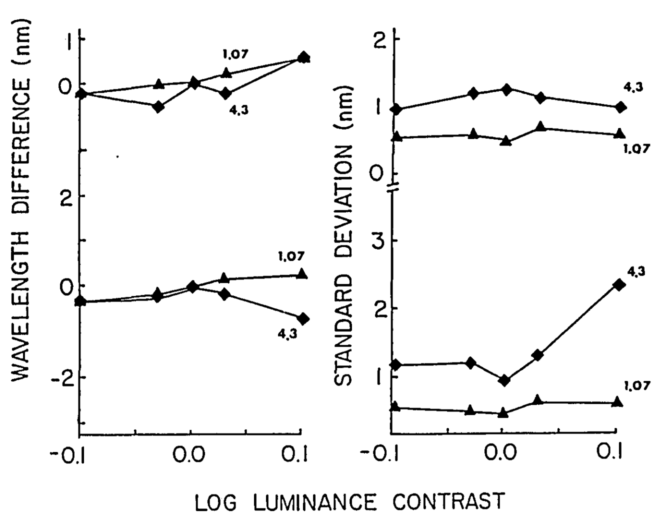

Fig. 2. Left panel: As in Fig. 1 for a 560-nm standard. Data with an additional spatial frequency and greater luminance contrast are shown in Fig. 3. Right panel: Hue discrimination for 560-nm standard, as in Fig. 1, but with only four sessions for observer SB.

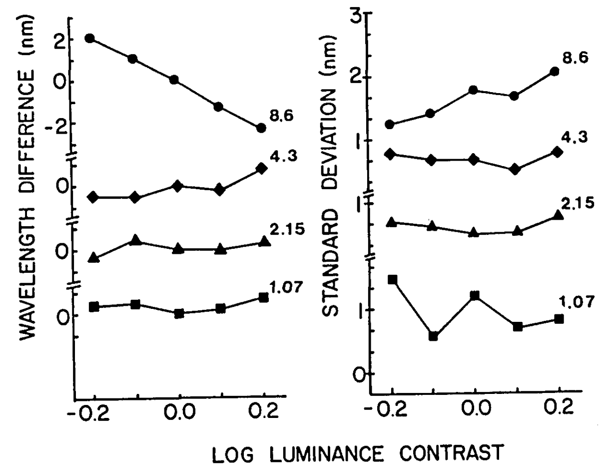

Fig. 3. Left panel: Bezold–Brücke hue shift for observer SB as a function of luminance contrast between variable and standard bars. The standard bars were 560 nm. Squares, 1.07 cpd; triangles, 2.15 cpd; diamonds, 4.3 cpd; and circles, 8.6 cpd. Right panel: Hue discrimination measured as the standard deviation. Symbols are as above.

Replications of the Finding That Luminance Contrast Does Not Improve Chromaticity Discrimination

We confirmed the generality of our findings in several ways. (1) Our results were replicated with a 2-deg bipartite field and several standards of midspectral wavelength on a laboratory wavelength-discrimination device.[41] (2) In a control study on the main apparatus, an experimenter naive about the purpose of this study varied the luminance of a 550-nm standard by inserting or removing calibrated 0.03-log-unit neutral-density filters in the variable channel. Thus the standard bars remained fixed in luminance, whereas the variable bars were brighter than, equal to, or dimmer than the standard bars. The observer made 50 settings in blocks of 10 at 4.3 cpd. Blocks were presented in random order. There were no statistically significant differences of the variances across conditions.[39] (3) The results were replicated with stimuli from 20 to 1000 Td. (4) The results were replicated with 2-deg gratings and white surrounds. (5) To test a range of hue and saturation wider than that induced by our luminance-contrast manipulations, we compared the effects of luminance contrast on hue matches of 520–590-nm standard bars of 1.0 colorimetric purity and variables with a colorimetric purity of 1.0, 0.79, or 0.63. Discrimination in the equal-luminance condition, regardless of saturation, was not statistically different from discrimination in other conditions.[38] For our conditions, the hue shifts obtained in the presence of luminance contrasts were smaller than the changes in hue with saturation, and both effects depended on spatial frequency. That is, the Abney effect[42] was greater than the Bezold–Brücke effect, and both depend on spatial frequency. (6) The results were replicated with another luminance matching procedure, the minimally distinct border criterion.

DISCUSSION

Hue Perception

As expected from published Bezold–Brücke data,[31] the matching wavelength changed as a function of both spatial frequency and luminance contrast between the variable and the standard (Figs. 1 and 3). A small difference in luminance between adjacent bars produced striking hue shifts at high spatial frequencies but not at low spatial frequencies, as van der Wildt and Bouman[31] have shown. In our data the shift in matching hue was sometimes in opposite directions for low versus higher spatial frequencies, e.g., shorter versus longer in wavelength at a given contrast. Thus, if our measure of wavelength discrimination were dependent on the adjusted wavelength *per se*, as opposed to the accuracy with which it was set, then these Bezold–Brücke effects would have introduced artifacts into our measure.

The change of matching hue across spatial frequency was not due to the precision of setting hue matches at higher spatial frequencies but was rather a change in color appearance, consistent with the properties of color vision mechanisms changing with spatial frequency. Chromatic contrast appeared greater at the higher spatial frequencies. We informally investigated this effect by adjusting the bars to be distinctly different. For the 530-nm standard, adjacent bars, which observers called "greenish-yellow" (although different) at low spatial frequencies, were called "red" and "blue" at high spatial frequencies. It is unlikely that these striking contrast effects were reflected in the standard deviations because our technique required that chromatic contrast be minimized. While collecting pilot data, for some conditions, we noted that at high spatial frequencies the entire grating changed in hue as the wavelength of the variable changed until the bars were discriminable, an assimilation phenomenon.[43] Thus the task was qualitatively different from matching bars in hue. It did not occur for the conditions used in this study.

Comparisons with Previous Studies

Previously it was reported that "Hue discrimination improves if a small luminance contrast is added."[14] For the discrimination of a 560-nm spot from a background, a two-alternative forced-choice paradigm was used, with the threshold defined as the wavelength interval between the 50 and 84% "longer" responses. Although the best discrimination in the experiment was for the largest spot on an equiluminant background, the performance for smaller spots was improved nonmonotonically by the addition of luminance contrast, sometimes by a factor of 5. For gratings, a just-noticeable difference between bars was adjusted by varying either wavelength or spatial frequency. A 0.045-log-unit luminance contrast improved performance by a factor of 2 to 4 times for spatial frequencies over 3 cpd. The improvement was nonmonotonic, however, with 0.09-log-unit luminance contrast leading to less improvement. In later studies,[8–11,15,44] most using a criterion-free two-alternative forced-choice detection of gratings, the observer varied the relative amounts of a red and a green primary. Typically, there is a very gradual, monotonic improvement in detection performance as the luminance of the primaries is varied to form a chromatic grating to a chromatic-plus-luminance-contrast grating, whether the primaries are broadband[10,11,15] or monochromatic.[44] Our data, although gathered with a different method, are more consistent with these later results in that gradual, monotonic changes in color appearance with luminance contrast were measured.

Let us now consider whether the difference between our results and previous results[13,14] for the equal-luminance condition can be explained by the difference in procedure for setting the relative radiances of the variable and standard bars. Suppose that there is a ratio of the relative luminances of standard and variable that produces the least discriminable grating bars but that this ratio is not produced by HFP. Then that ratio of luminances should be within the range of luminance contrasts that we tested, a 0.4-log-unit range. We used a very small luminance mismatch, 0.03 log unit. If this luminance mismatch decreased discrimination for a nonisomeric condition, then this should be reflected in either an increase in the standard deviation (a peak in the function) or a displaced mean (a wavelength shift). As can be seen in the section headed Results, no such peaks or nonmonotonic shifts in wavelength occurred. In contrast, the standard deviations were similar for all conditions at a given spatial frequency (Figs. 2 and 3). For equal-luminance conditions, both laboratories made isomeric matches. Our wavelength adjustment range was small, with little luminance change needed over the range of adjustments. Given that both laboratories used similar stimuli, square-wave gratings and a range of monochromatic standards, it is likely that the difference in the results depends on the task. Perhaps in the previous results the chromatic information in the equiluminant conditions could not be used by the observer because the spatial structure of the visual field could not be resolved without increasing the contrast, either luminance or chromatic. A luminance contrast of 0.045 log unit is sufficient to define grating bars for the range of frequencies that we explored. Any additional contrast serves only to increase the Bezold–Brücke hue shift and possibly contributes to the nonmonotonicity of the previous discrimination results.

In summary, we show that luminance contrast affects color appearance but does not necessarily lead to improved color discrimination. Previous reports of luminance contrast improving color discrimination may therefore be due either to changes in color appearance with luminance contrast or to assimilation under equiluminant conditions.

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