CHAPTER 11

Areas of Polygons and Circles

Then

You learned about circles and angles within circles.

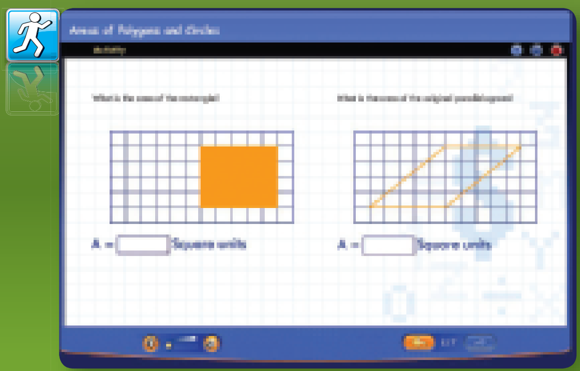
Now

In this chapter, you will:

- Find areas of polygons.
- Solve problems involving areas and sectors of circles.
- Find scale factors using similar figures.

Why?

ART Artisans and craftsmen use area to determine the amount of raw materials that they will need for a project.



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Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets

Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

The area and width of a rectangle are given. Find the length of the rectangle.

1. $A = 25$, $w = 5$

2. $A = 42$, $w = 6$

3. $A = 280$, $w = 14$

4. $A = 360$, $w = 60$

5. **GARDENS** Molly planted a garden with a length of 72 feet. If she bought enough fertilizer to cover 792 square feet, what width should she make the garden?

Evaluate each expression if $a = 9$, $b = 10$, $c = 12$, and $d = 13$.

6. $\frac{1}{2}a(b + c)$

7. $\frac{1}{2}(ab + cd)$

8. $\frac{1}{2}(a + bd)$

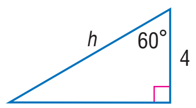
9. $\frac{1}{2}cd$

10. $\frac{1}{2}(ab + c)$

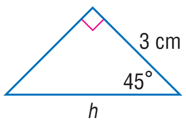
11. $\frac{1}{2}(a + d)$

Find h in each triangle.

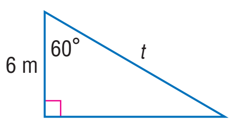
12.



13.



14. **LOOKOUT** The lookout on a pirate ship slides down a rope from the top of the mast 6 meters above the water. He can see the land at a 60° angle. How far does he slide?



QuickReview

Example 1

The area of a rectangle is 64 square units and the width is 4 units. Find the length.

$A = \ell w$	Area of rectangle
$64 = \ell(4)$	Substitution
$16 = \ell$	Divide each side by 4.

The length is 16 units.

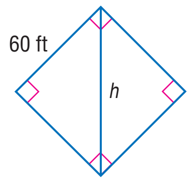
Example 2

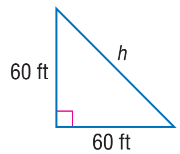
Evaluate $\frac{1}{2}x(2x + 3y)$ for $x = 4$ and $y = 12$.

$\frac{1}{2}x(2x + 3y) = \frac{1}{2}(4)[2(4) + 3(12)]$	Substitution
$= 2(8 + 36)$	Multiply.
$= 2(44)$	Add.
$= 88$	Multiply.

Example 3

Find the value of h.





Since h is the hypotenuse of the triangle, the triangle can be redrawn as shown.

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg.

$h = (\sqrt{2})60$

≈ 84.85

So, h is approximately 84.85 feet.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

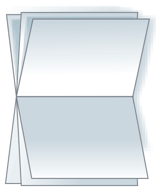
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 11. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

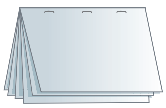
FOLDABLES StudyOrganizer

Areas of Polygons and Circles Make this Foldable to help you organize your Chapter 11 notes about areas of polygons and circles. Begin with three sheets of notebook paper.

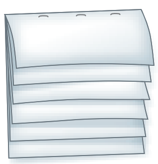
1 **Stack** three sheets of paper and fold them in half, lengthwise.



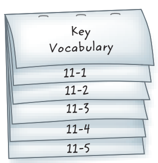
2 **Staple** the papers together one inch from the top fold.



3 **Cut** the top sheet two inches from the top fold and each following sheet one inch longer than the previous sheet.



4 **Label** as shown.



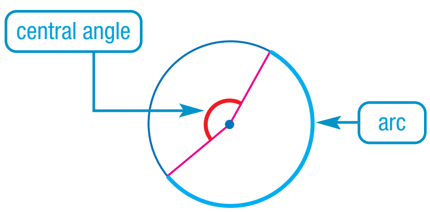
NewVocabulary

English		Español
base of a parallelogram	p. 779	base de un paralelogramo
height of a parallelogram	p. 779	altura de un paralelogramo
base of a triangle	p. 781	base de un triángulo
height of a triangle	p. 781	altura de un triángulo
height of a trapezoid	p. 789	altura de un trapecio
sector of a circle	p. 799	sector circular
center of a regular polygon	p. 807	centro de un polígono regular
radius of a regular polygon	p. 807	radio de un polígono regular
apothem	p. 807	apotema
central angle of a regular polygon	p. 807	ángulo central de un polígono regular

ReviewVocabulary

arc arco a part of a circle that is defined by two endpoints

central angle ángulo central an angle that intersects a circle in two points and has its vertex at the center of the circle



diagonal diagonal a segment that connects nonconsecutive vertices of a polygon

LESSON 11-1 Areas of Parallelograms and Triangles

Then

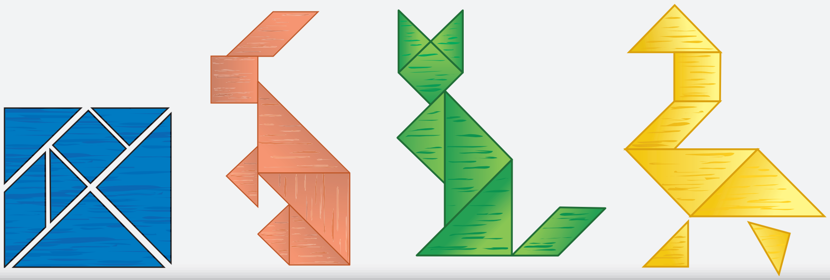
- You found areas of rectangles and squares.

Now

1. Find perimeters and areas of parallelograms.
2. Find perimeters and areas of triangles.

Why?

- A tangram is an ancient Chinese puzzle that can be rearranged to form different images, such as the animals shown. The area of the puzzle, before and after being rearranged, remains the same. It is the sum of all the areas of its pieces.



NewVocabulary

base of a parallelogram
height of a parallelogram
base of a triangle
height of a triangle

Common Core State Standards

Content Standards

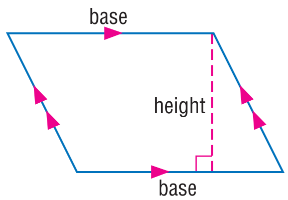
G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Mathematical Practices

1 Make sense of problems and persevere in solving them.
7 Look for and make use of structure.

1 Areas of Parallelograms

In Lesson 6-2, you learned that a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called the **base of a parallelogram**. The **height of a parallelogram** is the perpendicular distance between any two parallel bases.



You can use the following postulate to develop the formula for the area of a parallelogram.

Postulate 11.1 Area Addition Postulate

The area of a region is the sum of the areas of its nonoverlapping parts.

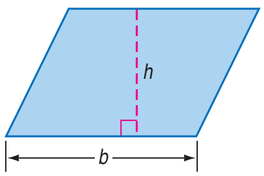
In the figures below, a right triangle is cut off from one side of a parallelogram and translated to the other side as shown to form a rectangle with the same base and height.

Recall from Lesson 1-6 that the area of a rectangle is the product of its base and height. By the Area Addition Postulate, a parallelogram with base b and height h has the same area as a rectangle with base b and height h.

KeyConcept Area of a Parallelogram

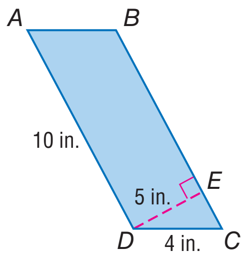
Words The area A of a parallelogram is the product of a base b and its corresponding height h.

Symbols $A = bh$



Example 1 Perimeter and Area of a Parallelogram

Find the perimeter and area of $\square ABCD$.



Perimeter

Since opposite sides of a parallelogram are congruent, $\overline{AB} \cong \overline{DC}$ and $\overline{BC} \cong \overline{AD}$. So $AB = 4$ inches and $BC = 10$ inches.

$$\begin{aligned} \text{Perimeter of } \square ABCD &= AB + BC + DC + AD \\ &= 4 + 10 + 4 + 10 \text{ or } 28 \text{ in.} \end{aligned}$$

Area

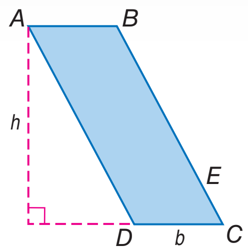
The height given, DE, is 5 inches. $\overline{BC}$ is the base, which measures 10 inches.

$A = bh$ — Area of a parallelogram

$= (10)(5)$ or 50 in^2 — $b = 10$ and $h = 5$

Study Tip

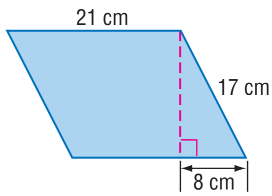
Heights of Figures The height of a figure can be measured by extending a base. In Example 1, the height of $\square ABCD$ that corresponds to base $\overline{DC}$ can be measured by extending $\overline{DC}$.



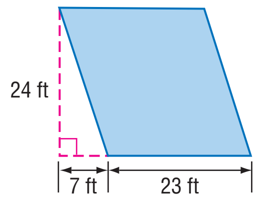
Guided Practice

Find the perimeter and area of each parallelogram.

1A.



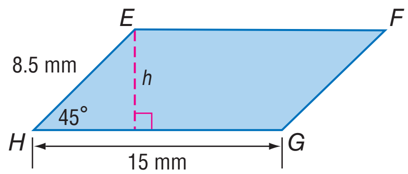
1B.



You may need to use trigonometry to find the area of a parallelogram.

Example 2 Area of a Parallelogram

Find the area of $\square EFGH$.



Step 1 Use a 45°-45°-90° triangle to find the height h of the parallelogram.

Recall that if the measure of the leg opposite the 45° angle is h, then the measure of the hypotenuse is $h\sqrt{2}$.

$h\sqrt{2} = 8.5$ — Substitute 8.5 for the measure of the hypotenuse.

$h = \dfrac{8.5}{\sqrt{2}}$ or about 6 mm — Divide each side by $\sqrt{2}$.

Step 2 Find the area.

$A = bh$ — Area of a parallelogram

$\approx (15)(6)$ or 90 mm^2 — $b = 15$ and $h \approx 6$

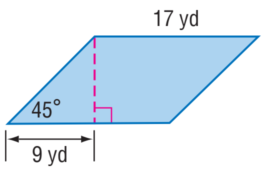
Watch Out!

CCSS Precision Remember that perimeter is measured in linear units such as inches and centimeters. Area is measured in square units such as square feet and square millimeters.

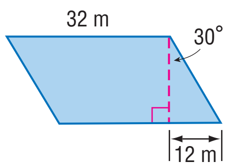
Guided Practice

Find the area of each parallelogram. Round to the nearest tenth if necessary.

2A.



2B.

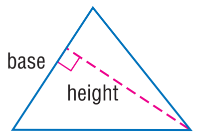


ReviewVocabulary

altitude of a triangle
a segment from a vertex of a triangle to the line containing the opposite side and perpendicular to the line containing that side

2 Areas of Triangles

Like the base of a parallelogram, the **base of a triangle** can be any side. The **height of a triangle** is the length of an altitude drawn to a given base.



You can use the following postulate to develop the formula for the area of a triangle.

Postulate 11.2 Area Congruence Postulate

If two figures are congruent, then they have the same area.

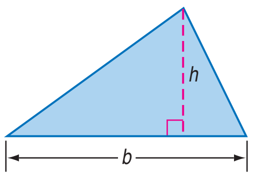
In the figures below, a parallelogram is cut in half along a diagonal to form two congruent triangles with the same base and height.

By the Area Congruence Postulate, the two congruent triangles have the same area. So, one triangle with base b and height h has half the area of a parallelogram with base b and height h.

KeyConcept Area of a Triangle

Words The area A of a triangle is one half the product of a base b and its corresponding height h.

Symbols $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$



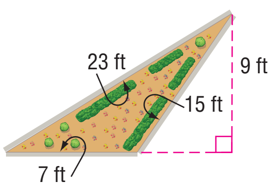
Real-WorldLink

Triangular gardens can serve as focal points in landscaping or simply result from intersecting walkways.

Janet Johnson/Gap Photos/age fotostock

Real-World Example 3 Perimeter and Area of a Triangle

GARDENING D'Andre needs enough mulch to cover the triangular garden shown and enough paving stones to border it. If one bag of mulch covers 12 square feet and one paving stone provides a 4-inch border, how many bags of mulch and how many stones does he need to buy?



Step 1 Find the perimeter of the garden.

Perimeter of garden = 23 + 15 + 7 or 45 ft

Step 2 Find the area of the garden.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(9)$ or 31.5 ft^2 $b = 7$ and $h = 9$

Step 3 Use unit analysis to determine how many of each item are needed.

Bags of Mulch

$31.5\ \cancel{\text{ft}^2} \cdot \frac{1 \text{ bag}}{12\ \cancel{\text{ft}^2}} = 2.625 \text{ bags}$

Paving Stones

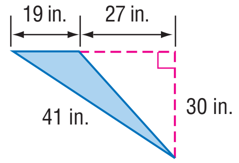
$45\ \cancel{\text{ft}} \cdot \frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft}}} \cdot \frac{1 \text{ stone}}{4\ \cancel{\text{in.}}} = 135 \text{ stones}$

Round the number of bags up so there is enough mulch. He will need 3 bags of mulch and 135 paving stones.

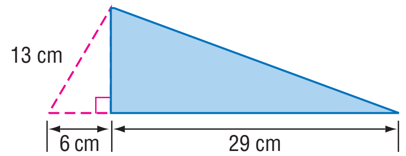
GuidedPractice

Find the perimeter and area of each triangle.

3A.



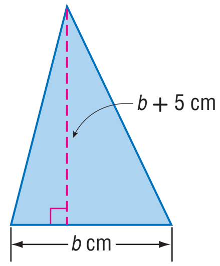
3B.



You can use algebra to solve for unknown measures in parallelograms and triangles.

Example 4 Use Area to Find Missing Measures

ALGEBRA **The height of a triangle is 5 centimeters more than its base. The area of the triangle is 52 square centimeters. Find the base and height.**



Step 1 Write expressions to represent each measure.

Let b represent the base of the triangle. Then the height is $b + 5$.

Step 2 Use the formula for the area of a triangle to find b.

$A = \frac{1}{2}bh$	Area of a triangle
$52 = \frac{1}{2}b(b + 5)$	Replace A with 52 and h with $b + 5$.
$104 = b(b + 5)$	Multiply each side by 2.
$104 = b^2 + 5b$	Distributive Property
$0 = b^2 + 5b - 104$	Subtract 104 from each side.
$0 = (b + 13)(b - 8)$	Factor.
$b + 13 = 0$ and $b - 8 = 0$	Zero Product Property
$b = -13$ $\quad$ $b = 8$	Solve for b.

StudyTip

Zero Product Property If the product of two factors is 0, then at least one of the factors must be 0.

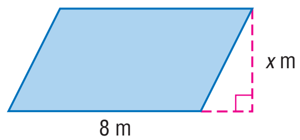
Step 3 Use the expressions from Step 1 to find each measure.

Since a length cannot be negative, the base measures 8 centimeters and the height measures 8 + 5 or 13 centimeters.

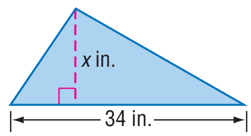
GuidedPractice

ALGEBRA **Find x.**

4A. $A = 148\text{ m}^2$



4B. $A = 357\text{ in}^2$



4C. **ALGEBRA** The base of a parallelogram is twice its height. If the area of the parallelogram is 72 square feet, find its base and height.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Examples 1–3 **Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.**

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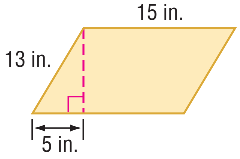

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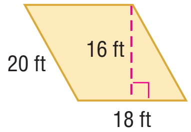

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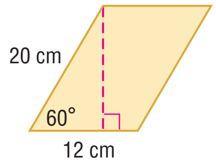

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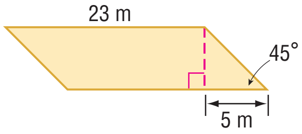

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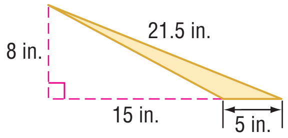

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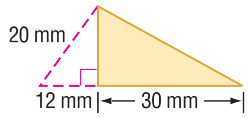
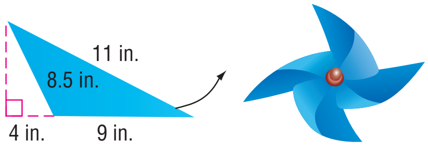
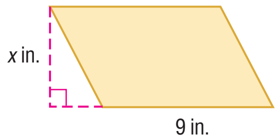
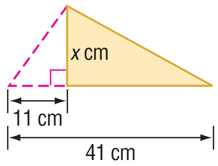


7. CRAFTS Marquez and Victoria are making pinwheels. Each pinwheel is composed of 4 triangles with the dimensions shown. Find the perimeter and area of one triangle.



Example 4 Find x.

8. $A = 153 \text{ in}^2$


9. $A = 165 \text{ cm}^2$


Practice and Problem Solving

Extra Practice is on page R11.

Examples 1–3 CCSS **STRUCTURE** **Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.**

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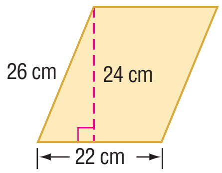

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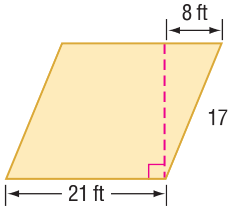

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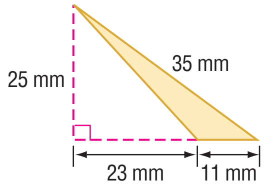

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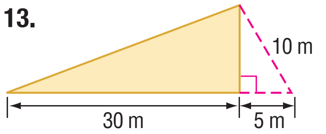

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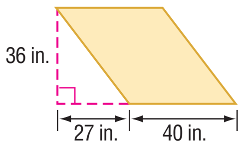
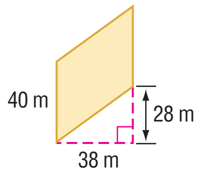
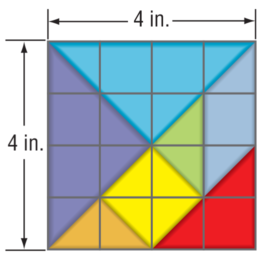


15.


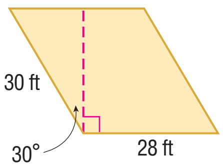
16. TANGRAMS The tangram shown is a 4-inch square.

a. Find the perimeter and area of the purple triangle. Round to the nearest tenth.

b. Find the perimeter and area of the blue parallelogram. Round to the nearest tenth.



Example 2 **CCSS STRUCTURE** **Find the area of each parallelogram. Round to the nearest tenth if necessary.**

17.


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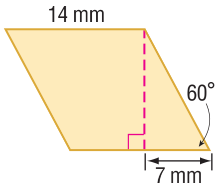

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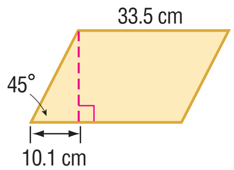

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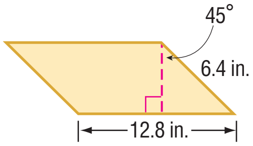

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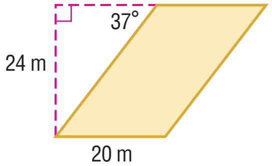

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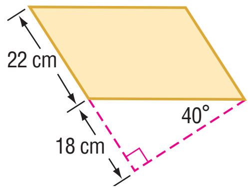
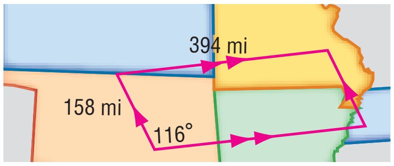


23 **WEATHER** Tornado watch areas are often shown on weather maps using parallelograms. What is the area of the region affected by the tornado watch shown? Round to the nearest square mile.



Example 4 **24.** The height of a parallelogram is 4 millimeters more than its base. If the area of the parallelogram is 221 square millimeters, find its base and height.

25. The height of a parallelogram is one fourth of its base. If the area of the parallelogram is 36 square centimeters, find its base and height.

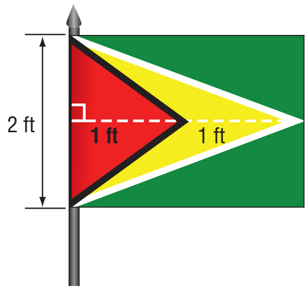
26. The base of a triangle is twice its height. If the area of the triangle is 49 square feet, find its base and height.

27. The height of a triangle is 3 meters less than its base. If the area of the triangle is 44 square meters, find its base and height.

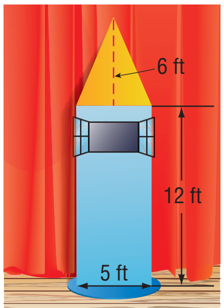
28. **FLAGS** Omar wants to make a replica of Guyana's national flag.

a. What is the area of the piece of fabric he will need for the red region? for the yellow region?

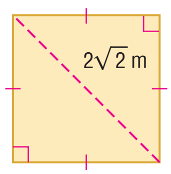
b. If the fabric costs $3.99 per square yard for each color and he buys exactly the amount of fabric he needs, how much will it cost to make the flag?



29. **DRAMA** Madison is in charge of the set design for her high school's rendition of *Romeo and Juliet*. One pint of paint covers 80 square feet. How many pints will she need of each color if the roof and tower each need 3 coats of paint?



Find the perimeter and area of each figure. Round to the nearest hundredth, if necessary.

30.


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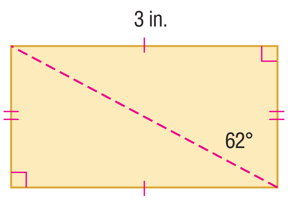
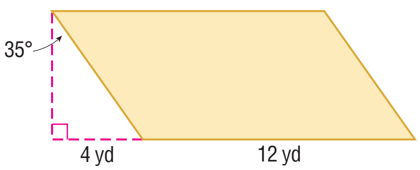


32.


COORDINATE GEOMETRY **Find the area of each figure. Explain the method that you used.**

33. ▱$ABCD$ with $A(4, 7)$, $B(2, 1)$, $C(8, 1)$, and $D(10, 7)$

34. $\triangle RST$ with $R(-8, -2)$, $S(-2, -2)$, and $T(-3, -7)$

35. **HERON'S FORMULA** Heron's Formula relates the lengths of the sides of a triangle to the area of the triangle. The formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the *semiperimeter*, or one half the perimeter, of the triangle and a, b, and c are the side lengths.

a. Use Heron's Formula to find the area of a triangle with side lengths 7, 10, and 4.

b. Show that the areas found for a 5-12-13 right triangle are the same using Heron's Formula and using the triangle area formula you learned earlier in this lesson.

36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the area and perimeter of a rectangle.

a. Algebraic A rectangle has a perimeter of 12 units. If the length of the rectangle is x and the width of the rectangle is y, write equations for the perimeter and area of the rectangle.

b. Tabular Tabulate all possible whole-number values for the length and width of the rectangle, and find the area for each pair.

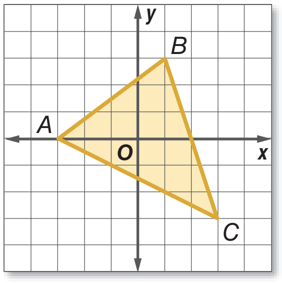
c. Graphical Graph the area of the rectangle with respect to its length.

d. Verbal Describe how the area of the rectangle changes as its length changes.

e. Analytical For what whole-number values of length and width will the area be greatest? least? Explain your reasoning.

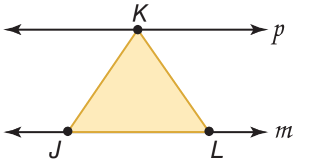
H.O.T. Problems Use Higher-Order Thinking Skills

37. **CHALLENGE** Find the area of $\triangle ABC$ graphed at the right. Explain your method.



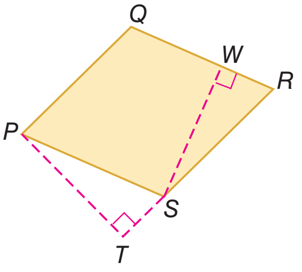
38. **CCSS ARGUMENTS** Will the perimeter of a nonrectangular parallelogram *always*, *sometimes*, or *never* be greater than the perimeter of a rectangle with the same area and the same height? Explain.

39. **WRITING IN MATH** Points J and L lie on line m. Point K lies on line p. If lines m and p are parallel, describe how the area of $\triangle JKL$ will change as K moves along line p.



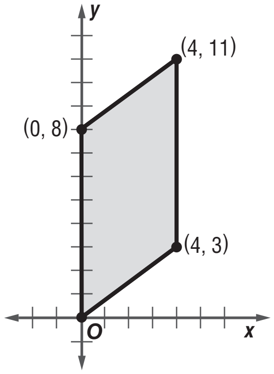
40. **OPEN ENDED** The area of a polygon is 35 square units. The height is 7 units. Draw three different triangles and three different parallelograms that meet these requirements. Label the base and height on each.

41. **WRITING IN MATH** Describe two different ways you could use measurement to find the area of parallelogram $PQRS$.



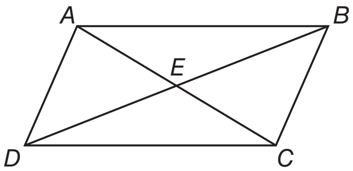
Standardized Test Practice

42. What is the area, in square units, of the parallelogram shown?

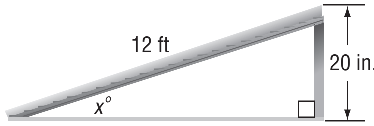


A 12 **C** 32
B 20 **D** 40

43. GRIDDED RESPONSE In parallelogram $ABCD$, $\overline{BD}$ and $\overline{AC}$ intersect at E. If $AE = 9$, $BE = 3x - 7$, and $DE = x + 5$, find x.



44. A wheelchair ramp is built that is 20 inches high and has a length of 12 feet as shown. What is the measure of the angle x that the ramp makes with the ground, to the *nearest* degree?



Note: Not drawn to scale.

F 8 **H** 37
G 16 **J** 53

45. SAT/ACT The formula for converting a Celsius temperature to a Fahrenheit temperature is $F = \frac{9}{5}C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius. Which of the following is the Celsius equivalent to a temperature of 86° Fahrenheit?

A 15.7° C **D** 122.8° C
B 30° C **E** 186.8° C
C 65.5° C

Spiral Review

Write the equation of each circle. (Lesson 10-8)

46. center at origin, $r = 3$

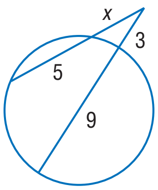
47. center at origin, $d = 12$

48. center at $(-3, -10)$, $d = 24$

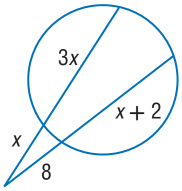
49. center at $(1, -4)$, $r = \sqrt{17}$

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent. (Lesson 10-7)

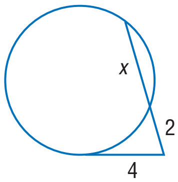
50.



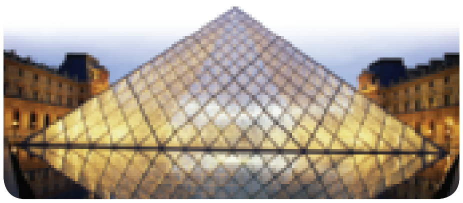
51.



52.



53. ARCHITECTURE The Louvre Pyramid is the main entrance to the Louvre Museum in Paris, France. The structure consists mainly of quadrilateral-shaped glass segments, as shown in the photo at the right. Describe one method that could be used to prove that the shapes of the segments are parallelograms. (Lesson 6-3)



Skills Review

Evaluate each expression if $a = 2$, $b = 6$, and $c = 3$.

54. $\frac{1}{2}ac$

55. $\frac{1}{2}cb$

56. $\frac{1}{2}b(2a + c)$

57. $\frac{1}{2}c(b + a)$

58. $\frac{1}{2}a(2c + b)$

EXPLORE 11-2

Graphing Technology Lab
Areas of Trapezoids, Rhombi, and Kites

You can use the TI-Nspire Technology to explore special quadrilaterals.

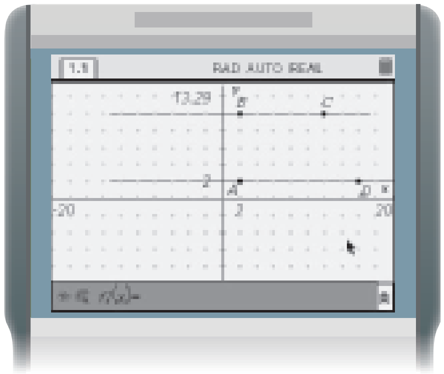
Common Core State Standards
Content Standards
Preparation for G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★
Mathematical Practices 5

PT

Activity 1

Step 1 Open a new **Graphs** page. Select **Show Grid** from the **View** menu so that points can be placed at integer coordinates.

Step 1:



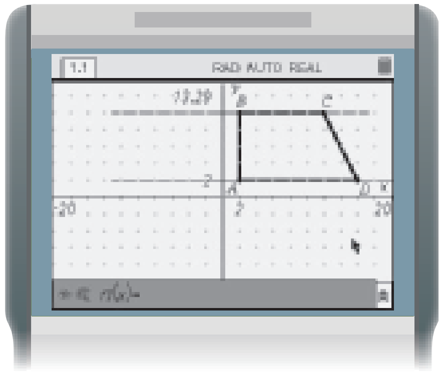
Step 2 Select **Line** from the **Points & Lines** menu, and draw a horizontal line.

Step 3 Select **Parallel** from the **Construction** menu to draw a line parallel to your original line through a point with the same *x*-coordinate as a point in Step 2.

Step 4 Place an additional point on the parallel line you just constructed using **Point on** from the **Points & Lines** menu. Label the four points as shown.

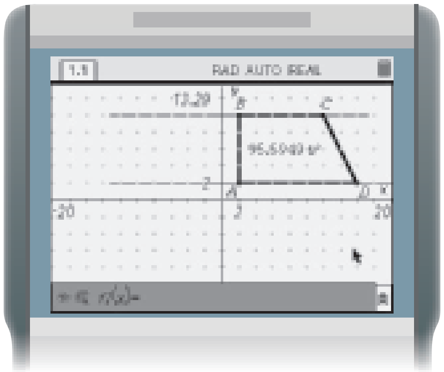
Step 5 From the **Shapes** menu, select **Polygon**, and draw a polygon using the four points you created. From the **Actions** menu, select **Attributes,** select the polygon, and increase the line thickness of the polygon.

Step 5:



Step 6 Display the area of the polygon using the **Area** tool from the **Measurement** menu. Move each of the points and observe the effect on the area.

Step 6:



Analyze the Results

1. What type of quadrilateral is *ABCD*? Explain your reasoning.

2. **MAKE A CONJECTURE** Using the formulas you learned in Lesson 11-1, make a conjecture about the formula for the area of this type of quadrilateral if *BC* is b_1, *AD* is b_2, and *AB* is *h*. Explain.

(continued on the next page)

Graphing Technology Lab
Areas of Trapezoids, Rhombi, and Kites *Continued*

Activity 2

Step 1 Open a new **Graphs** page. Select **Show Grid** from the **View** menu so that points can be placed at integer coordinates.

Step 2 Select **Line** from the **Points & Lines** menu, and draw a line.

Step 3 Place a point above the line by selecting **Point** from the **Points & Lines** menu.

Step 4 Reflect the point above the line by choosing **Reflection** from the **Transformation** menu, then select the point and then the line.

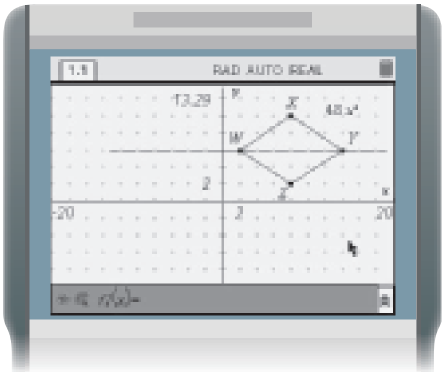
Step 1:

Step 5 Label the four points as shown.

Step 6 From the **Shapes** menu, select **Polygon**, and draw a polygon using points *W*, *X*, *Y*, and *Z*.

Step 7 Display the area of the polygon using the **Area** tool from the **Measurement** menu. Move points *W*, *X*, and *Y*, and observe the effect on the area.

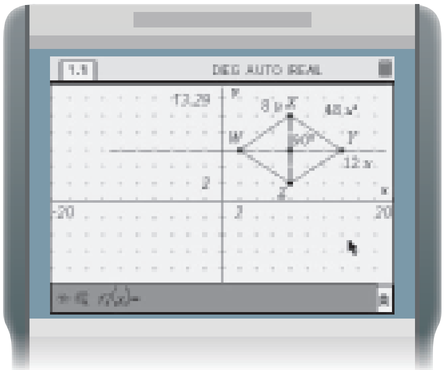
Step 6:



Step 8 Select **Segment** from the **Points & Lines** menu to draw the diagonals of *WXYZ*.

Step 9 Display the lengths of the diagonals using the **Length** tool from the **Measurement** menu, and display the angle between the diagonals using the **Angle** tool. Continue to move points *W*, *X*, and *Y*, and observe the effect on the area and the angle between the diagonals.

Step 8:



Analyze the Results

3. What type of quadrilateral is *WXYZ*? Explain your reasoning.

4. **MAKE A CONJECTURE** Using the formulas you learned in Lesson 11-1, develop a formula for the area of this type of quadrilateral. Let *WY* be d_1, and let *XZ* be d_2. Explain your reasoning.

5. **CHALLENGE** Construct a quadrilateral using two perpendicular lines and reflecting a point on each as you did in Step 4 of Activity 2. What type of quadrilateral is formed? Does the formula for the area you developed in Exercise 4 apply?

LESSON 11-2 Areas of Trapezoids, Rhombi, and Kites

Then

- You found areas of triangles and parallelograms.

Now

1. Find areas of trapezoids.
2. Find areas of rhombi and kites.

Why?

- Brianna has turned her hobby of making designer handbags and totes into a small business. Among her designs is a trapezoid-shaped handbag. To estimate the amount of material needed to produce each handbag, she needs to calculate the area of a trapezoid.

NewVocabulary
height of a trapezoid

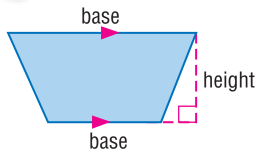
Common Core State Standards

Content Standards
G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

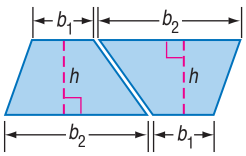
Mathematical Practices
1 Make sense of problems and persevere in solving them.
7 Look for and make use of structure.

1 Areas of Trapezoids

In Lesson 6-6, you learned that a *trapezoid* is a quadrilateral with exactly one pair of parallel sides. These parallel sides are called *bases*. The **height of a trapezoid** is the perpendicular distance between its bases.



In the figure below, a glide reflection of the first trapezoid results in two congruent trapezoids that fit together to form a parallelogram.

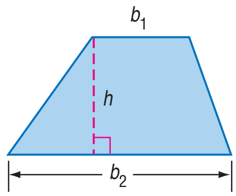


The area of the parallelogram is the product of the height h and the sum of the two bases, b_1 and b_2. The area of one trapezoid is one half the area of the parallelogram.

KeyConcept Area of a Trapezoid

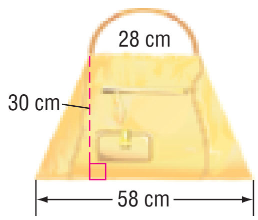
Words The area A of a trapezoid is one half the product of the height h and the sum of its bases, b_1 and b_2.

Symbols $A = \frac{1}{2}h(b_1 + b_2)$



Real-World Example 1 Area of a Trapezoid

CRAFTS **One of Brianna's trapezoid-shaped totes is shown. Find the amount of material used to make the side shown.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

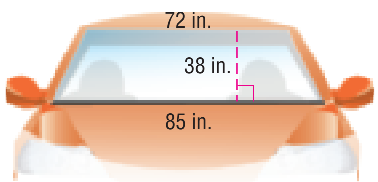
$= \frac{1}{2}(30)(28 + 58)$ $h = 30$, $b_1 = 28$, $b_2 = 58$

$= 1290$ Simplify.

The tote requires 1290 square centimeters.

GuidedPractice

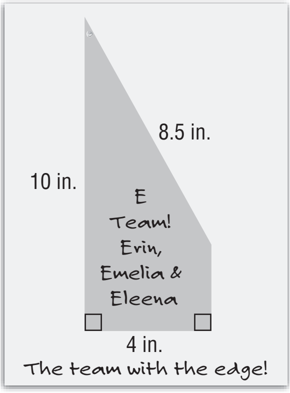
1. **AUTOMOBILES** Find the area of glass used to make the windshield of a van shown at the right.



Juice Images/age fotostock

Standardized Test Example 2 Area of a Trapezoid

SHORT RESPONSE **Emelia designed the pennant shown for her team. Find the area of the shaded portion of her team's pennant.**



Read the Test Item

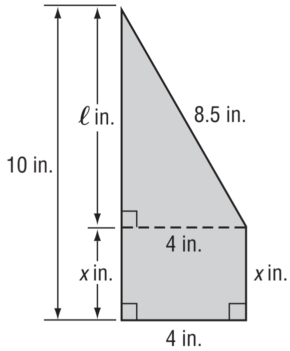
You are given a trapezoid with one base measuring 10 inches, a height of 4 inches, and a third side measuring 8.5 inches. To find the area of the trapezoid, first find the measure of the other base.

Test-Taking Tip

Separating Figures To solve some area problems, you need to draw in parallel and/or perpendicular lines to find information not provided.

Solve the Test Item

Draw the segment shown to form a right triangle and a rectangle. The triangle has a hypotenuse of 8.5 inches and legs of 4 and ℓ inches. The rectangle has a length of 4 inches and a width of x inches.



Use the Pythagorean Theorem to find ℓ.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$\ell^2 + 4^2 = 8.5^2$	$a = \ell$, $b = 4$, and $c = 8.5$
$\ell^2 + 16 = 72.25$	Simplify.
$\ell^2 = 56.25$	Subtract 16 from each side.
$\ell = 7.5$	Take the positive square root of each side.

By Segment Addition, $\ell + x = 10$. So, $7.5 + x = 10$ and $x = 2.5$. The width of the rectangle is also the measure of the second base of the trapezoid.

$A = \frac{1}{2}h(b_1 + b_2)$	Area of a trapezoid
$= \frac{1}{2}(4)(10 + 2.5)$	$h = 4$, $b_1 = 10$, and $b_2 = 2.5$
$= 25$	Simplify.

So the pennant has an area of 25 square inches.

CHECK The area of the trapezoid is the sum of the areas of the right triangle and rectangle. The area of the triangle is $\frac{1}{2}(4)(7.5)$ or 15 square inches. The area of the rectangle is (4)(2.5) or 10 square inches. So the area of the trapezoid is 15 + 10 or 25 square inches. ✓

Real-World Career

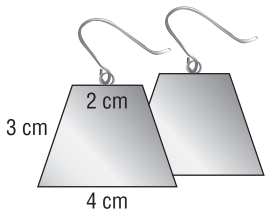
Craft Artist Craft artists create their art by hand to sell or exhibit. They work with a wide variety of materials including textiles, woods, metal, and ceramics.

Most artists receive some type of postsecondary training, and about 63% are self-employed. Craft artists make up about 3% of all artists.

Richard Hutchings/Digital Light Source

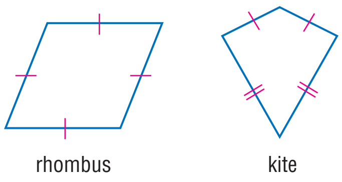
Guided Practice

2. SHORT RESPONSE Owen designed the silver earrings shown that are shaped like isosceles trapezoids. What is the area of each earring?



2 Areas of Rhombi and Kites

Recall from Lessons 6-5 and 6-6 that a *rhombus* is a parallelogram with all four sides congruent and a *kite* is a quadrilateral with exactly two pairs of consecutive congruent sides.



The areas of rhombi and kites are related to the lengths of their diagonals.

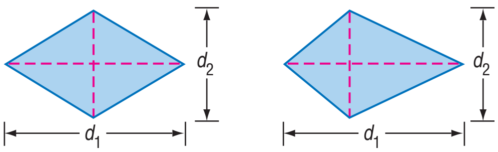
ReviewVocabulary

diagonal a segment that connects any two nonconsecutive vertices in a polygon

KeyConcept Area of a Rhombus or Kite

Words The area A of a rhombus or kite is one half the product of the lengths of its diagonals, d_1 and d_2.

Symbols $A = \frac{1}{2}d_1d_2$

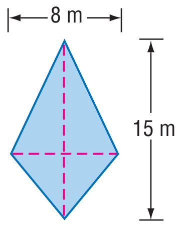


You will derive the formulas for the area of a kite and the area of a rhombus in Exercises 23 and 24.

Example 3 Area of a Rhombus and a Kite

Find the area of each rhombus or kite.

a.

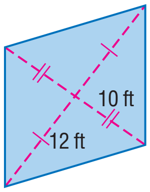


$A = \frac{1}{2}d_1d_2$ Area of a kite

$= \frac{1}{2}(8)(15)$ $d_1 = 8$ and $d_2 = 15$

$= 60 \text{ m}^2$ Simplify.

b.



Step 1 Find the length of each diagonal.

Since the diagonals of a rhombus bisect each other, then lengths of the diagonals are 12 + 12 or 24 feet and 10 + 10 or 20 feet.

Step 2 Find the area of the rhombus.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

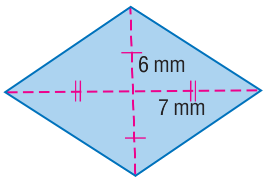
$= \frac{1}{2}(24)(20)$ $d_1 = 24$ and $d_2 = 20$

$= 240 \text{ ft}^2$ Simplify.

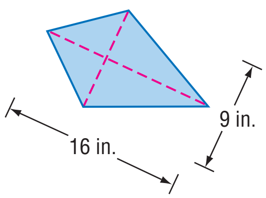
GuidedPractice

Find the area of each rhombus or kite.

3A.



3B.



Apic/Hulton Archive/Getty Images

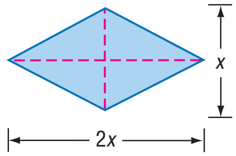
Math HistoryLink

Heron of Alexandria (c. 10–70 A.D.) Heron was a mathematician and engineer in Roman Egypt. He developed a formula for finding the area of a triangle if the lengths of the sides are known.

You can use algebra to solve for unknown measures in trapezoids, rhombi, and kites.

Example 4 Use Area to Find Missing Measures

ALGEBRA One diagonal of a rhombus is twice as long as the other diagonal. If the area of the rhombus is 169 square millimeters, what are the lengths of the diagonals?



Step 1 Write an expression to represent each measure.

Let x represent the length of one diagonal. Then the length of the other diagonal is $2x$.

Step 2 Use the formula for the area of a rhombus to find x.

$A = \frac{1}{2}d_1d_2$ — Area of a rhombus

$169 = \frac{1}{2}(x)(2x)$ — $A = 169$, $d_1 = x$, and $d_2 = 2x$

$169 = x^2$ — Simplify.

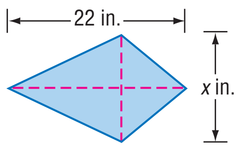
$13 = x$ — Take the positive square root of each side.

So the lengths of the diagonals are 13 millimeters and 2(13) or 26 millimeters.

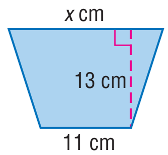
GuidedPractice

ALGEBRA Find x.

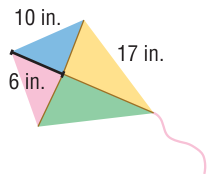
4A. $A = 92 \text{ in}^2$



4B. $A = 177 \text{ cm}^2$



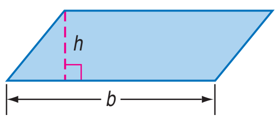
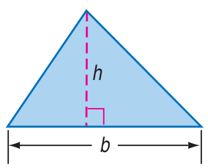
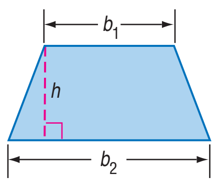
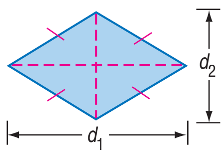
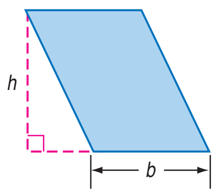
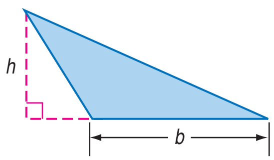
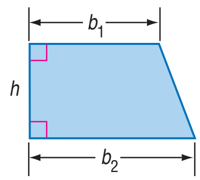
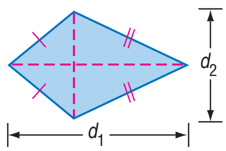
4C. ALGEBRA What is the area of the kite shown?



StudyTip

Kites Recall from Lesson 6-6 that the diagonals of kites are perpendicular.

ConceptSummary Areas of Polygons

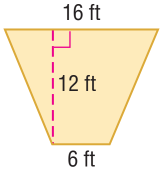
Parallelogram	Triangles	Trapezoids	Rhombi and Kites
			
			
$A = bh$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(b_1 + b_2)$	$A = \frac{1}{2}d_1d_2$

Check Your Understanding

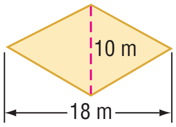
= Step-by-Step Solutions begin on page R14.

Examples 1–3 **Find the area of each trapezoid, rhombus, or kite.**

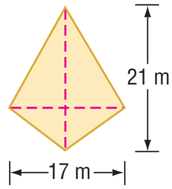
1.



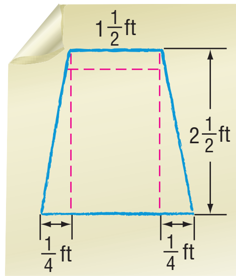
2.



3.

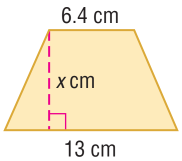


4. SHORT RESPONSE Suki is doing fashion design at 4-H Club. Her first project is to make a simple A-line skirt. How much fabric will she need according to the design at the right?

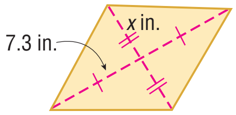


Example 4 **ALGEBRA** **Find x.**

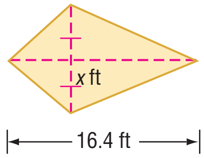
5. $A = 78 \text{ cm}^2$



6. $A = 96 \text{ in}^2$



7. $A = 104 \text{ ft}^2$

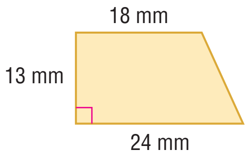


Practice and Problem Solving

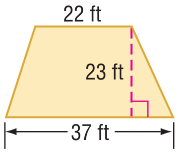
Extra Practice is on page R11.

Examples 1–3 CCSS **STRUCTURE** **Find the area of each trapezoid, rhombus, or kite.**

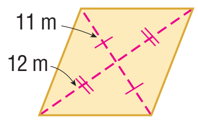
8.



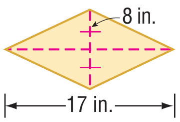
9.



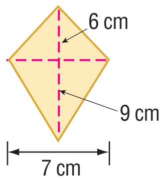
10.



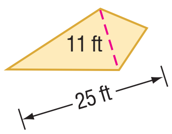
11.



12.

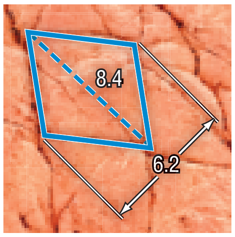


13.

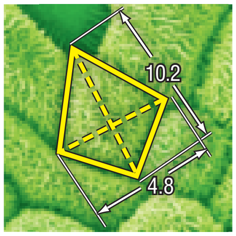


MICROSCOPES **Find the area of the identified portion of each magnified image. Assume that the identified portion is either a trapezoid, rhombus, or kite. Measures are provided in microns.**

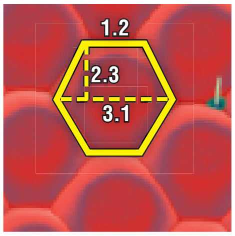
14. human skin



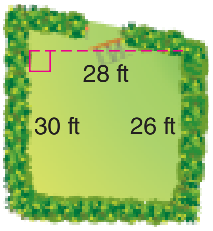
15 heartleaf plant



16. eye of a fly



17. JOBS Jimmy works on his neighbors' yards after school to earn extra money to buy a car. He is going to plant grass seed in Mr. Troyer's yard. What is the area of the yard?



Example 4 **ALGEBRA** **Find each missing length.**

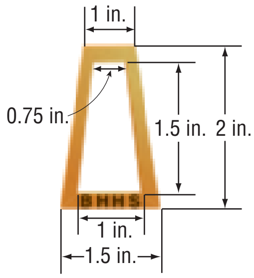
18. One diagonal of a kite is twice as long as the other diagonal. If the area of the kite is 240 square inches, what are the lengths of the diagonals?

19 The area of a rhombus is 168 square centimeters. If one diagonal is three times as long as the other, what are the lengths of the diagonals?

20. A trapezoid has base lengths of 12 and 14 feet with an area of 322 square feet. What is the height of the trapezoid?

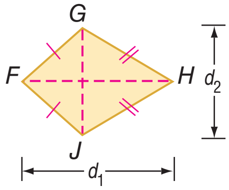
21. A trapezoid has a height of 8 meters, a base length of 12 meters, and an area of 64 square meters. What is the length of the other base?

22. HONORS Estella has been asked to join an honor society at school. Before the first meeting, new members are asked to sand and stain the front side of a piece of wood in the shape of an isosceles trapezoid. What is the surface area that Estella will need to sand and stain?

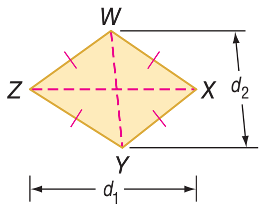


For each figure, provide a justification showing that $A = \frac{1}{2}d_1d_2$.

23.



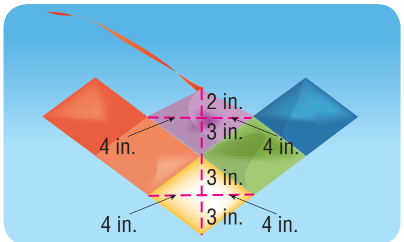
24.



25. CRAFTS Ashanti is in a kite competition. The yellow, red, orange, green, and blue pieces of her kite design shown are congruent rhombi.

a. How much fabric of each color does she need to buy?

b. Competition rules require that the total area of each kite be no greater than 200 square inches. Does Ashanti's kite meet this requirement? Explain.



CCSS SENSE-MAKING **Find the area of each quadrilateral with the given vertices.**

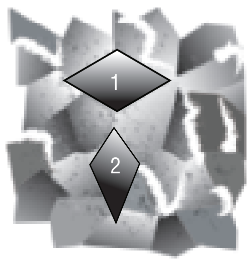
26. $A(-8, 6)$, $B(-5, 8)$, $C(-2, 6)$, and $D(-5, 0)$

27. $W(3, 0)$, $X(0, 3)$, $Y(-3, 0)$, and $Z(0, -3)$

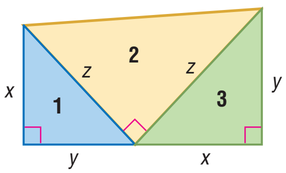
28. METALS When magnified in very powerful microscopes, some metals are composed of grains that have various polygonal shapes.

a. What is the area of figure 1 if the grain has a height of 4 microns and bases with lengths of 5 and 6 microns?

b. If figure 2 has perpendicular diagonal lengths of 3.8 microns and 4.9 microns, what is the area of the grain?

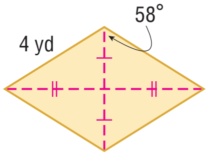


29. **PROOF** The figure at the right is a trapezoid that consists of two congruent right triangles and an isosceles triangle. In 1876, James A. Garfield, the 20th president of the United States, discovered a proof of the Pythagorean Theorem using this diagram. Prove that $x^2 + y^2 = z^2$.

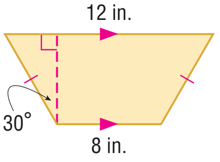


DIMENSIONAL ANALYSIS **Find the perimeter and area of each figure in feet. Round to the nearest tenth, if necessary.**

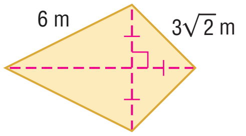
30.



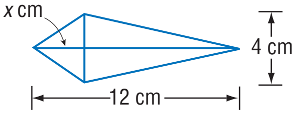
31.



32.



33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate perimeters of kites.



a. Geometric Draw a kite like the one shown if $x = 2$.

b. Geometric Repeat the process in part **a** for three x-values between 2 and 10 and for an x-value of 10.

c. Tabular Measure and record in a table the perimeter of each kite, along with the x-value.

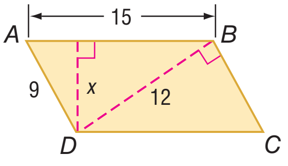
d. Graphical Graph the perimeter versus the x-value using the data from your table.

e. Analytical Make a conjecture about the value of x that will minimize the perimeter of the kite. What is the significance of this value?

H.O.T. Problems Use Higher-Order Thinking Skills

34. CCSS **CRITIQUE** Antonio and Madeline want to draw a trapezoid that has a height of 4 units and an area of 18 square units. Antonio says that only one trapezoid will meet the criteria. Madeline disagrees and thinks that she can draw several different trapezoids with a height of 4 units and an area of 18 square units. Is either of them correct? Explain your reasoning.

35. **CHALLENGE** Find x in parallelogram $ABCD$.



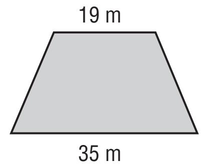
36. **OPEN ENDED** Draw a kite and a rhombus with an area of 6 square inches. Label and justify your drawings.

37. **REASONING** If the areas of two rhombi are equal, are the perimeters *sometimes*, *always*, or *never* equal? Explain.

38. **WRITING IN MATH** How can you use trigonometry to find the area of a figure?

Standardized Test Practice

39. The lengths of the bases of an isosceles trapezoid are shown below.



If the perimeter is 74 meters, what is its area?

A 162 m^2 **C** 332.5 m^2
B 270 m^2 **D** 342.25 m^2

40. SHORT RESPONSE One diagonal of a rhombus is three times as long as the other diagonal. If the area of the rhombus is 54 square millimeters, what are the lengths of the diagonals?

41. ALGEBRA What is the effect on the graph of the equation $y = \frac{1}{2}x$ when the equation is changed to $y = -2x$?

F The graph is moved 1 unit down.
G The graph is moved 1 unit up.
H The graph is rotated 45° about the origin.
J The graph is rotated 90° about the origin.

42. A regular hexagon is divided into 6 congruent triangles. If the perimeter of the hexagon is 48 centimeters, what is the height of each triangle?

A 4 cm **C** $6\sqrt{3}$ cm **E** $8\sqrt{3}$ cm
B $4\sqrt{3}$ cm **D** 8 cm

Spiral Review

COORDINATE GEOMETRY **Find the area of each figure.** (Lesson 11-1)

43. $\triangle JKL$ with $J(-4, 3)$, $K(-9, -1)$, and $L(-4, -4)$

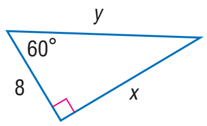
44. $\square RSTV$ with $R(-5, 7)$, $S(2, 7)$, $T(0, 2)$, and $V(-7, 2)$

45. WEATHER Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring? (Lesson 10-8)

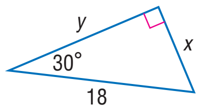
NOAA

Find x and y. (Lesson 8-3)

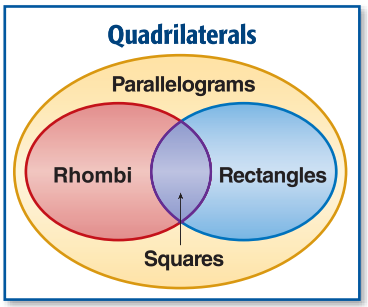
46.



47.



Use the Venn diagram to determine whether each statement is *always*, *sometimes*, or *never* true. (Lesson 6-5)



48. A parallelogram is a square.

49. A square is a rhombus.

50. A rectangle is a parallelogram.

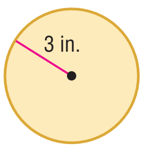
51. A rhombus is a rectangle but not a square.

52. A rhombus is a square.

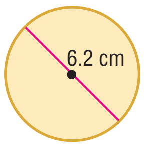
Skills Review

Find the circumference and area of each figure. Round to the nearest tenth.

53.



54.



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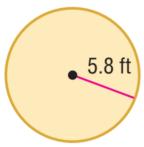


EXTEND 11-2 Geometry Lab
Population Density

After data are collected for the U.S. census, the population density is calculated for states, major cities, and other areas. **Population density** is the measurement of population per unit of area.

Common Core State Standards
Content Standards
G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★
Mathematical Practices 1

Activity 1 Calculate Population Density

Find the population density for the borough of Queens using the data in the table.

Calculate population density with the formula population density $= \frac{\text{population}}{\text{land area}}$.

The population density of Queens would be $\frac{2{,}229{,}379}{109.24}$ or about 20,408 people per square mile.

Borough	Population	Land Area (mi^2)
Brooklyn	2,465,326	70.61
Manhattan	1,537,195	22.96
Queens	2,229,379	109.24
Staten Island	443,728	58.48
The Bronx	1,332,650	42.03

Model and Analyze

1. Find the population densities for Brooklyn, Manhattan, Staten Island and the Bronx. Round to the nearest person. Of the five boroughs, which have the highest and the lowest population densities?

Activity 2 Use Population Density

PT

In a proposal to establish a new rustic campground at Yellowstone National Park, there is a concern about the number of wolves in the area. At last report, there were 98 wolves in the park. The new campground will be accepted if there are fewer than 2 wolves in the campground. Use the data in the table to determine if the new campground can be established.

Location	Size
Area of park	3472 mi^2
Area of new campground	10 acres

Step 1 Find the density of wolves in the park.
$98 \div 3472 = 0.028$ wolves per square mile

Step 2 Find the density of wolves in the proposed campground. First convert the size of the campground to square miles. If 1 acre is equivalent to 0.0015625 square mile, then 10 acres is 0.015625 square mile. The potential number of wolves in the proposed site is $0.015625 \cdot 0.028$ or 0.0004375 wolves.

Step 3 Since 0.0004375 is fewer than 2, the proposed campground can be accepted.

Exercises

2. Find the population density of gaming system owners if there are 436,000 systems in the United States and the area of the United States is 3,794,083 square miles.

3. The population density of the burrowing owl in Cape Coral, Florida, is 8.3 pairs per square mile. A new golf club is planned for a 2.4-square-mile site where the owl population is estimated to be 17 pairs. Would Lee County approve the proposed club if their policy is to decline when the estimated population density of owls is below the average density? Explain.

LESSON 11-3

Areas of Circles and Sectors

Then	Now	Why?
You found the circumference of a circle.	1 Find areas of circles. 2 Find areas of sectors of circles.	To determine whether a medium or large pizza is a better value, you can compare the cost per square inch. Divide the cost of each pizza by its area.

NewVocabulary
sector of a circle
segment of a circle

Common Core State Standards

Content Standards

G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

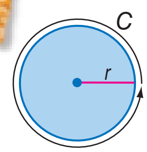
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Mathematical Practices

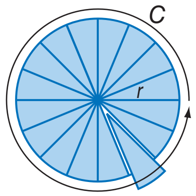
1 Make sense of problems and persevere in solving them.
6 Attend to precision.

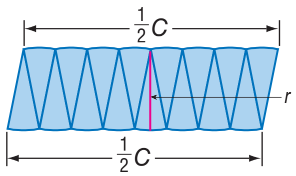
1 Areas of Circles

In Lesson 10-1, you learned that the formula for the circumference C of a circle with radius r is given by $C = 2\pi r$. You can use this formula to develop the formula for the area of a circle.



Below, a circle with radius r and circumference C has been divided into congruent pieces and then rearranged to form a figure that resembles a parallelogram.



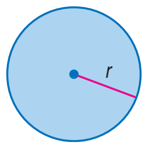


As the number of congruent pieces increases, the rearranged figure more closely approaches a parallelogram. The base of the parallelogram is $\frac{1}{2}C$ and the height is r, so its area is $\frac{1}{2}C \cdot r$. Since $C = 2\pi r$, the area of the parallelogram is also $\frac{1}{2}(2\pi r)r$ or πr^2.

KeyConcept Area of a Circle

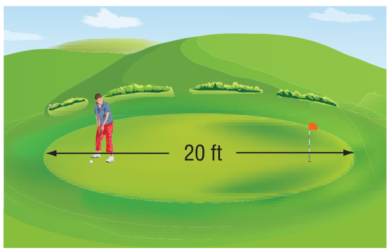
Words The area A of a circle is equal to π times the square of the radius r.

Symbols $A = \pi r^2$



Real-World Example 1 Area of a Circle

SPORTS What is the area of the circular putting green shown to the nearest square foot?



The diameter is 20 feet, so the radius is 10 feet.

$A = \pi r^2$	Area of a circle
$= \pi(10)^2$	$r = 10$
≈ 314	Use a calculator.

So, the area is about 314 square feet.

GuidedPractice

1. **SPORTS** An archery target has a radius of 12 inches. What is the area of the target to the nearest square inch?

Example 2 Use the Area of a Circle to Find a Missing Measure

ALGEBRA Find the radius of a circle with an area of 95 square centimeters.

$A = \pi r^2$	Area of a circle
$95 = \pi r^2$	$A = 95$
$\frac{95}{\pi} = r^2$	Divide each side by π.
$5.5 \approx r$	Use a calculator. Take the positive square root of each side.

The radius of the circle is about 5.5 centimeters.

GuidedPractice

2. **ALGEBRA** The area of a circle is 196π square yards. Find the diameter.

ReviewVocabulary

central angle an angle with a vertex in the center of a circle and with sides that contain two radii of the circle

arc a portion of a circle defined by two endpoints

2 Areas of Sectors

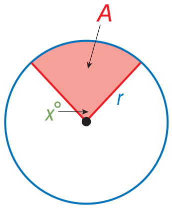
A slice of a circular pizza is an example of a sector of a circle. A **sector of a circle** is a region of a circle bounded by a central angle and its intercepted major or minor arc. The formula for the area of a sector is similar to the formula for arc length.

KeyConcept Area of a Sector

The ratio of the **area A of a sector** to the **area of the whole circle, πr^2**, is equal to the ratio of the **degree measure of the intercepted arc x** to 360.

Proportion: $\frac{A}{\pi r^2} = \frac{x}{360}$

Equation: $A = \frac{x}{360} \cdot \pi r^2$



Picturenet/Blend Images/Getty Images

Real-WorldLink

About 3 billion pizzas are sold each year in the United States. That is equivalent to about 46 slices per person annually.

Source: ThinkQuest Library

Real-World Example 3 Area of a Sector

PIZZA A circular pizza has a diameter of 12 inches and is cut into 8 congruent slices. What is the area of one slice to the nearest hundredth?

Step 1 Find the arc measure of a pizza slice.

Since the pizza is equally divided into 8 slices, each slice will have an arc measure of 360 ÷ 8 or 45.

Step 2 Find the radius of the pizza. Use this measure to find the area of the sector, or slice.

The diameter is 12 inches, so the radius is 6 inches.

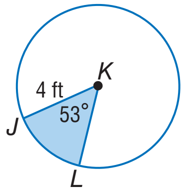
$A = \frac{x}{360} \cdot \pi r^2$	Area of a sector
$= \frac{45}{360} \cdot \pi (6)^2$	$x = 45$ and $r = 6$
≈ 14.14	Use a calculator.

So, the area of one slice of this pizza is about 14.14 square inches.

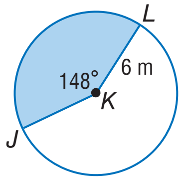
GuidedPractice

Find the area of the shaded sector. Round to the nearest tenth.

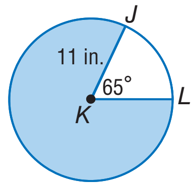
3A.



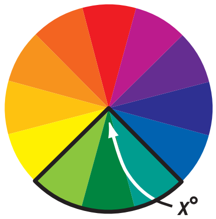
3B.



3C.



3D. **CRAFTS** The color wheel at the right is a tool that artists use to organize color schemes. If the diameter of the wheel is 10 inches and each of the 12 sections is congruent, find the approximate area covered by green hues.

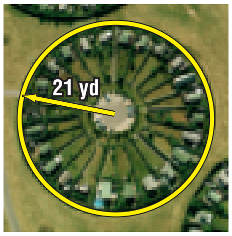


Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Example 1 **CONSTRUCTION** **Find the area of each circle. Round to the nearest tenth.**

1.



2.

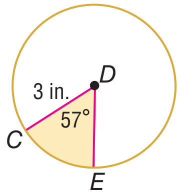
Example 2 **Find the indicated measure. Round to the nearest tenth.**

3. Find the diameter of a circle with an area of 74 square millimeters.

4. The area of a circle is 88 square inches. Find the radius.

Example 3 **Find the area of each shaded sector. Round to the nearest tenth.**

5.



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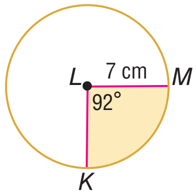
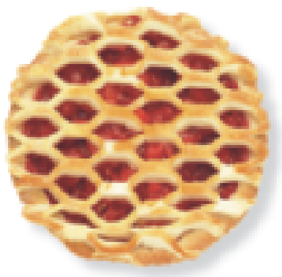


7. **BAKING** Chelsea is baking pies for a fundraiser at her school. She divides each 9-inch pie into 6 equal slices.

a. What is the area, in square inches, for each slice of pie?

b. If each slice costs $0.25 to make and she sells 8 pies at $1.25 for each slice, how much money will she raise?



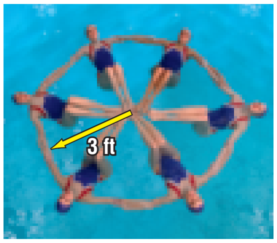
(tl)Tom Nebbia/CORBIS, (tr)Imageplus/CORBIS, (b)Stockbyte/Punchstock

Practice and Problem Solving

Extra Practice is on page R11.

Example 1 CCSS **MODELING** **Find the area of each circle. Round to the nearest tenth.**

8.



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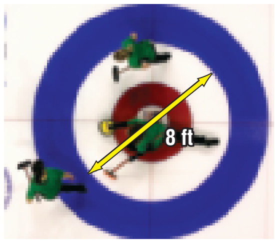

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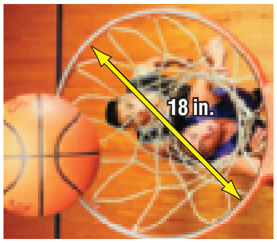

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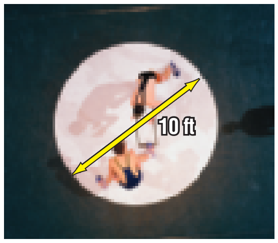


Example 2 **Find the indicated measure. Round to the nearest tenth, if necessary.**

14. The area of a circle is 68 square centimeters. Find the diameter.

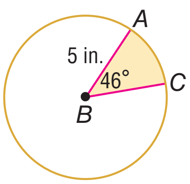
15. Find the diameter of a circle with an area of 94 square millimeters.

16. The area of a circle is 112 square inches. Find the radius.

17. Find the radius of a circle with an area of 206 square feet.

Example 3 **Find the area of each shaded sector. Round to the nearest tenth, if necessary.**

18.



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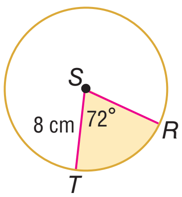

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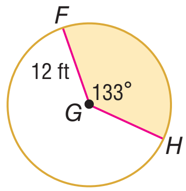

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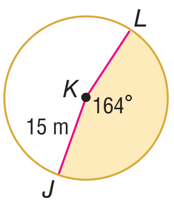

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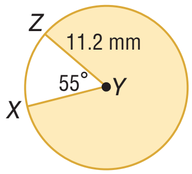

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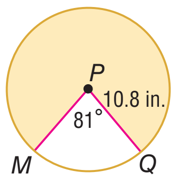
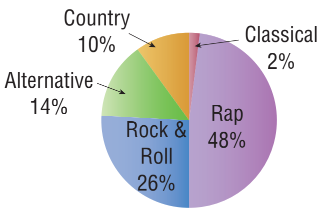


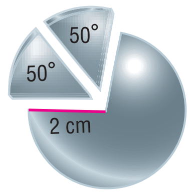
24. **MUSIC** The music preferences of students at Thomas Jefferson High are shown in the circle graph. Find the area of each sector and the degree measure of each intercepted arc if the radius of the circle is 1 unit.



25. **JEWELRY** A jeweler makes a pair of earrings by cutting two 50° sectors from a silver disk.

a. Find the area of each sector.

b. If the weight of the silver disk is 2.3 grams, how many milligrams does the silver wedge for each earring weigh?



26. **PROM** The table shows the results of a survey of students to determine their preference for a prom theme.

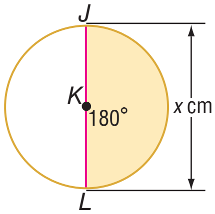
a. Create a circle graph with a diameter of 2 inches to represent these data.

b. Find the area of each theme's sector in your graph. Round to the nearest hundredth of an inch.

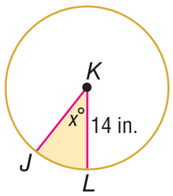
Theme	Percent
An Evening of Stars	11
Mardi Gras	32
Springtime in Paris	8
Night in Times Square	47
Undecided	2

CCSS SENSE-MAKING **The area A of each shaded region is given. Find x.**

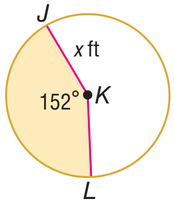
27. $A = 66\text{ cm}^2$



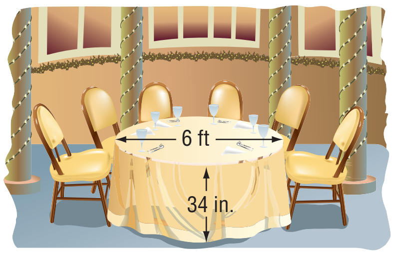
28. $A = 94\text{ in}^2$



29. $A = 128\text{ ft}^2$



30. **CRAFTS** Luna is making tablecloths with the dimensions shown for a club banquet. Find the area of each tablecloth in square feet if each one is to just reach the floor.



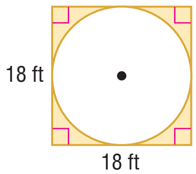
31 **TREES** The age of a living tree can be determined by multiplying the diameter of the tree by its growth factor, or rate of growth.

a. What is the diameter of a tree with a circumference of 2.5 feet?

b. If the growth factor of the tree is 4.5, what is the age of the tree?

Find the area of the shaded region. Round to the nearest tenth.

32.



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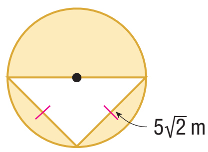

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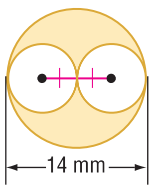

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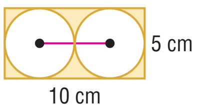

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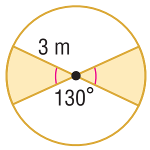

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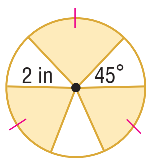
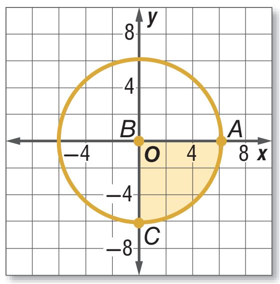
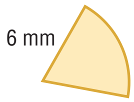


38. **COORDINATE GEOMETRY** What is the area of sector ABC shown on the graph?

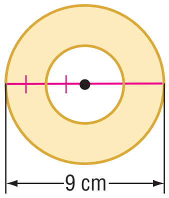


39. **ALGEBRA** The figure shown below is a sector of a circle. If the perimeter of the figure is 22 millimeters, find its area in square millimeters.



Find the area of each shaded region.

40.



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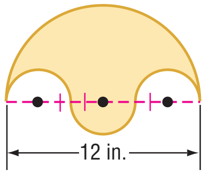

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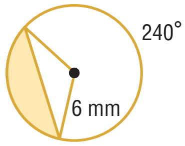
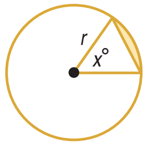


43. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate segments of circles. A **segment of a circle** is the region bounded by an arc and a chord.

a. Algebraic Write an equation for the area A of a segment of a circle with a radius r and a central angle of $x°$. (*Hint*: Use trigonometry to find the base and height of the triangle.)



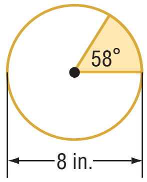
b. Tabular Calculate and record in a table ten values of A for x-values ranging from 10 to 90 if r is 12 inches. Round to the nearest tenth.

c. Graphical Graph the data from your table with the x-values on the horizontal axis and the A-values on the vertical axis.

d. Analytical Use your graph to predict the value of A when x is 63. Then use the formula you generated in part **a** to calculate the value of A when x is 63. How do the values compare?

H.O.T. Problems Use Higher-Order Thinking Skills

44. **ERROR ANALYSIS** Kristen and Chase want to find the area of the shaded region in the circle shown. Is either of them correct? Explain your reasoning.



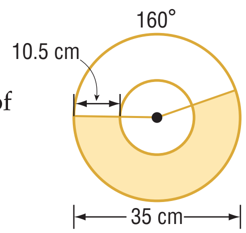
Kristen

$$A = \frac{x}{360} \cdot \pi r^2$$
$$= \frac{58}{360} \cdot \pi(8)^2$$
$$= 32.4 \text{ in}^2$$

Chase

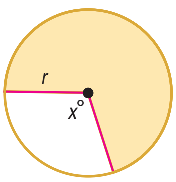
$$A = \frac{x}{360} \cdot \pi r^2$$
$$= \frac{58}{360} \cdot \pi(4)^2$$
$$= 8.1 \text{ in}^2$$

45. **CHALLENGE** Find the area of the shaded region. Round to the nearest tenth.



46. **CCSS ARGUMENTS** Refer to Exercise 43. Is the area of a sector of a circle *sometimes*, *always*, or *never* greater than the area of its corresponding segment?

47. **WRITING IN MATH** Describe two methods you could use to find the area of the shaded region of the circle. Which method do you think is more efficient? Explain your reasoning.

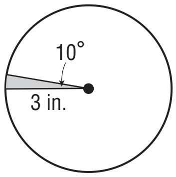


48. **CHALLENGE** Derive the formula for the area of a sector of a circle using the formula for arc length.

49. **WRITING IN MATH** If the radius of a circle doubles, will the measure of a sector of that circle double? Will it double if the arc measure of that sector doubles?

Standardized Test Practice

50. What is the area of the sector?



A $\frac{9\pi}{10}$ in^2

B $\frac{3\pi}{5}$ in^2

C $\frac{\pi}{4}$ in^2

D $\frac{\pi}{6}$ in^2

51. SHORT RESPONSE $\overleftrightarrow{MN}$ and $\overleftrightarrow{PQ}$ intersect at T. Find the value of x for which $m\angle MTQ = 2x + 5$ and $m\angle PTM = x + 7$. What are the degree measures of $\angle MTQ$ and $\angle PTM$?

52. ALGEBRA Raphael bowled 4 games and had a mean score of 130. He then bowled two more games with scores of 180 and 230. What was his mean score for all 6 games?

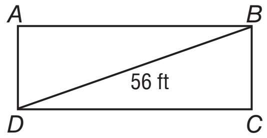
F 90

G 155

H 180

J 185

53. SAT/ACT The diagonals of rectangle $ABCD$ each have a length of 56 feet. If $m\angle BAC = 42°$, what is the length of $\overline{AB}$ to the nearest tenth of a foot?



A 80.5

B 75.4

C 56.3

D 50.4

E 41.6

Spiral Review

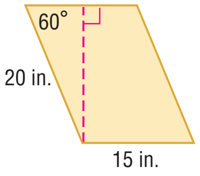
Find each missing length. (Lesson 11-2)

54. One diagonal of a kite is half as long as the other diagonal. If the area of the kite is 188 square inches, what are the lengths of the diagonals?

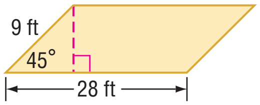
55. The area of a rhombus is 175 square centimeters. If one diagonal is two times as long as the other, what are the lengths of the diagonals?

Find the area of each parallelogram. Round to the nearest tenth if necessary. (Lesson 11-1)

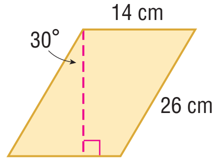
56.



57.



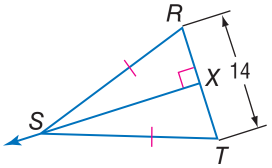
58.



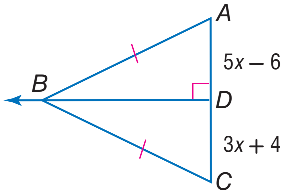
Skills Review

Find each measure.

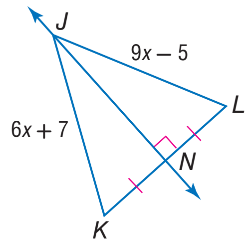
59. XT



60. AC



61. JK



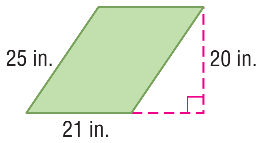
CHAPTER 11

Mid-Chapter Quiz

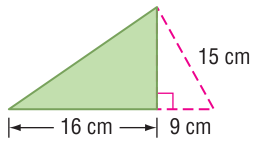
Lessons 11-1 through 11-3

Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary. (Lesson 11-1)

1.



2.



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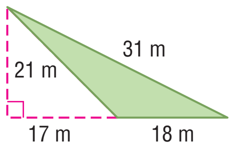
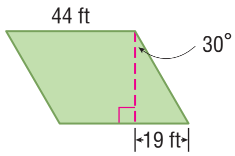


4.



5. The height of a triangle is 8 inches more than its base. The area of the triangle is 104.5 square inches. Find the base and height. (Lesson 11-1)

6. **DESIGN** A plaque is made with a rhombus in the middle. If the diagonals of the rhombus measure 7 inches and 9 inches, how much space is available for engraving text onto the award? (Lesson 11-2)

7. **MULTIPLE CHOICE** The area of a kite is 4 square feet. If the tail is to be 3 times longer than the kite's long diagonal, and the short diagonal measures 2 feet, how long should the kite's tail be? (Lesson 11-2)

A 4 feet

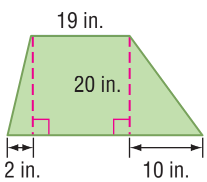
B 6 feet

C 7 feet

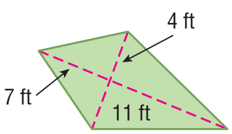
D 12 feet

Find the area of each trapezoid, rhombus, or kite. (Lesson 11-2)

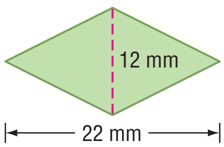
8.



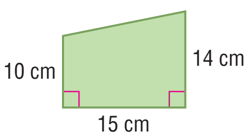
9.



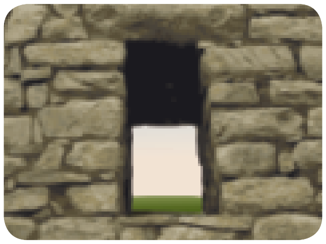
10.



11.



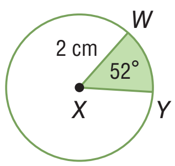
12. **ARCHAEOLOGY** The most predominant shape in Incan architecture is the trapezoid. The doorway pictured below is 3 feet wide at the top and 4 feet wide at the bottom. A person who is 5 feet 8 inches tall can barely pass through the doorway. How much fabric would be necessary to make a curtain for the doorway? (Lesson 11-2)



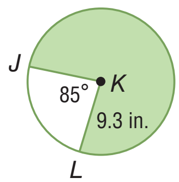
13. **ALGEBRA** A sector of a circle has a central angle measure of 30° and radius r. Write an expression for the perimeter of the sector in terms of r. (Lesson 11-3)

Find the area of each shaded sector. Round to the nearest tenth. (Lesson 11-3)

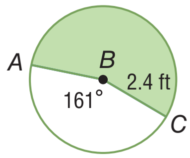
14.



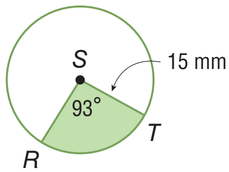
15.



16.



17.

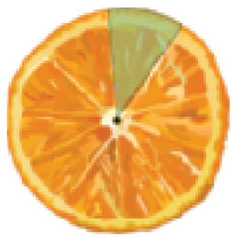


Find the indicated measure. Round to the nearest tenth. (Lesson 11-3)

18. The area of a circle is 52 square inches. Find the diameter.

19. Find the radius of a circle with an area of 104 square meters.

20. **FRUIT** The diameter of the orange slice shown is 9 centimeters. If each of the orange's 10 sections are congruent, find the approximate area covered by 8 sections. (Lesson 11-3)



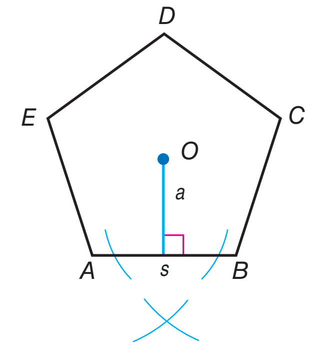
EXPLORE 11-4 Geometry Lab
Investigating Areas of Regular Polygons

The point in the interior of a regular polygon that is equidistant from all of the vertices is the *center* of the polygon. A segment from the center that is perpendicular to a side of the polygon is an **apothem**.

Activity

Step 1 Copy regular pentagon $ABCDE$ and its center O.

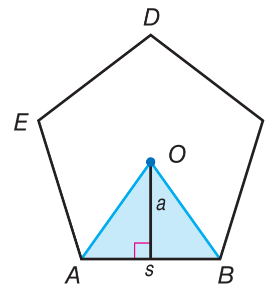
Step 2 Draw the apothem from O to side $\overline{AB}$ by constructing the perpendicular bisector of $\overline{AB}$. Label the apothem measure as a. Label the measure of $\overline{AB}$ as s.



Step 3 Use a straightedge to draw $\overline{OA}$ and $\overline{OB}$.

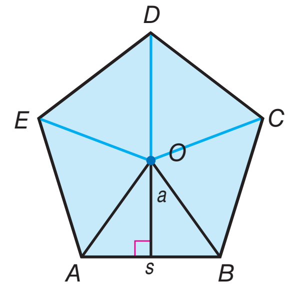
Step 4 What measure in $\triangle AOB$ represents the base of the triangle? What measure represents the height?

Step 5 Find the area of $\triangle AOB$ in terms of s and a.



Step 6 Draw $\overline{OC}$, $\overline{OD}$, and $\overline{OE}$. What is true of the five small triangles formed?

Step 7 How do the areas of the five triangles compare?



Analyze the Results

1. The area of a pentagon $ABCDE$ can be found by adding the areas of the given triangles that make up the pentagonal region.

$$A = \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa$$

$$A = \frac{1}{2}(sa + sa + sa + sa + sa) \text{ or } \frac{1}{2}(5sa)$$

What does $5s$ represent?

2. Write a formula for the area of a pentagon in terms of perimeter P.

LESSON 11-4

Areas of Regular Polygons and Composite Figures

Then

- You used inscribed and circumscribed figures and found the areas of circles.

Now

1. Find areas of regular polygons.
2. Find areas of composite figures.

Why?

- The top of the table shown is a regular hexagon. Notice that the top is composed of six congruent triangular sections. To find the area of the table top, you can find the sum of the areas of the sections.

NewVocabulary

center of a regular polygon
radius of a regular polygon
apothem
central angle of a regular polygon
composite figure

Common Core State Standards

Content Standards

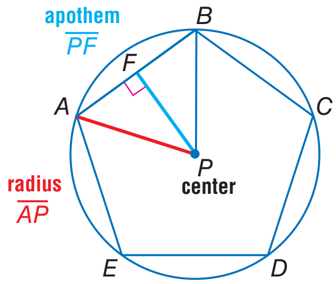
G.MG.3 Apply geometric methods to solve problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Mathematical Practices

1 Make sense of problems and persevere in solving them.
6 Attend to precision.

1 Areas of Regular Polygons

In the figure, a regular pentagon is *inscribed* in ⊙P, and ⊙P is *circumscribed* about the pentagon. The **center of a regular polygon** and the **radius of a regular polygon** are also the center and the radius of its circumscribed circle.



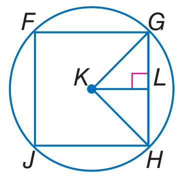
$\angle APB$ is a central angle of regular pentagon *ABCDE*.

A segment drawn from the center of a regular polygon perpendicular to a side of the polygon is called an **apothem**. Its length is the height of an isosceles triangle that has two radii as legs.

A **central angle of a regular polygon** has its vertex at the center of the polygon and its sides pass through consecutive vertices of the polygon. The measure of each central angle of a regular n-gon is $\frac{360}{n}$.

Example 1 Identify Segments and Angles in Regular Polygons

Square *FGHJ* is inscribed in ⊙*K*. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.

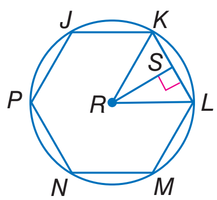


center: point K radius: $\overline{KG}$ or $\overline{KH}$

apothem: $\overline{KL}$ central angle: $\angle GKH$

A square is a regular polygon with 4 sides. Thus, the measure of each central angle of square *FGHJ* is $\frac{360}{4}$ or 90.

GuidedPractice



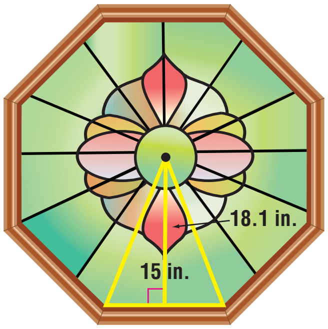
1. In the figure, regular hexagon *JKLMNP* is inscribed in ⊙R. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.

You can find the area of any regular n-gon by dividing the polygon into congruent isosceles triangles. This strategy is sometimes called *decomposing the polygon into triangles.*

Lindsey Stock/Alamy

Real-World Example 2 Area of a Regular Polygon

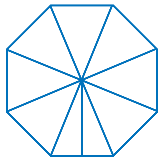
ART **Kang created the stained glass window shown. The window is a regular octagon with a side length of 15 inches and an apothem of 18.1 inches. What is the area covered by the window?**



Step 1 Divide the polygon into congruent isosceles triangles.

Since the polygon has 8 sides, the polygon can be divided into 8 congruent isosceles triangles, each with a base of 15 inches and a height of 18.1 inches.

Step 2 Find the area of one triangle.



$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(15)(18.1)$ $b = 15$ and $h = 18.1$

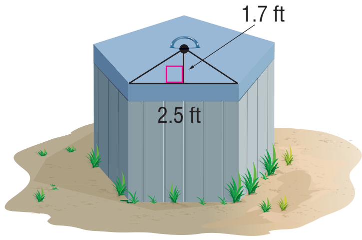
$= 135.75 \text{ in}^2$ Simplify.

Step 3 Multiply the area of one triangle by the total number of triangles.

Since there are 8 triangles, the area of the stained glass is $135.75 \cdot 8$ or 1086 square inches.

GuidedPractice

2. **HOT TUBS** The cover of the hot tub shown is a regular pentagon. If the side length is 2.5 feet and the apothem is 1.7 feet, find the area of the lid to the nearest tenth.



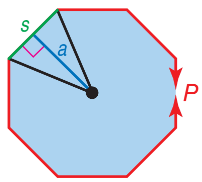
ReadingMath

Apothem Like the *radius* of a circle, the *apothem* of a polygon refers to the length of any apothem of the polygon.

WatchOut!

Area of Regular Polygon this approach can only be applied to *regular* polygons.

From Example 2, we can develop a formula for the area of a regular n-gon with side length s and apothem a.



A = area of one triangle • number of triangles

$= \frac{1}{2} \cdot$ base • height • number of triangles

$= \frac{1}{2} \cdot s \cdot a \cdot n$ Base of triangle is s and height is a. The number of triangles is n.

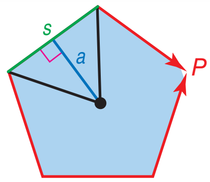
$= \frac{1}{2} \cdot a \cdot (n \cdot s)$ Commutative and Associative Properties

$= \frac{1}{2} \cdot a \cdot P$ The perimeter P of the polygon is $n \cdot s$.

KeyConcept Area of a Regular Polygon

Words The area A of a regular n-gon with side length s is one half the product of the apothem a and perimeter P.

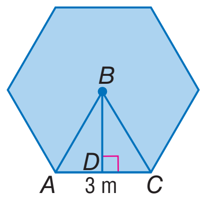
Symbols $A = \frac{1}{2}a(ns)$ or $A = \frac{1}{2}aP$.



Example 3 Use the Formula for the Area of a Regular Polygon

Find the area of each regular polygon. Round to the nearest tenth.

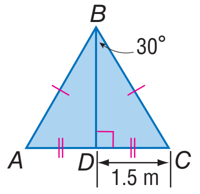
a. regular hexagon



Step 1 Find the measure of a central angle.

A regular hexagon has 6 congruent central angles, so $m\angle ABC = \frac{360}{6}$ or 60.

Step 2 Find the apothem.



Apothem $\overline{BD}$ is the height of isosceles $\triangle ABC$. It bisects $\angle ABC$, so $m\angle DBC = 30$. It also bisects $\overline{AC}$, so $DC = 1.5$ meters.

$\triangle BDC$ is a 30°-60°-90° triangle with a shorter leg that measures 1.5 meters, so $BD = 1.5\sqrt{3}$ meters.

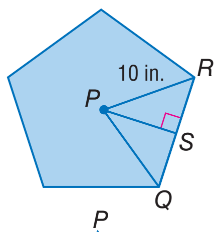
Step 3 Use the apothem and side length to find the area.

$A = \frac{1}{2}aP$ — Area of a regular polygon

$= \frac{1}{2}(1.5\sqrt{3})(18)$ — $a = 1.5\sqrt{3}$ and $P = 6(3)$ or 18

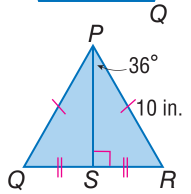
$\approx 23.4 \text{ m}^2$ — Use a calculator.

b. regular pentagon



Step 1 A regular pentagon has 5 congruent central angles, so $m\angle QPR = \frac{360}{5}$ or 72.

Step 2 Apothem $\overline{PS}$ is the height of isosceles $\triangle RPQ$. It bisects $\angle RPQ$, so $m\angle RPS = 36$. Use trigonometric ratios to find the side length and apothem of the polygon.



$\sin 36° = \frac{SR}{10}$ $\qquad$ $\cos 36° = \frac{PS}{10}$

$10 \sin 36° = SR$ $\qquad$ $10 \cos 36° = PS$

$QR = 2SR$ or $2(10 \sin 36°)$. So the pentagon's perimeter is $5 \cdot 2(10 \sin 36°)$ or $10(10 \sin 36°)$. The length of the apothem $\overline{PS}$ is $10 \cos 36°$.

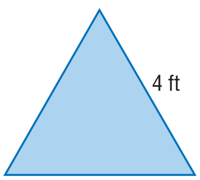
Step 3 $A = \frac{1}{2}aP$ — Area of a regular polygon

$= \frac{1}{2}(10 \cos 36°)[10(10 \sin 36°)]$ — $a = 10 \cos 36°$, $P = 10(10 \sin 36°)$

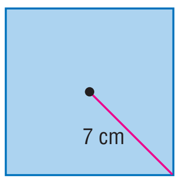
$\approx 237.8 \text{ in}^2$ — Use a calculator.

Guided Practice

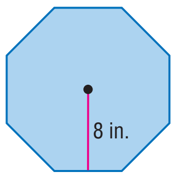
3A.



3B.



3C.



> **Study Tip**
>
> **CCSS Precision** The altitude of an isosceles triangle from its vertex to its base is also an angle bisector and median of the triangle.

2 Areas of Composite Figures

A **composite figure** is a figure that can be separated into regions that are basic figures, such as triangles, rectangles, trapezoids, and circles. To find the area of a composite figure, find the area of each basic figure and then use the Area Addition Postulate.

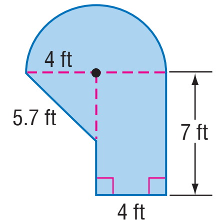
Real-WorldLink

The first miniature golf course was built in Pinehurst, North Carolina, on a private estate owned by James Barber. There are currently between 5000 and 7500 miniature golf courses in the United States.

Source: Miniature Golf Association of the United States

Example 4 Find the Area of a Composite Figure by Adding

MINIATURE GOLF The dimensions of a putting green at a miniature golf course are shown. How many square feet of carpet are needed to cover this green?



The area to be carpeted can be separated into a rectangle with a length of 4 feet and a width of 7 feet, a right triangle with a hypotenuse of 5.7 feet and a leg measuring 4 feet, and a semicircle with a radius of 4 feet.

Using the Pythagorean Theorem, the other leg of the right triangle is $\sqrt{5.7^2 - 4^2}$ or about 4.1 feet.

Area of green = **area of rectangle** + **area of triangle** + **area of semicircle.**

$$= \ell \cdot w + \frac{1}{2} \cdot b \cdot h + \frac{180}{360} \cdot \pi \cdot r^2$$

$$\approx 4 \cdot 7 + \frac{1}{2} \cdot 4 \cdot 4.1 + \frac{180}{360} \cdot \pi \cdot 4^2$$

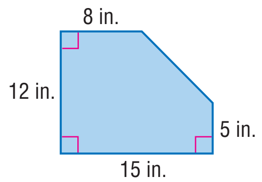
$$\approx 28 + 8.2 + 8\pi \text{ or about } 61.3 \text{ ft}^2$$

So, about 62 square feet of carpet is needed.

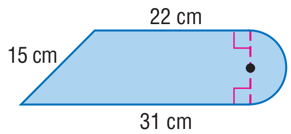
GuidedPractice

Find the area of each figure. Round to the nearest tenth if necessary.

4A.



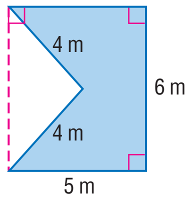
4B.



The areas of some figures can be found by subtracting the areas of basic figures.

Example 5 Find the Area of a Composite Figure by Subtracting

Find the area of the figure. Round to the nearest tenth if necessary.



To find the area of the figure, subtract the area of the triangle from the area of the rectangle.

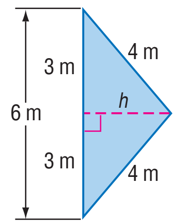
Using the Pythagorean Theorem, the height h of the triangle is $\sqrt{4^2 - 3^2}$ or $\sqrt{7}$ meters.

Area of figure = **Area of rectangle** − **Area of triangle**

$$= b \cdot h - \frac{1}{2}bh$$

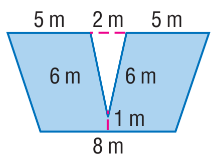
$$= 5 \cdot 6 - \frac{1}{2}(6)(\sqrt{7})$$

$$\approx 30 - 7.9 \text{ or about } 22.1 \text{ m}^2$$

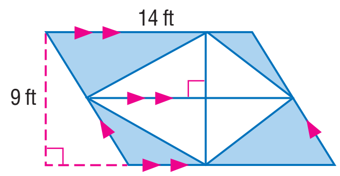


GuidedPractice

5A.



5B.



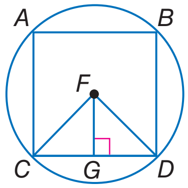
Exactostock/SuperStock

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

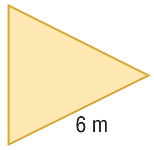
Example 1

1. In the figure, square $ABDC$ is inscribed in $\odot F$. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.

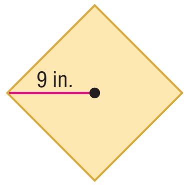


Examples 2–3 **Find the area of each regular polygon. Round to the nearest tenth.**

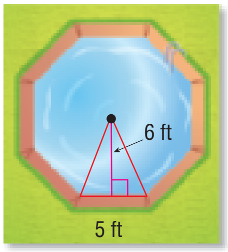
2.



3.

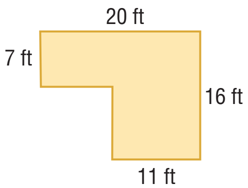


4. **POOLS** Kenton's job is to cover the community pool during fall and winter. Since the pool is in the shape of an octagon, he needs to find the area in order to have a custom cover made. If the pool has the dimensions shown at the right, what is the area of the pool?

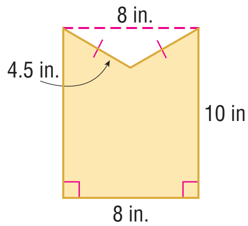


Examples 4–5 CCSS **SENSE-MAKING** **Find the area of each figure. Round to the nearest tenth if necessary.**

5.



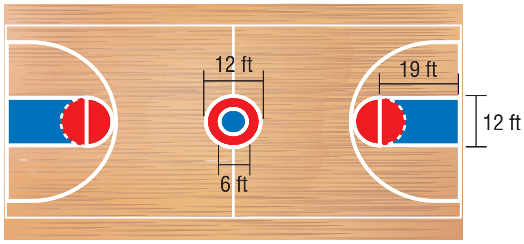
6.



7 **BASKETBALL** The basketball court in Jeff's school is painted as shown.

a. What area of the court is blue? Round to the nearest square foot.

b. What area of the court is red? Round to the nearest square foot.



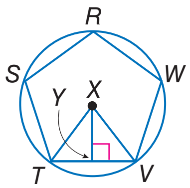
Note: Art not drawn to scale.

Practice and Problem Solving

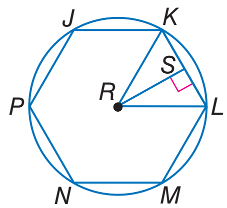
Extra Practice is on page R11.

Example 1 **In each figure, a regular polygon is inscribed in a circle. Identify the center, a radius, an apothem, and a central angle of each polygon. Then find the measure of a central angle.**

8.

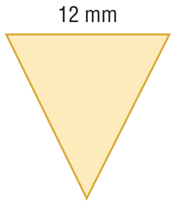


9.

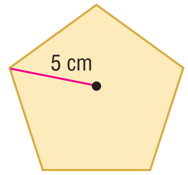


Examples 2–3 **Find the area of each regular polygon. Round to the nearest tenth.**

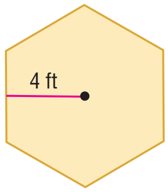
10.



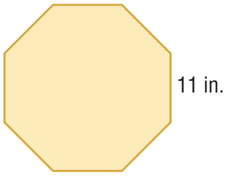
11.



12.

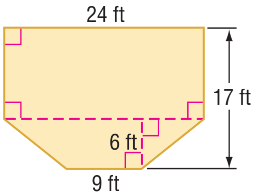


13.



Example 4

14. **CARPETING** Ignacio's family is getting new carpet in their family room, and they want to determine how much the project will cost.

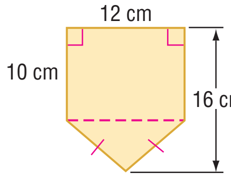


a. Use the floor plan shown to find the area to be carpeted.

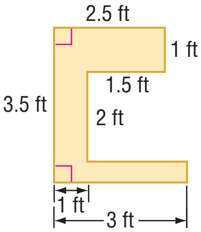
b. If the carpet costs $4.86 per square yard, how much will the project cost?

Examples 4–5 CCSS **SENSE-MAKING** **Find the area of each figure. Round to the nearest tenth if necessary.**

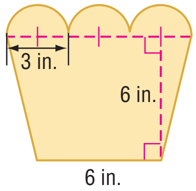
15



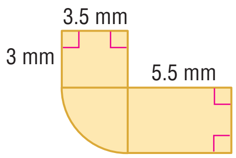
16.



17.



18.



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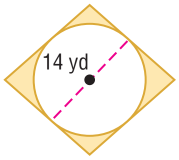

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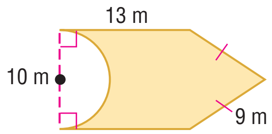
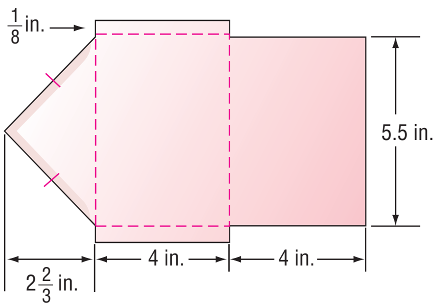


21. **CRAFTS** Latoya's greeting card company is making envelopes for a card from the pattern shown.

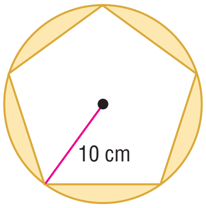
a. Find the perimeter and area of the pattern. Round to the nearest tenth.

b. If Latoya orders sheets of paper that are 2 feet by 4 feet, how many envelopes can she make per sheet?

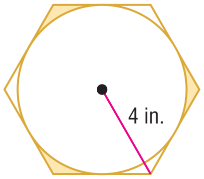


Find the area of each shaded region formed by each circle and regular polygon. Round to the nearest tenth.

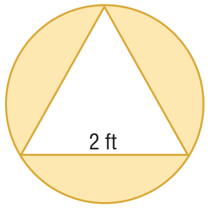
22.



23.



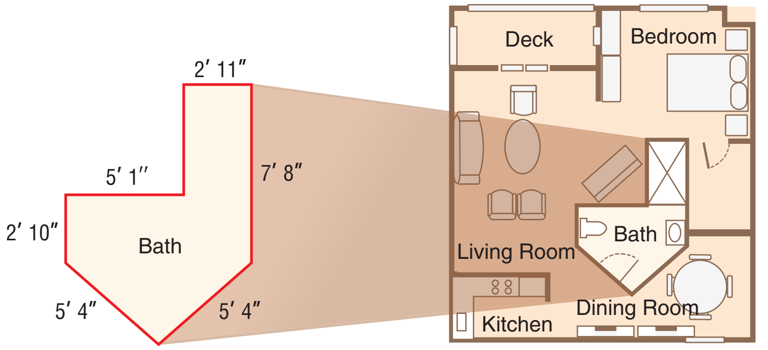
24.



25. **FLOORING** JoAnn wants to lay 12″ × 12″ tile on her bathroom floor.

a. Find the area of the bathroom floor in her apartment floor plan.

b. If the tile comes in boxes of 15 and JoAnn buys no extra tile, how many boxes will she need?



Find the perimeter and area of each figure. Round to the nearest tenth, if necessary.

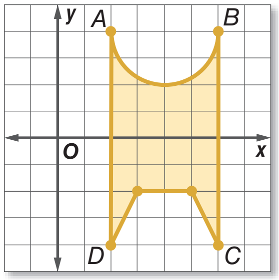
26. a regular hexagon with a side length of 12 centimeters

27. a regular pentagon circumscribed about a circle with a radius of 8 millimeters

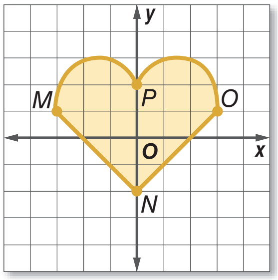
28. a regular octagon inscribed in a circle with a radius of 5 inches

CCSS **PERSEVERANCE** **Find the area of each shaded region. Round to the nearest tenth.**

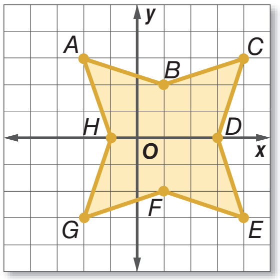
29.



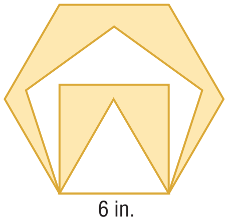
30.



31.

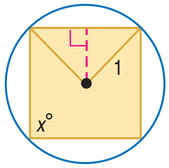


32. Find the total area of the shaded regions. Round to the nearest tenth.



33. **CHANGING DIMENSIONS** Calculate the area of an equilateral triangle with a perimeter of 3 inches. Calculate the areas of a square, a regular pentagon, and a regular hexagon with perimeters of 3 inches. How does the area of a regular polygon with a fixed perimeter change as the number of sides increases?

34. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the areas of regular polygons inscribed in circles.



a. Geometric Draw a circle with a radius of 1 unit and inscribe a square. Repeat twice, inscribing a regular pentagon and hexagon.

b. Algebraic Use the inscribed regular polygons from part **a** to develop a formula for the area of an inscribed regular polygon in terms of angle measure x and number of sides n.

c. Tabular Use the formula you developed in part **b** to complete the table below. Round to the nearest hundredth.

Number of Sides, n	4	5	6	8	10	20	50	100
Interior Angle Measure, x								
Area of Inscribed Regular Polygon								

d. Verbal Make a conjecture about the area of an inscribed regular polygon with a radius of 1 unit as the number of sides increases.

H.O.T. Problems Use Higher-Order Thinking Skills

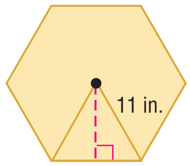
35. **ERROR ANALYSIS** Chloe and Flavio want to find the area of the hexagon shown. Is either of them correct? Explain your reasoning.

Chloe

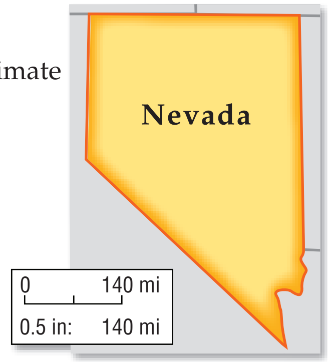
$$A = \frac{1}{2}Pa$$
$$= \frac{1}{2}(66)(9.5)$$
$$= 313.5 \text{ in}^2$$

Flavio

$$A = \frac{1}{2}Pa$$
$$= \frac{1}{2}(33)(9.5)$$
$$= 156.8 \text{ in}^2$$

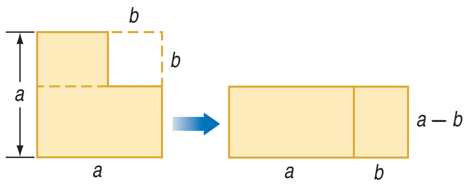


36. **SENSE-MAKING** Using the map of Nevada shown, estimate the area of the state. Explain your reasoning.



37. **OPEN ENDED** Draw a pair of composite figures that have the same area. Make one composite figure out of a rectangle and a trapezoid, and make the other composite figure out of a triangle and a rectangle. Show the area of each basic figure.

38. **WRITING IN MATH** Consider the sequence of area diagrams shown.



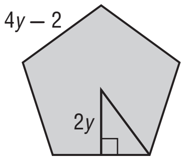
a. What algebraic theorem do the diagrams prove? Explain your reasoning.

b. Create your own sequence of diagrams to prove a different algebraic theorem.

39. **WRITING IN MATH** How can you find the area of any figure?

Standardized Test Practice

40. Which polynomial best represents the area of the regular pentagon shown below?



A $10y^2 - 5$

B $10y^2 + 5y$

C $20y^2 + 10$

D $20y^2 - 10y$

41. What is $27^{-\frac{2}{3}}$ in radical form?

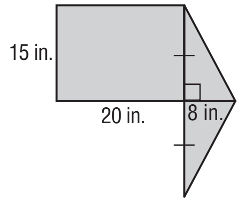
F $\dfrac{1}{(\sqrt[3]{27})^2}$

G $(\sqrt[3]{27})^2$

H $\dfrac{1}{(\sqrt{27})^2}$

J $(\sqrt{27})^3$

42. SHORT RESPONSE Find the area of the shaded figure in square inches. Round to the nearest tenth.



43. SAT/ACT If the cos $\theta = \frac{12}{13}$ what is the value of tan θ?

A $\frac{13}{5}$

B $\frac{12}{5}$

C $\frac{13}{12}$

D $\frac{5}{12}$

E $\frac{5}{13}$

Spiral Review

Find the indicated measure. Round to the nearest tenth. (Lesson 11-3)

44. The area of a circle is 95 square feet. Find the radius.

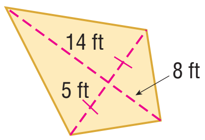
45. Find the area of a circle whose radius is 9 centimeters.

46. The area of a circle is 256 square inches. Find the diameter.

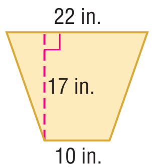
47. Find the area of a circle whose diameter is 25 millimeters.

Find the area of each trapezoid, rhombus, or kite. (Lesson 11-2)

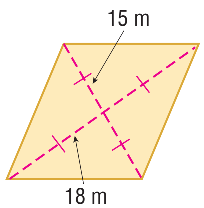
48.



49.



50.

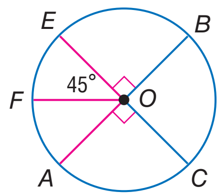


$\overline{EC}$ **and** $\overline{AB}$ **are diameters of ⊙*O*. Identify each arc as a *major arc*, *minor arc*, or *semicircle* of the circle. Then find its measure.** (Lesson 10-2)

51. $m\widehat{ACB}$

52. $m\widehat{EB}$

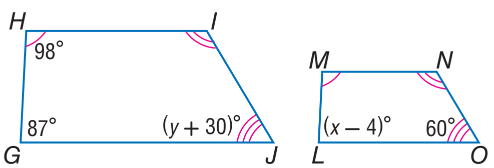
53. $m\widehat{ACE}$



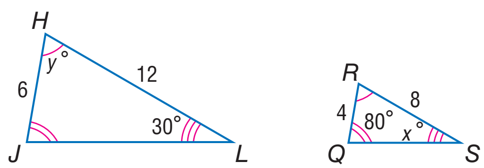
Skills Review

Each pair of polygons is similar. Find *x*.

54.



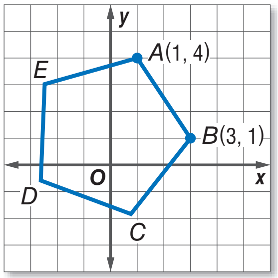
55.



EXTEND 11-4

Geometry Lab
Regular Polygons on the Coordinate Plane

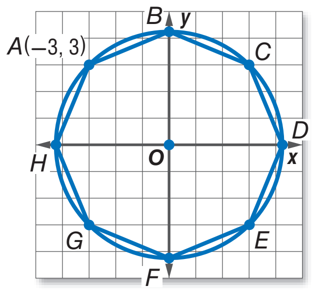
If you know the coordinates of two consecutive vertices of a regular polygon, you can use the Distance Formula to find the length of each side. For example, in the figure shown, the length of $\overline{AB}$ is $\sqrt{(3-1)^2 + (1-4)^2}$ or $\sqrt{13}$. Using this measure, you can then find the perimeter and area of the figure using the techniques presented in Lesson 11-4.



You can also use the Distance Formula to find the perimeter and area of a regular polygon inscribed in a circle given the coordinates of the endpoints of a radius.

Activity 1 Inscribed Polygon

Find the perimeter and area of octagon *ABCDEFGH*, which is inscribed in ⊙*O*. Round to the nearest tenth, if necessary.

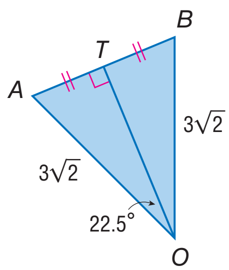


Step 1 Use the Distance Formula to find a radius of ⊙*O*.

$$OA = \sqrt{(-3-0)^2 + (3-0)^2} \qquad x_2 = -3,\ x_1 = 0,\ y_2 = 3, \text{ and } y_1 = 0$$

$$= \sqrt{18} \text{ or } 3\sqrt{2} \qquad \text{Simplify.}$$

Step 2 Find the perimeter and area.



Because the octagon is inscribed in ⊙*O*, $\overline{OA}$ and $\overline{OB}$ are both radii of ⊙*O*. Therefore, $OA = OB = 3\sqrt{2}$. Let $\overline{OT}$ be an apothem of the octagon with length a. Then $\overline{OT}$ is also the height of isosceles $\triangle AOB$. Since the octagon is regular, $m\angle AOB$ is $360 \div 8$ or 45. Since $\overline{OT}$ bisects $\angle AOB$ and side $\overline{AB}$, $m\angle AOT = 45 \div 2$ or 22.5, and $AB = 2(AT)$.

Use trigonometric ratios to find a and AT.

$$\cos 22.5° = \frac{a}{3\sqrt{2}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$a = 3\sqrt{2}\cos 22.5° \qquad \text{Solve for } a.$$

$$\sin 22.5° = \frac{AT}{3\sqrt{2}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}}$$

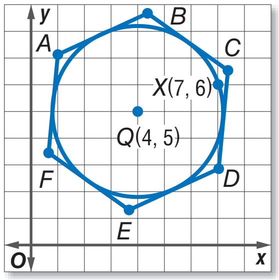
$$AT = 3\sqrt{2}\sin 22.5° \qquad \text{Solve for } AT.$$

$AB = 2(AT)$, so $AB = 2(3\sqrt{2}\sin 22.5°)$ and the perimeter P of the octagon is $8(2)3\sqrt{2}\sin 22.5°$ or about 26.0 units. The area of the octagon is $\frac{1}{2}aP$, which is $\frac{1}{2}3\sqrt{2}\cos 22.5° \cdot 8(2)3\sqrt{2}\sin 22.5°$ or about 50.9 units^2.

You can also use the Distance Formula to find the perimeter and area of a regular polygon circumscribed about a circle given the coordinates of the endpoints of a radius.

Activity 2 Circumscribed Polygon

Find the perimeter and area of hexagon $ABCDEF$, which is circumscribed about $\odot Q$. Round to the nearest tenth, if necessary.

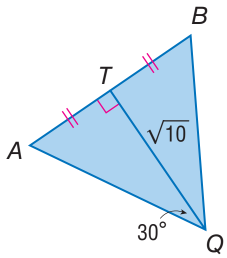


Step 1 Use the Distance Formula to find a radius of $\odot Q$.

$QX = \sqrt{(7-4)^2 + (6-5)^2}$ or $\sqrt{10}$ $x_2 = 7, x_1 = 4, y_2 = 6$, and $y_1 = 5$

Step 2 Find the perimeter and area of hexagon $ABCDEF$.

Because the hexagon is circumscribed about $\odot Q$, $\overline{AB}$ is tangent to the circle. Let the point of tangency be T. Since all radii of a circle are congruent, radius $\overline{QT}$ also measures $\sqrt{10}$. $\overline{QT}$ is an apothem of the hexagon, so $a = \sqrt{10}$.



The apothem is also the height of isosceles $\triangle AQB$. Since the hexagon is regular, $m\angle AQB$ is $360 \div 6$ or 60. Since $\overline{QT}$ bisects $\angle AQB$ and side $\overline{AB}$ $m\angle AQT = 60 \div 2$ or 30, and $AB = 2(AT)$. Use trigonometric ratios to find AT. Then find AB.

$\tan 30° = \frac{AT}{\sqrt{10}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$AT = \sqrt{10} \tan 30°$ Solve for AT.

$AT = \sqrt{10}\left(\frac{\sqrt{3}}{3}\right)$ or $\frac{\sqrt{30}}{3}$ $\tan 30° = \frac{\sqrt{3}}{3}$

$$AB = 2(AT) = 2\left(\frac{\sqrt{30}}{3}\right) = \frac{2\sqrt{30}}{3}$$

The perimeter P of the hexagon is $6 \cdot \frac{2\sqrt{30}}{3}$ or $4\sqrt{30}$, which is about 21.9 units. The area of the hexagon is $\frac{1}{2}aP$, which is $\frac{1}{2}\sqrt{10}(4\sqrt{30})$ or about 34.6 units^2.

Exercises

Find the perimeter and area of each regular polygon with the given consecutive vertices. Round to the nearest tenth, if necessary.

1. pentagon $ABCDE$; $A(1, 4)$, $B(3, 1)$

2. hexagon $ABCDEF$; $A(-4, 2)$, $B(0, 5)$

Find the perimeter and area of each regular polygon inscribed in $\odot O$, centered at the origin, and containing the given point. Round to the nearest tenth, if necessary.

3. pentagon $ABCDE$; $E(-4, -1)$

4. hexagon $ABCDEF$; $D(4, -5)$

Find the perimeter and area of each regular polygon circumscribed about $\odot Q$, with the given center and point X on the circle. Round to the nearest tenth, if necessary.

5. pentagon $ABCDE$; $Q(-2, 1)$; $X(-1, 3)$

6. octagon $ABCDEFGH$; $Q(3, -1)$; $X(1, -3)$

LESSON 11-5

Areas of Similar Figures

Then

- You used scale factors and proportions to solve problems involving the perimeters of similar figures.

Now

- **1** Find areas of similar figures by using scale factors.
  2 Find scale factors or missing measures given the areas of similar figures.

Why?

- Architecture firms often hire model makers to make scale models of projects that are used to sell their designs. Since the base of a model is geometrically similar to the base of the actual building it represents, their areas are related.

CCSS Common Core State Standards

Content Standards

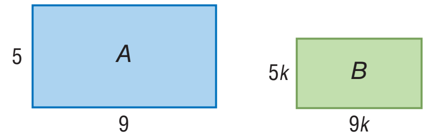
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★

Mathematical Practices

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.

1 Areas of Similar Figures

In Lesson 7-2, you learned that if two polygons are similar, then their perimeters are proportional to the scale factor between them. The areas of two similar polygons share a different relationship.



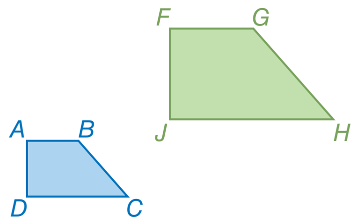
$$\frac{\text{perimeter of figure } B}{\text{perimeter of figure } A} = \frac{28k}{28} \text{ or } k$$

$$\frac{\text{area of figure } B}{\text{area of figure } A} = \frac{45k^2}{45} \text{ or } k^2$$

Theorem 11.1 Areas of Similar Polygons

Words If two polygons are similar, then their areas are proportional to the square of the scale factor between them.

Example If $ABCD \sim FGHJ$, then $\frac{\text{area of } FGHJ}{\text{area of } ABCD} = \left(\frac{FG}{AB}\right)^2$.

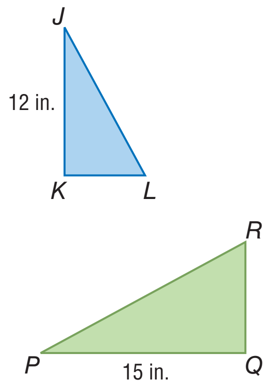


You will prove Theorem 11.1 for triangles in Exercise 22.

Example 1 Find Areas of Similar Polygons

PT

If $\triangle JKL \sim \triangle PQR$ and the area of $\triangle JKL$ is 30 square inches, find the area of $\triangle PQR$.



The scale factor between $\triangle PQR$ and $\triangle JKL$ is $\frac{15}{12}$ or $\frac{5}{4}$, so the ratio of their areas is $\left(\frac{5}{4}\right)^2$.

$\frac{\text{area of } \triangle PQR}{\text{area of } \triangle JKL} = \left(\frac{5}{4}\right)^2$ — Write a proportion.

$\frac{\text{area of } \triangle PQR}{30} = \frac{25}{16}$ — Area of $\triangle JKL = 30$ and $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$

$\text{area of } \triangle PQR = \frac{25}{16} \cdot 30$ — Multiply each side by 30.

$\text{area of } \triangle PQR = 46.875$ — Simplify.

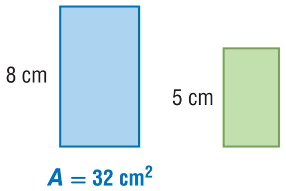
So the area of $\triangle PQR$ is about 46.9 square inches.

Nicholas Cope/Stone/Getty Images

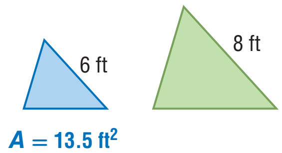
GuidedPractice

For each pair of similar figures, find the area of the green figure.

1A.



1B.

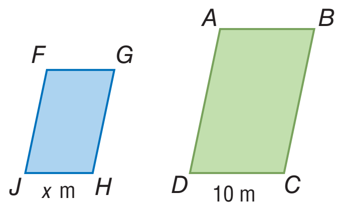


2 Scale Factors and Missing Measures in Similar Figures

You can use the areas of similar figures to find the scale factor between them or a missing measure.

Example 2 Use Areas of Similar Figures

The area of ▱*ABCD* is 150 square meters. The area of ▱*FGHJ* is 54 square meters. If ▱*ABCD* ~ ▱*FGHJ*, find the scale factor of ▱*FGHJ* to ▱*ABCD* and the value of *x*.



Let k be the scale factor between ▱*FGHJ* and ▱*ABCD*.

$\frac{\text{area of ▱}FGHJ}{\text{area of ▱}ABCD} = k^2$ — Theorem 11.1

$\frac{54}{150} = k^2$ — Substitution

$\frac{9}{25} = k^2$ — Simplify.

$\frac{3}{5} = k$ — Take the positive square root of each side.

So the scale factor of ▱*FGHJ* to ▱*ABCD* is $\frac{3}{5}$. Use this scale factor to find the value of x.

$\frac{JH}{DC} = k$ — The ratio of corresponding lengths of similar polygons is equal to the scale factor between the polygons.

$\frac{x}{10} = \frac{3}{5}$ — Substitution

$x = \frac{3}{5} \cdot 10$ or 6 — Multiply each side by 10.

CHECK Confirm that $\frac{JH}{DC}$ is equal to the scale factor.

$\frac{JH}{DC} = \frac{6}{10} = \frac{3}{5}$ ✓

WatchOut!

Writing Ratios When finding the ratio of the area of Figure *A* to the area of Figure *B*, be sure to write your ratio as $\frac{\text{area of figure } A}{\text{area of figure } B}$.

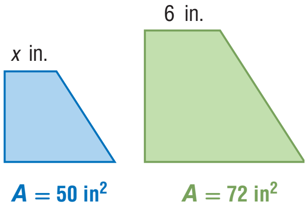
ReadingMath

Ratios Ratios can be written in different ways. For example, x to y, $x : y$, and $\frac{x}{y}$ are all representations of the ratio of x and y.

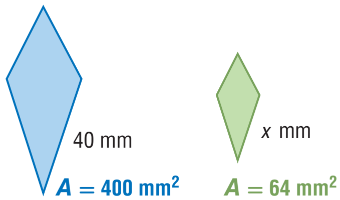
GuidedPractice

For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find *x*.

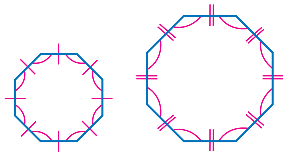
2A.



2B.



In Lesson 7-2, you learned that if all corresponding angles are congruent and all corresponding sides are proportional, then two polygons are similar. For this reason, all regular polygons with the same number of sides are similar.



Real-World Example 3 Scale Models

CRAFTS Use the information at the left. Orlando and Mia are making a scale model of the Pentagon. If the area of the base of their model is approximately 50 square inches, about how many times the length of each outer wall of the Pentagon is the length of the outer wall of the model?

Understand All regular pentagons are similar, so the base of the model is similar to the base of the Pentagon. You need to find the scale factor from the Pentagon to their model.

Plan The ratio of the areas of the bases of the two figures is equal to the square of the scale factor between them. Before comparing the two areas, write them so that they have the same units.

Solve Convert the area of the model's base to square feet.

$$50\ \cancel{\text{in}^2} \cdot \frac{1\ \text{ft}^2}{144\ \cancel{\text{in}^2}} \approx 0.3472\ \text{ft}^2$$

Next, write an equation using the ratio of the two areas in square feet. Let k represent the scale factor between the two bases.

$\dfrac{\text{area of model}}{\text{area of Pentagon}} = k^2$	Theorem 11.1
$\dfrac{0.3472\ \text{ft}^2}{1{,}481{,}000\ \text{ft}^2} \approx k^2$	Substitution
$2.34 \cdot 10^{-7} \approx k^2$	Simplify using a calculator.
$4.84 \cdot 10^{-4} \approx k$	Take the positive square root of each side.
$0.0005 \approx k$	Write in standard form.
$\dfrac{1}{2000} \approx k$	Write as a simplified fraction.

So the model's outer walls are about $\frac{1}{2000}$ the length of each outer wall of the Pentagon.

Check Multiply the area of the Pentagon's base by the square of this scale factor and compare to the given area of the model's base.

$$\frac{1{,}481{,}000\ \cancel{\text{ft}^2}}{1} \cdot \frac{144\ \text{in}^2}{1\ \cancel{\text{ft}^2}} \cdot \left(\frac{1}{2000}\right)^2 \approx 53\ \text{in}^2$$

This is close to the given area of 50 square inches, so our scale factor is reasonable. ✓

Guided Practice

3. CRAFTS Miyoki is crocheting two circles. The area of the larger circle is to be 2.5 times the size of the smaller. If the area of the smaller circle is about 50.2 square centimeters, what is the diameter of the larger circle?

Real-World Link

The Pentagon building, including its center courtyard, occupies approximately 34 acres or 1,481,000 square feet of land. Each outer wall of the regular pentagonal building is 921 feet in length.

Source: U.S. Department of Defense

Reading Math

Similar Circles Since all circles have the same shape, all circles are similar. Therefore, the areas of two circles are also related by the square of the scale factor between them.

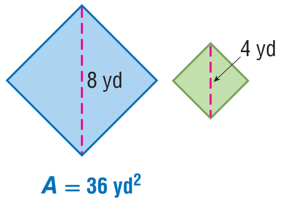
Digital Vision/Photodisc/Getty Images

Check Your Understanding

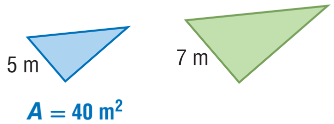
Step-by-Step Solutions begin on page R14.

Example 1 For each pair of similar figures, find the area of the green figure.

1.

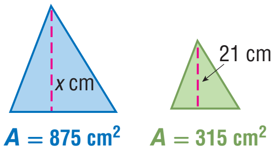


2.

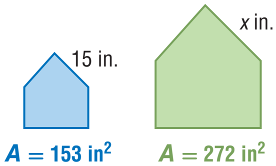


Example 2 For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find x.

3.



4.



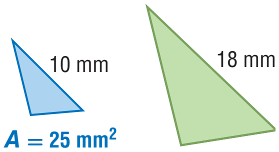
Example 3 **5. MEMORIES** Zola has a picture frame that holds all of her school pictures. Each small opening is similar to the large opening in the center. If the center opening has an area of 33 square inches, what is the area of each small opening?

Practice and Problem Solving

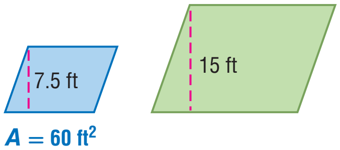
Extra Practice is on page R11.

Example 1 For each pair of similar figures, find the area of the green figure.

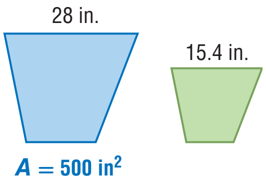
6.



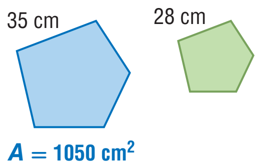
7.



8.

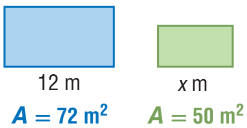


9.

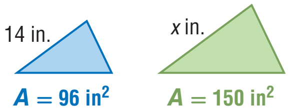


Example 2 CCSS **STRUCTURE** For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find x.

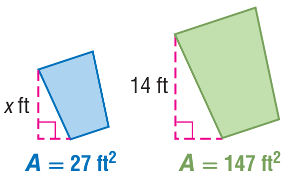
10.



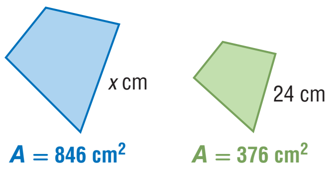
11.



12.

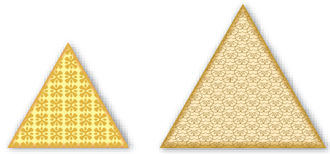


13.



Example 3

14. CRAFTS Marina crafts unique trivets and other kitchenware. Each trivet is an equilateral triangle. The perimeter of the small trivet is 9 inches, and the perimeter of the large trivet is 12 inches. If the area of the small trivet is about 3.9 square inches, what is the approximate area of the large trivet?



15. BAKING Kaitlyn wants to use one of two regular hexagonal cake pans for a recipe she is making. The side length of the larger pan is 4.5 inches, and the area of the base of the smaller pan is 41.6 square inches.

a. What is the side length of the smaller pan?

b. The recipe that Kaitlyn is using calls for a circular cake pan with an 8-inch diameter. Which pan should she choose? Explain your reasoning.

16. CHANGING DIMENSIONS A polygon has an area of 144 square meters.

a. If the area is doubled, how does each side length change?

b. How does each side length change if the area is tripled?

c. What is the change in each side length if the area is increased by a factor of x?

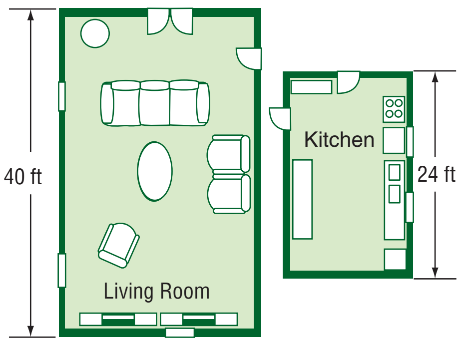
17. CHANGING DIMENSIONS A circle has a radius of 24 inches.

a. If the area is doubled, how does the radius change?

b. How does the radius change if the area is tripled?

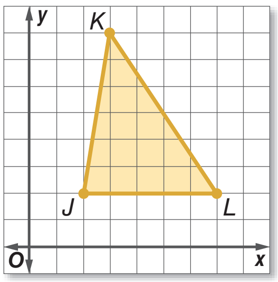
c. What is the change in the radius if the area is increased by a factor of x?

18. CCSS MODELING Federico's family is putting hardwood floors in the two geometrically similar rooms shown. If the cost of flooring is constant and the flooring for the kitchen cost \$2000, what will be the total flooring cost for the two rooms? Round to the nearest hundred dollars.

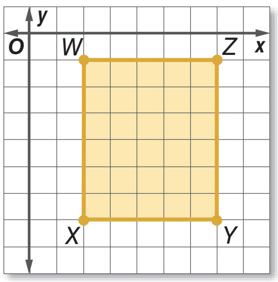


COORDINATE GEOMETRY Find the area of each figure. Use the segment length given to find the area of a similar polygon.

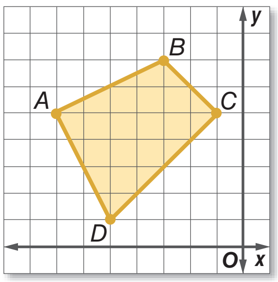
19. $J'L' = 3$



20. $W'X' = 8$



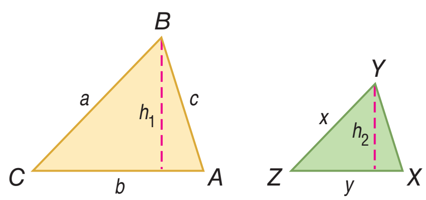
21. $B'C' = 5$



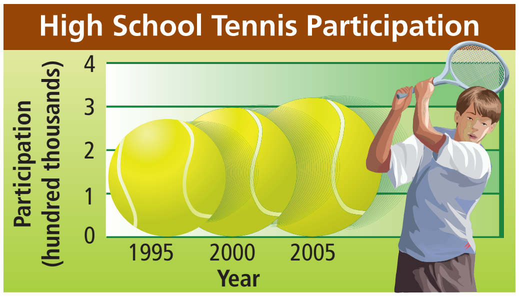
22. PROOF Write a paragraph proof.

Given: $\triangle ABC \sim \triangle XYZ$

Prove: $\dfrac{\text{area of } \triangle ABC}{\text{area of } \triangle XYZ} = \dfrac{a^2}{x^2}$



23. **STATISTICS** The graph shows the increase in high school tennis participation from 1995 to 2005.



a. Explain why the graph is misleading.

b. How could the graph be changed to more accurately represent the growth in high school tennis participation?

24. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate changing dimensions proportionally in three-dimensional figures.

a. Tabular Copy and complete the table below for each scale factor of a rectangular prism that is 2 inches by 3 inches by 5 inches.

Scale Factor	Length (in.)	Width (in.)	Height (in.)	Volume (in^3)	Ratio of Scaled Volume to Initial Volume
1	3	2	5		
2					
3					
4					
5					
10					

b. Verbal Make a conjecture about the relationship between the scale factor and the ratio of the scaled volume to the initial volume.

c. Graphical Make a scatter plot of the scale factor and the ratio of the scaled volume to the initial volume using the **STAT PLOT** feature on your graphing calculator. Then use the **STAT CALC** feature to approximate the function represented by the graph.

d. Algebraic Write an algebraic expression for the ratio of the scaled volume to the initial volume in terms of scale factor k.

H.O.T. Problems Use Higher-Order Thinking Skills

25. **CCSS CRITIQUE** Violeta and Gavin are trying to come up with a formula that can be used to find the area of a circle with a radius r after it has been enlarged by a scale factor k. Is either of them correct? Explain your reasoning.

Violeta	Gavin
$A = k\pi r^2$	$A = \pi(r^2)^k$

26. **CHALLENGE** If you want the area of a polygon to be x% of its original area, by what scale factor should you multiply each side length?

27. **REASONING** A regular n-gon is enlarged, and the ratio of the area of the enlarged figure to the area of the original figure is R. Write an equation relating the perimeter of the enlarged figure to the perimeter of the original figure Q.

28. **OPEN ENDED** Draw a pair of similar figures with areas that have a ratio of 4:1. Explain.

29. **WRITING IN MATH** Explain how to find the area of an enlarged polygon if you know the area of the original polygon and the scale factor of the enlargement.

Standardized Test Practice

30. $\triangle ABC \sim \triangle PRT$, $AC = 15$ inches, $PT = 6$ inches, and the area of $\triangle PRT$ is 24 square inches. Find the area of $\triangle ABC$.

A 9.6 in^2 **C** 66.7 in^2
B 60 in^2 **D** 150 in^2

31. ALGEBRA Which of the following shows $2x^2 - 18xy - 72y^2$ factored completely?

F $(2x - 18y)(x + 4y)$ **H** $(2x - 9y)(x + 4y)$
G $2(x - 9y)(x + 4y)$ **J** $2(x - 12y)(x + 3y)$

32. EXTENDED RESPONSE The measures of two complementary angles are represented by $2x + 1$ and $5x - 9$.

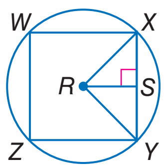
a. Write an equation that represents the relationship between the two angles.
b. Find the degree measure of each angle.

33. SAT/ACT Which of the following are the values of x for which $(x + 5)(x - 4) = 10$?

A -5 and 4 **D** 6 and -5
B 5 and 6 **E** -6 and 5
C -4 and 5

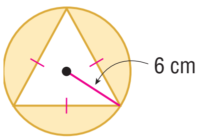
Spiral Review

34. In the figure, square $WXYZ$ is inscribed in $\odot R$. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle. (Lesson 11-4)

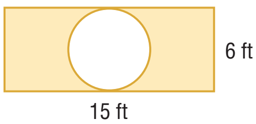


Find the area of the shaded region. Round to the nearest tenth. (Lesson 11-3)

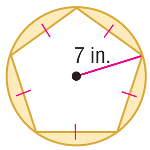
35.



36.

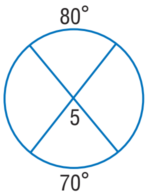


37.

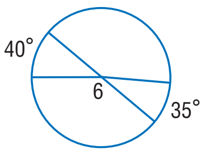


Find each measure. (Lesson 10-6)

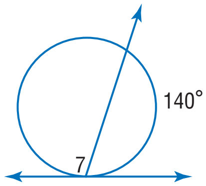
38. $m\angle 5$



39. $m\angle 6$



40. $m\angle 7$



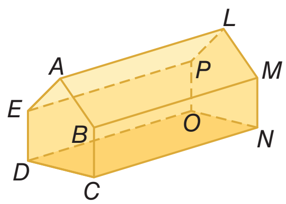
41. State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*. (Lesson 9-5)

42. YEARBOOKS Tai resized a photograph that was 8 inches by 10 inches so that it would fit in a 4-inch by 4-inch area on a yearbook page. (Lesson 7-7)

a. Find the maximum dimensions of the reduced photograph.
b. What is the percent of reduction of the length?

Skills Review

Refer to the figure at the right to identify each of the following.



43. Name all segments parallel to $\overline{AE}$.

44. Name all planes intersecting plane BCN.

45. Name all segements skew to $\overline{DC}$.

CHAPTER 11

Study Guide and Review

Study Guide

KeyConcepts

Areas of Parallelograms and Triangles (Lesson 11-1)

- The area A of a parallelogram is the product of a base b and its corresponding height h. $A = bh$
- The area A of a triangle is one half the product of a base b and its corresponding height h. $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Areas of Trapezoids, Rhombi, and Kites (Lesson 11-2)

- The area A of a trapezoid is one half the product of the height h and the sum of its bases, b_1 and b_2.
$A = \frac{1}{2}h(b_1 + b_2)$
- The area A of a rhombus or kite is one half the product of the lengths of its diagonals, d_1 and d_2.
$A = \frac{1}{2}d_1 d_2$

Areas of Circles and Sectors (Lesson 11-3)

- The area A of a circle is equal to π times the square of the radius r. $A = \pi r^2$
- The ratio of the area A of a sector to the area of the whole circle, πr^2, is equal to the ratio of the degree measure of the intercepted arc x to 360.
Proportion: $\frac{A}{\pi r^2} = \frac{x}{360}$ Equation: $A = \frac{x}{360} \cdot \pi r^2$

Areas of Regular Polygons and Composite Figures (Lesson 11-4)

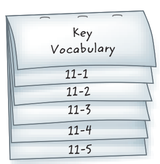
- The area A of a regular n-gon with side length s is one half the product of the apothem a and perimeter P.
$A = \frac{1}{2}a(ns)$ or $A = \frac{1}{2}aP$

Areas of Similar Figures (Lesson 11-5)

- If two polygons are similar, then their areas are proportional to the square of the scale factor between them.
If $ABCD \sim FGHJ$, then $\frac{\text{area of } FGHJ}{\text{area of } ABCD} = \left(\frac{FG}{AB}\right)^2$.

FOLDABLES® StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

apothem (p. 807)

base of a parallelogram (p. 779)

base of a triangle (p. 781)

center of a regular polygon (p. 807)

central angle of a regular polygon (p. 807)

composite figure (p. 809)

height of a parallelogram (p. 779)

height of a trapezoid (p. 789)

height of a triangle (p. 781)

radius of a regular polygon (p. 807)

sector of a circle (p. 799)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The <u>center</u> of a trapezoid is the perpendicular distance between the bases.

2. A slice of pizza is a <u>sector</u> of a circle.

3. The <u>center</u> of a regular polygon is the distance from the middle to the circle circumscribed around the polygon.

4. The segment from the center of a square to the corner can be called the <u>radius</u> of the square.

5. A segment drawn perpendicular to a side of a regular polygon is called an <u>apothem</u> of the polygon.

6. The measure of each <u>radial</u> angle of a regular n-gon is $\frac{360}{n}$.

7. The <u>apothem of a polygon</u> is the perpendicular distance between any two parallel bases.

8. The <u>height of a triangle</u> is the length of an altitude drawn to a given base.

9. Any side of a parallelogram can be called the <u>height</u> of a parallelogram.

10. The <u>center</u> of a regular polygon is also the center of its circumscribed circle.

CHAPTER 11

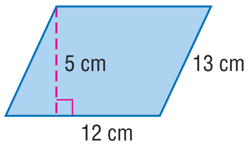
Study Guide and Review *Continued*

Lesson-by-Lesson Review

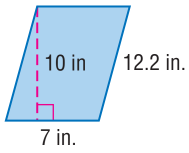
11-1 Areas of Parallelograms and Triangles

Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

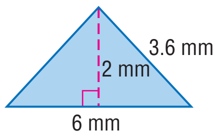
11.



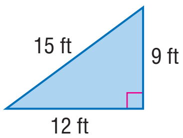
12.



13.



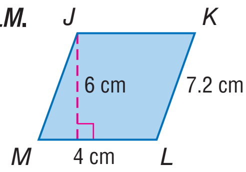
14.



15. **PAINTING** Two of the walls of an attic in an A-frame house are triangular, each with a height of 12 feet and a width of 22 feet. How much paint is needed to paint one end of the attic?

Example 1

Find the perimeter and area of ▱*JKLM*.



Perimeter

Perimeter of ▱$JKLM = JK + KL + LM + JM$

$= 4 + 7.2 + 4 + 7.2$ or 22.4 cm

Area

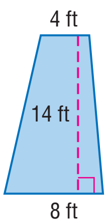
$A = bh$ — Area of a parallelogram

$= (4)(6)$ or 24 cm^2 — $b = 4$ and $h = 6$

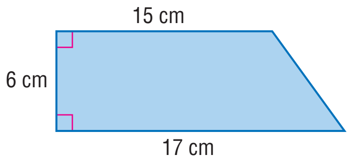
11-2 Areas of Trapezoids, Rhombi, and Kites

Find the area of each trapezoid, rhombus, or kite.

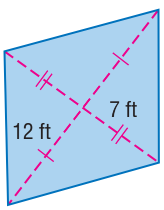
16.



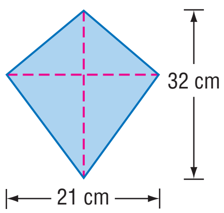
17.



18.



19.

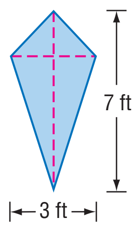


20. **KITES** Team Dragon's kite is 4 feet long and 3 feet across. How much fabric does it take to make their kite?

Example 2

Find the area of each rhombus or kite.

a.

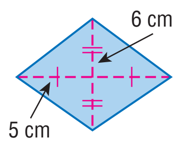


$A = \frac{1}{2}d_1d_2$ — Area of a kite

$= \frac{1}{2}(7)(3)$ — $d_1 = 7$ and $d_2 = 3$

$= 10.5 \text{ ft}^2$ — Simplify.

b.



Since the diagonals of a rhombus bisect each other, the lengths of the diagonals are 6 + 6 or 12 centimeters and 5 + 5 or 10 centimeters.

$A = \frac{1}{2}d_1d_2$ — Area of a rhombus

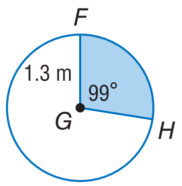
$= \frac{1}{2}(10)(12)$ — $d_1 = 10$ and $d_2 = 12$

$= 60 \text{ cm}^2$ — Simplify.

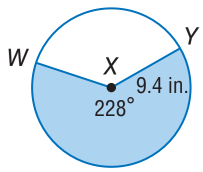
11-3 Areas of Circles and Sectors

Find the area of each shaded sector. Round to the nearest tenth.

21.



22.



23. **BICYCLES** A bicycle tire decoration covers $\frac{1}{9}$ of the circle formed by the tire. If the tire has a diameter of 26 inches, what is the area of the decoration?

24. **PIZZA** Charlie and Kris ordered a 16-inch pizza and cut the pizza into 12 slices.

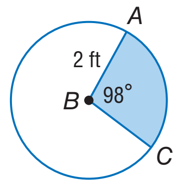
a. If Charlie ate 3 pieces, what area of the pizza did he eat?

b. If Kris ate 2 pieces, what area of the pizza did she eat?

c. What is the area of leftover pizza?

Example 3

Find the area of the shaded sector. Round to the nearest tenth.



$A = \frac{x}{360} \cdot \pi r^2$ Area of a sector

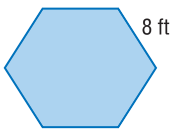
$= \frac{98}{360} \cdot \pi(2)^2$ Substitution

$\approx 3.4 \text{ ft}^2$ Simplify.

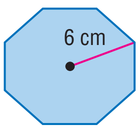
11-4 Areas of Regular Polygons and Composite Figures

Find the area of each regular polygon or composite figure. Round to the nearest tenth.

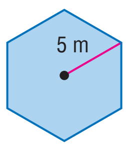
25.



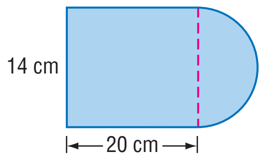
26.



27.



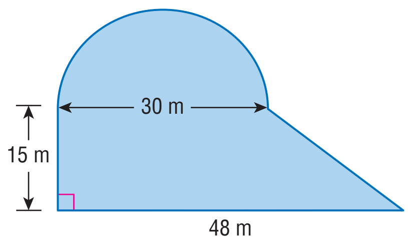
28.



29. **SIGNS** Find the area of the stop sign below in square inches.

Example 4

Find the area of the figure.



The composite shape is made up of a semicircle and a trapezoid.

Area = Area of semicircle + Area of trapezoid

$= \frac{180}{360} \cdot \pi \cdot r^2 + \frac{1}{2} \cdot h \cdot (b_1 + b_1)$

$\approx \frac{180}{360} \cdot \pi \cdot 15^2 + \frac{1}{2} \cdot 15 \cdot (30 + 48)$

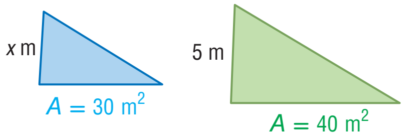
$\approx 112.5\pi + 585$ or about 938.4 m^2

CHAPTER 11 Study Guide and Review *Continued*

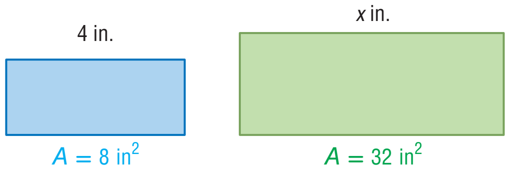
11-5 Areas of Similar Figures

For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find *x*.

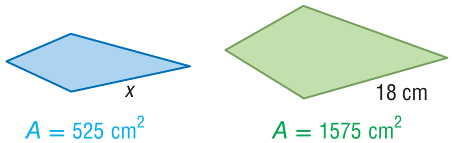
30.



31.

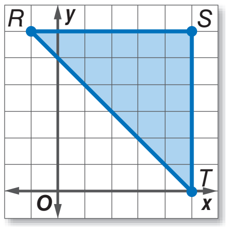


32.

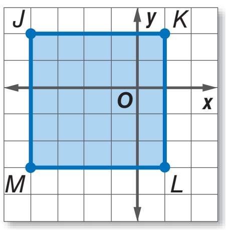


COORDINATE GEOMETRY Find the area of each figure. Use the segment length given to find the area of a similar polygon.

33. $R'S' = 3$



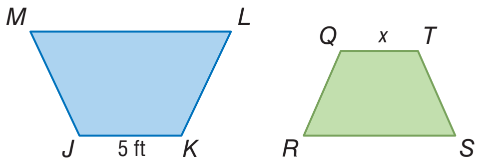
34. $K'L' = 15$



35. LAND OWNERSHIP Joshua's land is 600 square miles. A map of his land is 5 square feet. If one side of the map is 1.5 feet, how long is the corresponding side of the land?

Example 5

The area of trapezoid *JKLM* is 138 square feet. The area of trapezoid *QRST* is 5.52 square feet. If trapezoid *JKLM* ~ trapezoid *QRST*, find the scale factor from trapezoid *JKLM* to trapezoid *QRST* and the value of *x*.



Let *k* be the scale factor between trapezoid *JKLM* and trapezoid *QRST*.

$\frac{\text{Area of trapezoid } JKLM}{\text{Area of trapezoid } QRST} = k^2$ Theorem 11.1

$\frac{138}{5.52} = k^2$ Substitution

$5 = k$ Take the positive square root of each side.

So, the scale factor from trapezoid *JKLM* to trapezoid *QRST* is 5. Use this scale factor to find the value of *x*.

$\frac{JK}{QT} = k$ The ratio of corresponding lengths of similar polygons is equal to the scale factor between the polygons.

$\frac{5}{x} = 5$ Substitution

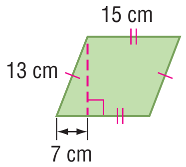
$1 = x$ Simplify.

CHAPTER 11

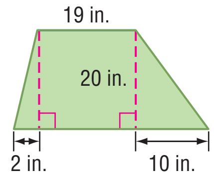
Practice Test

Find the area and perimeter of each figure. Round to the nearest tenth if necessary.

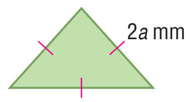
1.



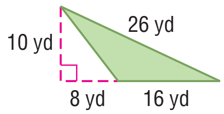
2.



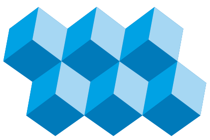
3.



4.

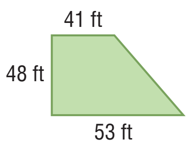


5. **ARCHAELOGY** The tile pattern shown was used in Pompeii for paving. If the diagonals of each rhombus are 2 and 3 inches, what area makes up each "cube" in the pattern?

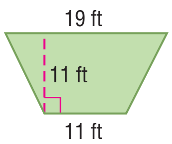


Find the area of each figure. Round to the nearest tenth if necessary.

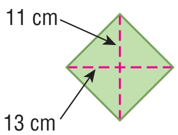
6.



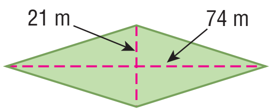
7.



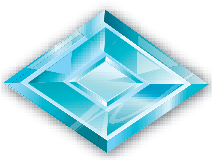
8.



9.



10. **GEMOLOGY** A gem is cut in a kite shape. It is 6.2 millimeters wide at its widest point and 5 millimeters long. What is the area?

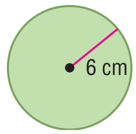


11. **ALGEBRA** The area of a triangle is 16 square units. The base of the triangle is $x + 4$ and the height is x. Find x.

12. **ASTRONOMY** A large planetarium in the shape of a dome is being built. When it is complete, the base of the dome will have a circumference of 870 meters. How many square meters of land were required for this planetarium?

Find the area of each circle or sector. Round to the nearest tenth.

13.



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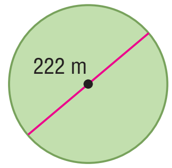

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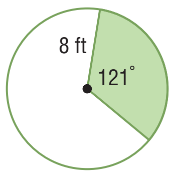
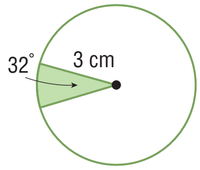


16.



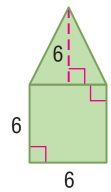
17. **MURALS** An artisan is creating a circular street mural for an art festival. The mural is going to be 50 feet wide.

a. Find the area of the mural to the nearest square foot.

b. One sector of the mural spans 38°. What is the area of this sector to the nearest square foot?

Find the perimeter and area of each figure. Round to the nearest tenth if necessary.

18.



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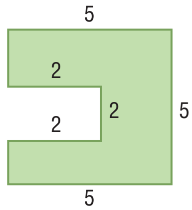
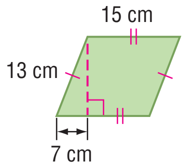


20. **BAKING** Todd wants to make a cheesecake for a birthday party. The recipe calls for a 9-inch diameter round pan. Todd only has square pans. He has an 8-inch square pan, a 9-inch square pan, and a 10-inch square pan. Which pan comes closest in area to the one that the recipe suggests?

CHAPTER 11 Practice Test

Find the area and perimeter of each figure. Round to the nearest tenth if necessary.

1.



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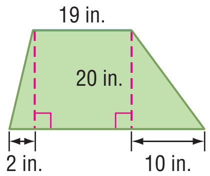

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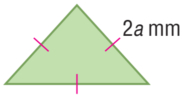

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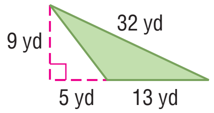
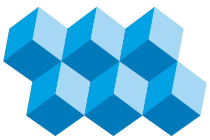
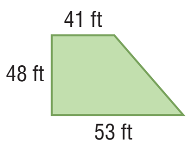


5. **ARCHAEOLOGY** The tile pattern shown was used in Pompeii for paving. If the diagonals of each rhombus are 2 and 3 inches, what area makes up each "cube" in the pattern?



Find the area of each figure. Round to the nearest tenth if necessary.

6.



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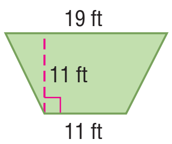

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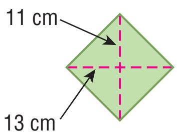

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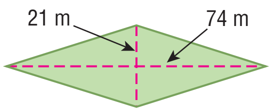
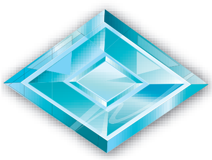


10. **GEMOLOGY** A gem is cut in a kite shape. It is 6.2 millimeters wide at its widest point and 5 millimeters long. What is the area?

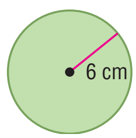


11. **ALGEBRA** The area of a triangle is 16 square units. The base of the triangle is $x + 4$ and the height is x. Find x.

12. **ASTRONOMY** A large planetarium in the shape of a dome is being built. When it is complete, the base of the dome will have a circumference of 870 meters. How many square meters of land were required for this planetarium?

Find the area of each circle or sector. Round to the nearest tenth.

13.



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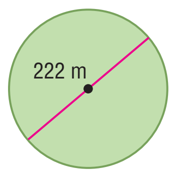

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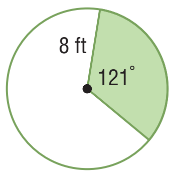

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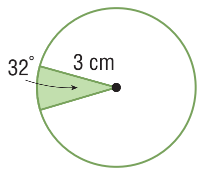

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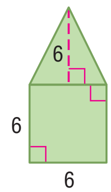

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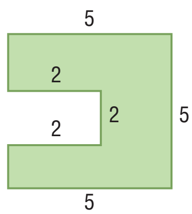

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CHAPTER 11 Preparing for Standardized Tests

Solve Multi-Step Problems

Some problems that you will encounter on standardized tests require you to solve multiple parts in order to come up with the final solution. Use this lesson to practice these types of problems.

Strategies for Solving Multi-Step Problems

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve? What information is given?
- Are there any intermediate steps that need to be completed before I can solve the problem?

Step 2

Organize your approach.

- List the steps you will need to complete in order to solve the problem.
- Remember that there may be more than one possible way to solve the problem.

Step 3

Solve and check.

- Work as efficiently as possible to complete each step and solve.
- If time permits, check your answer.

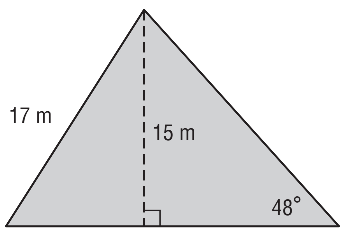
Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

What is the area of the triangle? Round your answer to the nearest tenth if necessary.

A 137.4 m^2 **C** 170.5 m^2

B 161.3 m^2 **D** 186.9 m^2



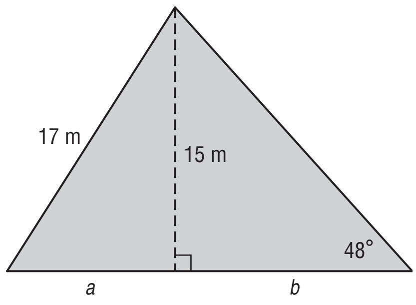
Purestock/Getty Images

Read the problem statement and study the figure carefully. At first glance, the problem may appear fairly straightforward. Notice, however, that you must first find the base of the triangle before you can find its area. Organize an approach to solve the problem.

Step 1 Use the Pythagorean Theorem to find a.

Step 2 Use trigonometry to find b.

Step 3 Find the area of the triangle.



Step 1 Find a.

$$a^2 + 15^2 = 17^2$$
$$a^2 = 289 - 225$$
$$a^2 = 64$$
$$a = 8$$

Step 2 Find b.

$$\tan 48° = \frac{15}{b}$$
$$b = \frac{15}{\tan 48°}$$
$$b \approx 13.506$$

Step 3 Find the area of the triangle.

The base of the triangle is $a + b$ or about 21.506 meters.

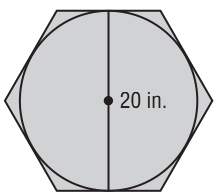
$$A \approx \frac{1}{2}(21.506)(15)$$
$$\approx 161.3$$

So, the area of the triangle is about 161.3 square meters. The answer is B.

Exercises

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

1. What is the area of the figure? Round to the nearest tenth.



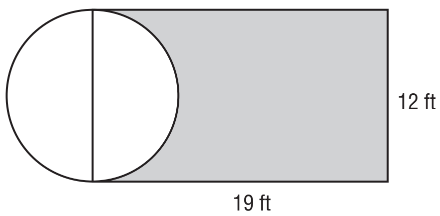
A 346.4 in^2

B 372.1 in^2

C 383.2 in^2

D 564.7 in^2

2. Kaleb is painting just the shaded part of the basketball key shown below. How much area will he need to cover? Round to the nearest tenth.



F 114.9 ft^2

G 142.4 ft^2

H 159.9 ft^2

J 171.5 ft^2

CHAPTER 11

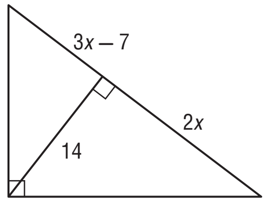
Standardized Test Practice

Cumulative, Chapters 1 through 11

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the value of x in the figure below?



A 5

B 7

C 8

D 10

2. Which of the following is *not* a property of parallelograms?

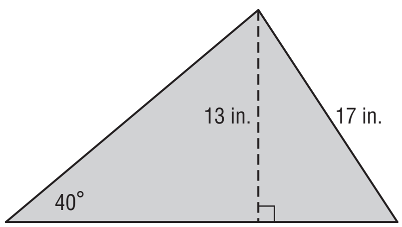
F The opposite angles of a parallelogram are congruent.

G The opposite sides of a parallelogram are congruent.

H The consecutive angles of a parallelogram are supplementary.

J The consecutive angles of a parallelogram are complementary.

3. What is the area of the triangle below? Round your answer to the nearest tenth if necessary.



A 110.5 in^2

B 144.2 in^2

C 164.5 in^2

D 171.9 in^2

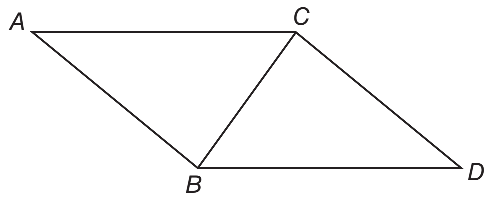
Test-Taking Tip

Question 3 Some problems require multiple steps to solve. You can use the Pythagorean Theorem and trigonometry to find the base of the triangle.

4. Given: $\overline{AC} \cong \overline{BD}$

$\overline{AC} \parallel \overline{BD}$

Prove: $\triangle ABC \cong \triangle DCB$



Statements	Reasons
1. $\overline{AC} \cong \overline{BD}$	1. Given
2. $\overline{AC} \parallel \overline{BD}$	2. Given
3. $\angle ACB \cong \angle DCB$	3. ___?___
4. $\overline{BC} \cong \overline{CB}$	4. Reflexive Property of Congruence
5. $\triangle ABC \cong \triangle DCB$	5. SAS

What is the missing line needed to complete the proof?

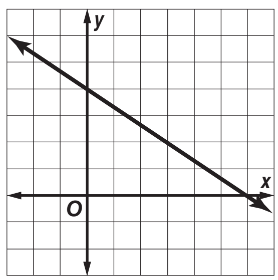
F Same side exterior angles are congruent.

G Vertical angles are congruent.

H Corresponding parts of congruent triangles are congruent.

J Alternate interior angles are congruent.

5. What is the slope of the line?



A $-\frac{3}{2}$

B $-\frac{2}{3}$

C $\frac{2}{3}$

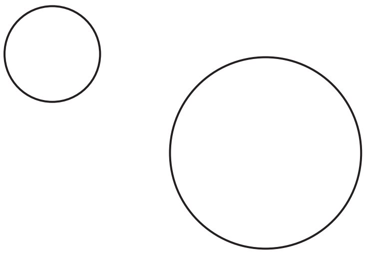
D $\frac{3}{2}$

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

6. **GRIDDED RESPONSE** Suppose two similar rectangles have a scale factor of 3:5. The perimeter of the smaller rectangle is 21 millimeters. What is the perimeter of the larger rectangle? Express your answer in millimeters.

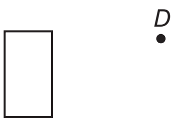
7. Copy the circles below on a sheet of paper and draw the common tangents, if any exist.



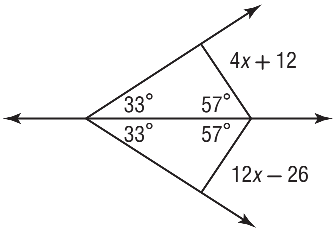
8. What is the contrapositive of the statement below?

If a chord contains the center of a circle, then the chord is a diameter.

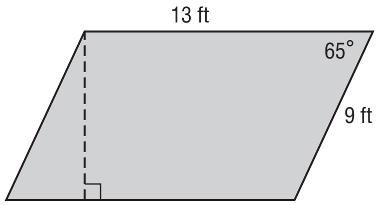
9. Copy the figure and point *D*. Then use a ruler to draw the image of the figure under a dilation with center *D* and a scale factor of 2.



10. **GRIDDED RESPONSE** Solve for x in the figure below.



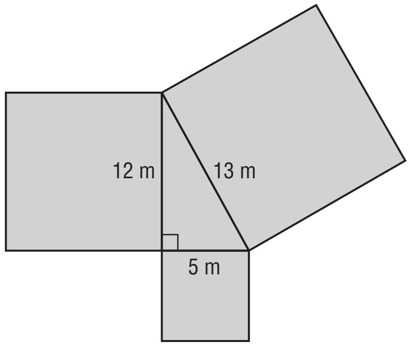
11. **GRIDDED RESPONSE** What is the area of the parallelogram below? Express your answer in square feet. Round to the nearest whole number if necessary.



Extended Response

Record your answers on a sheet of paper. Show your work.

12. Use the figure below to answer each question.



   a. Find the area of each square and the area of the triangle.

   b. What is the total area of the figure?

   c. Explain how the areas of the squares model the Pythagorean Theorem.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson...	8-1	6-2	11-1	4-4	3-3	7-2	10-5	2-3	9-6	5-1	11-1	11-4