CHAPTER 4 Congruent Triangles

Then

You learned about segments, angles, and discovered relationships between their measures.

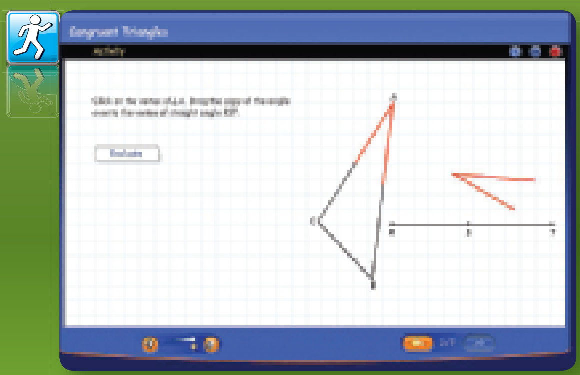
Now

In this chapter, you will:

- Apply special relationships about the interior and exterior angles of triangles.
- Identify corresponding parts of congruent triangles and prove triangles congruent.
- Learn about the special properties of isosceles and equilateral triangles

Why? ▲

FITNESS Triangles are used to add strength to many structures, including fitness equipment such as bike frames.



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Animation | Vocabulary | eGlossary | Personal Tutor | Virtual Manipulatives | Graphing Calculator | Audio | Foldables | Self-Check Practice | Worksheets

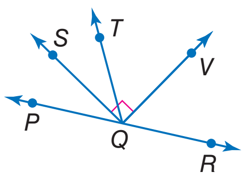
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option Take the Quick Check below. Refer to the Quick Review for help.

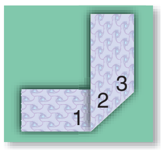
QuickCheck

Classify each angle as *right*, *acute*, or *obtuse*.

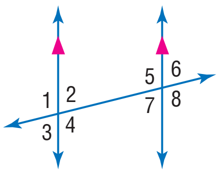


1. $m\angle VQS$
2. $m\angle TQV$
3. $m\angle PQV$

4. **ORIGAMI** The origami fold involves folding a strip of paper so that the lower edge of the strip forms a right angle with itself. Identify each angle as *right*, *acute*, or *obtuse*.



ALGEBRA Use the figure to find the indicated variable(s). Explain your reasoning.



5. Find x if $m\angle 3 = x - 12$ and $m\angle 6 = 72$.
6. If $m\angle 4 = 2y + 32$ and $m\angle 5 = 3y - 3$, find y.

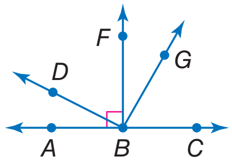
Find the distance between each pair of points.

7. $F(3, 6)$, $G(7, -4)$
8. $X(-2, 5)$, $Y(1, 11)$
9. $R(8, 0)$, $S(-9, 6)$
10. $A(14, -3)$, $B(9, -9)$
11. **MAPS** Miranda laid a coordinate grid on a map of a state where each 1 unit is equal to 10 miles. If her city is located at $(-8, -12)$ and the state capital is at $(0, 0)$, find the distance from her city to the capital to the nearest tenth of a mile.

QuickReview

Example 1

Classify each angle as *right*, *acute*, or *obtuse*.



a. $m\angle ABG$

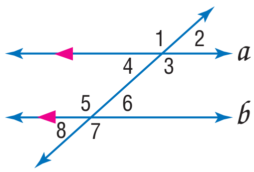
Point G on angle $\angle ABG$ lies on the exterior of right angle $\angle ABF$, so $\angle ABG$ is an obtuse angle.

b. $m\angle DBA$

Point D on angle $\angle DBA$ lies on the interior of right angle $\angle FBA$, so $\angle DBA$ is an acute angle.

Example 2

In the figure, $m\angle 4 = 42$. Find $m\angle 7$.



$\angle 7$ and $\angle 1$ are alternate interior angles, so they are congruent. $\angle 1$ and $\angle 4$ are a linear pair, so they are supplementary. Therefore, $\angle 7$ is supplementary to $\angle 1$. The measure of $\angle 7$ is $180 - 42$ or 138.

Example 3

Find the distance between $J(5, 2)$ and $K(11, -7)$.

$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(11 - 5)^2 + [(-7) - 2]^2}$	Substitute.
$= \sqrt{6^2 + (-9)^2}$	Subtract.
$= \sqrt{36 + 81}$ or $\sqrt{117}$	Simplify.

2 Online Option Take an online self-check Chapter Readiness Quiz at connectED.mcgraw-hill.com.

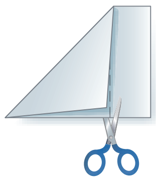
Get Started on the Chapter

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 4. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

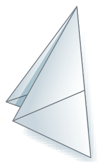
FOLDABLES® **Study**Organizer

Congruent Triangles Make this Foldable to help you organize your Chapter 4 notes about congruent triangles. Begin with a sheet of $8\frac{1}{2}" \times 11"$ paper.

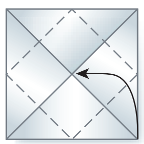
1 **Fold** into a taco forming a square. Cut off the excess paper strip formed by the square.



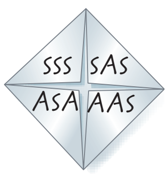
2 **Open** the fold and refold it the opposite way forming another taco and an X fold pattern.



3 **Open** and fold the corners toward the center point of the X forming a small square.



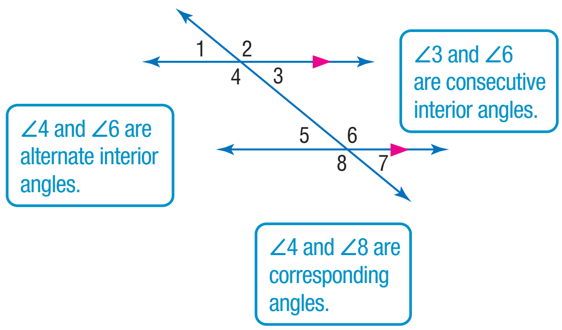
4 **Label** the flaps as shown.



NewVocabulary

English		Español
equiangular triangle	p. 237	triángulo equiangular
equilateral triangle	p. 238	triángulo equilátero
isosceles triangle	p. 238	triángulo isósceles
scalene triangle	p. 238	triángulo escaleno
auxiliary line	p. 246	linea auxiliar
congruent	p. 255	congruente
congruent polygons	p. 255	polígonos congruentes
corresponding parts	p. 255	partes correspondientes
included angle	p. 266	ángulo incluido
included side	p. 275	lado incluido
base angle	p. 285	ángulo de la base
transformation	p. 296	transformación
preimage	p. 296	preimagen
image	p. 296	imagen
reflection	p. 296	reflexión
translation	p. 296	traslación
rotation	p. 296	rotación

ReviewVocabulary



LESSON 4-1 Classifying Triangles

Then

- You measured and classified angles.

Now

1. Identify and classify triangles by angle measures.
2. Identify and classify triangles by side measures.

Why?

- Radio transmission towers are designed to support antennas for broadcasting radio or television signals. The structure of the tower shown reveals a pattern of triangular braces.

NewVocabulary

acute triangle
equiangular triangle
obtuse triangle
right triangle
equilateral triangle
isosceles triangle
scalene triangle

Common Core State Standards

Content Standards

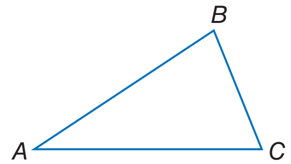
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Mathematical Practices

2 Reason abstractly and quantitatively.
6 Attend to precision.

1 Classify Triangles by Angles

Recall that a triangle is a three-sided polygon. Triangle *ABC*, written $\triangle ABC$, has parts that are named using *A*, *B*, and *C*.



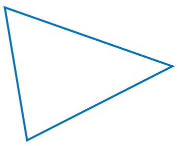
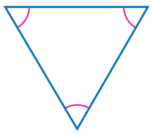
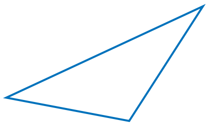
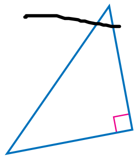
The sides of $\triangle ABC$ are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

The vertices are points *A*, *B*, and *C*.

The angles are $\angle BAC$ or $\angle A$, $\angle ABC$ or $\angle B$, and $\angle BCA$ or $\angle C$.

Triangles can be classified in two ways—by their angles ~~or by their~~ sides. All triangles have at least two acute angles, but the third angle is used to classify the triangle.

KeyConcept Classifications of Triangles by Angles

acute triangle	equiangular triangle	obtuse triangle	right triangle
			
3 acute angles	3 congruent acute angles	1 obtuse angle	1 right angle

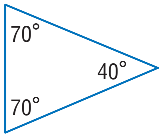
An equiangular triangle is a special kind of acute triangle.

When classifying triangles, be as specific as possible. While a triangle with three congruent acute angles is an acute triangle, it is more specific to classify it as an equiangular triangle.

Example 1 Classify Triangles by Angles

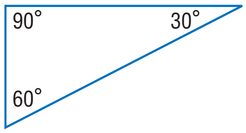
Classify each triangle as *acute, equiangular, obtuse,* or *right*.

a.



The triangle has three acute angles that are not all equal. It is an acute triangle.

b.



One angle of the triangle measures 90, so it is a right angle. Since the triangle has a right angle, it is a right triangle.

ReviewVocabulary

acute angle an angle with a degree measure less than 90

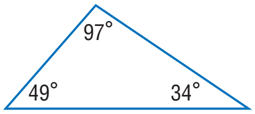
right angle an angle with a degree measure of 90

obtuse angle an angle with a degree measure greater than 90

GuidedPractice

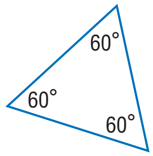
Classify each triangle as *acute, equiangular, obtuse,* or *right*.

1A.



Obtuse Triangle
because of the 97 degree obtuse angle

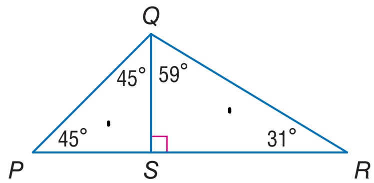
1B.



equiangular - all angles same
acute - all angles acute

Example 2 Classify Triangles by Angles Within Figures

Classify $\triangle PQR$ as *acute, equiangular, obtuse,* or *right*. Explain your reasoning.



Point S is in the interior of $\angle PQR$, so by the Angle Addition Postulate, $m\angle PQR = m\angle PQS + m\angle SQR$. By substitution, $m\angle PQR = 45 + 59$ or 104.

Since $\triangle PQR$ has one obtuse angle, it is an obtuse triangle.

GuidedPractice

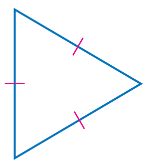
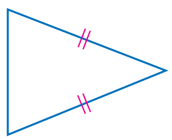
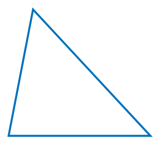
2. Use the diagram to classify $\triangle PQS$ as *acute, equiangular, obtuse* or *right*. Explain your reasoning.

Right Triangle

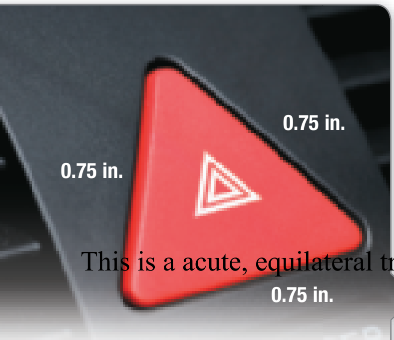
2 Classify Triangles by Sides

Triangles can also be classified according to the number of congruent sides they have. To indicate that sides of a triangle are congruent, an equal number of hash marks is drawn on the corresponding sides.

KeyConcept Classifications of Triangles by Sides

equilateral triangle	isosceles triangle	scalene triangle
		
3 congruent sides	at least 2 congruent sides	no congruent sides

An equilateral triangle is a special kind of isosceles triangle.



This is a acute, equilateral triangle

Real-WorldLink

In many cars, hazard lights are activated by pushing a small button located near the steering column. The switch is usually an icon shaped like an equilateral triangle.

Source: General Motors

Real-World Example 3 Classify Triangles by Sides

MUSIC Classify the sound box of the Russian lute below as *equilateral, isosceles,* or *scalene*.

Two sides have the same measure, 16 inches, so the triangle has two congruent sides. The triangle is isosceles.

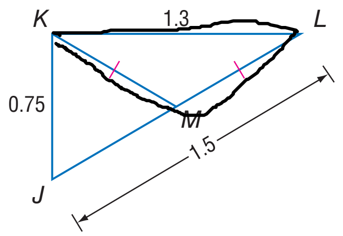
GuidedPractice

3. DRIVING SAFETY Classify the button in the picture at the left by its sides.

Equilateral triangle (all sides congruent)

Example 4 Classify Triangles by Sides Within Figures

If point M is the midpoint of $\overline{JL}$, classify $\triangle JKM$ as *equilateral*, *isosceles*, or *scalene*. Explain your reasoning.



By the definition of midpoint, $JM = ML$.

$JM + ML = JL$	Segment Addition Postulate
$ML + ML = 1.5$	Substitution
$2ML = 1.5$	Simplify.
$ML = 0.75$	Divide each side by 2.

$JM = ML$ or 0.75. Since $\overline{KM} \cong \overline{ML}$, $KM = ML$ or 0.75.

Since $KJ = JM = KM = 0.75$, the triangle has three sides with the same measure. Therefore, the triangle has three congruent sides, so it is equilateral.

GuidedPractice

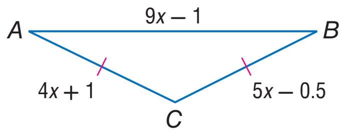
4. Classify $\triangle KML$ as *equilateral*, *isosceles*, or *scalene*. Explain your reasoning.

Tri KML is ISOS. (Because KM=ML)

You can also use the properties of isosceles and equilateral triangles to find missing values.

Example 5 Finding Missing Values

ALGEBRA Find the measures of the sides of isosceles triangle ABC.



Step 1 Find x.

$AC = CB$	Given
$4x + 1 = 5x - 0.5$	Substitution
$1 = x - 0.5$	Subtract $4x$ from each side.
$1.5 = x$	Add 0.5 to each side.

Step 2 Substitute to find the length of each side.

$AC = 4x + 1$	Given
$= 4(1.5) + 1$ or 7	$x = 1.5$
$CB = AC$	Given
$= 7$	$AC = 7$
$AB = 9x - 1$	Given
$= 9(1.5) - 1$	$x = 1.5$
$= 12.5$	Simplify.

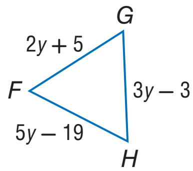
StudyTip

CCSS Perseverance In Example 5, to check your answer, test to see if $CB = AC$ when 1.5 is substituted for x in the expression for CB, $5x - 0.5$.

$CB = 5x - 0.5$
$= 5(1.5) - 0.5$ or 7 ✔

GuidedPractice

5. Find the measures of the sides of equilateral triangle FGH.



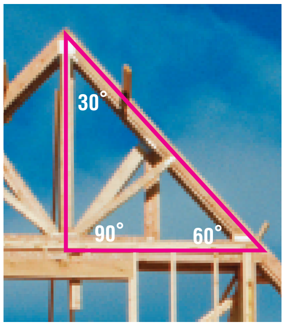
5y-19=3y-3

Check Your Understanding

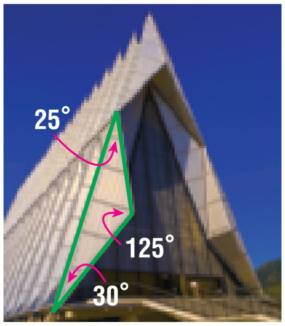
= Step-by-Step Solutions begin on page R14.

Example 1 **ARCHITECTURE** Classify each triangle as *acute, equiangular, obtuse,* or *right*.

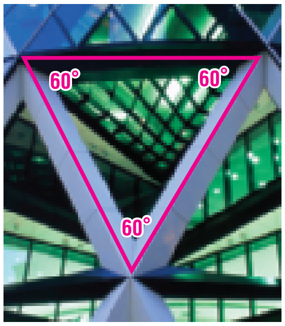
1.



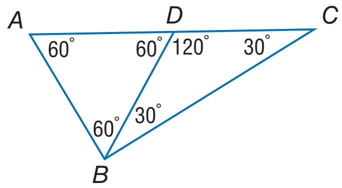
2.



3.



Example 2 Classify each triangle as *acute, equiangular, obtuse,* or *right*. Explain your reasoning.



4. $\triangle ABD$

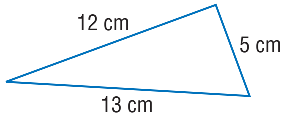
5. $\triangle BDC$

6. $\triangle ABC$

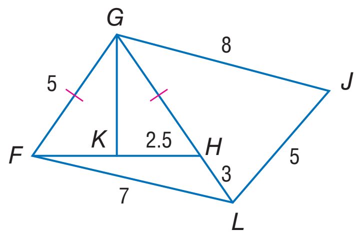
Example 3 **CCSS PRECISION** Classify each triangle as *equilateral, isosceles,* or *scalene*.

7.

8.



Example 4 If point K is the midpoint of $\overline{FH}$, classify each triangle in the figure at the right as *equilateral, isosceles,* or *scalene*.



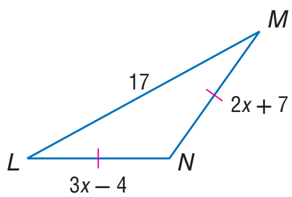
9 $\triangle FGH$

10. $\triangle GJL$

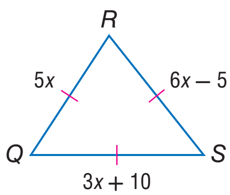
11. $\triangle FHL$

Example 5 **ALGEBRA** Find x and the measures of the unknown sides of each triangle.

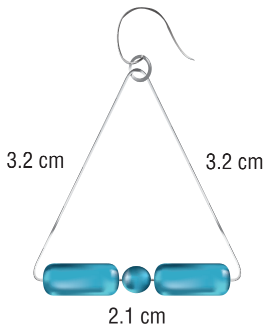
12.



13.



14. **JEWELRY** Suppose you are bending stainless steel wire to make the earring shown. The triangular portion of the earring is an isosceles triangle. If 1.5 centimeters are needed to make the hook portion of the earring, how many earrings can be made from 45 centimeters of wire? Explain your reasoning.

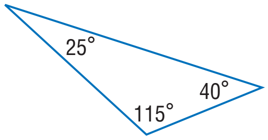


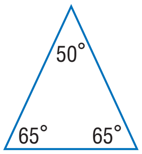
Try all the even questions by Friday end of class

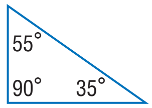
Practice and Problem Solving

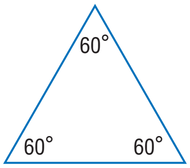
Extra Practice is on page R4.

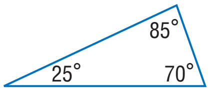
Example 1 Classify each triangle as *acute, equiangular, obtuse,* or *right*.

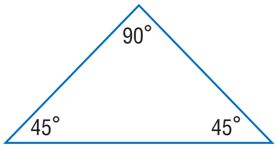
15. 

16. 

17. 

18. 

19. 

20. 

Example 2 **CCSS PRECISION** Classify each triangle as *acute, equiangular, obtuse,* or *right*.

21. $\triangle UYZ$

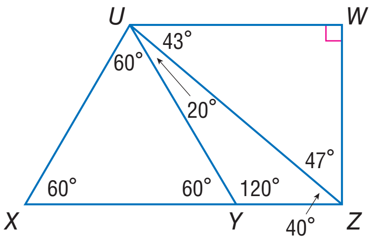
22. $\triangle BCD$

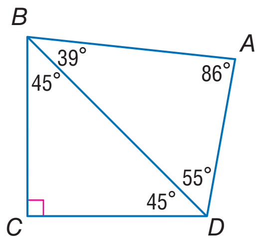
23. $\triangle ADB$

24. $\triangle UXZ$

25. $\triangle UWZ$

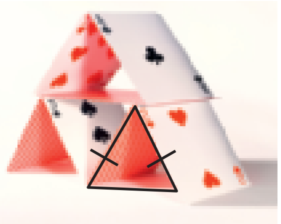
26. $\triangle UXY$

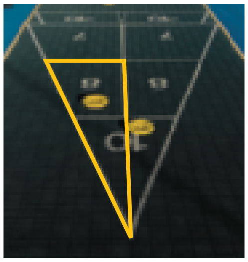




Example 3 Classify each triangle as *equilateral, isosceles,* or *scalene*.

27. 

28. 

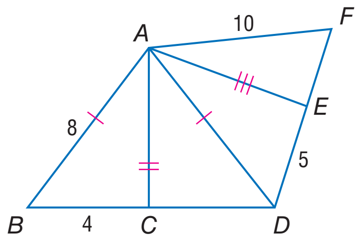
29. 

Example 4 If point C is the midpoint of $\overline{BD}$ and point E is the midpoint of $\overline{DF}$, classify each triangle as *equilateral, isosceles,* or *scalene*.

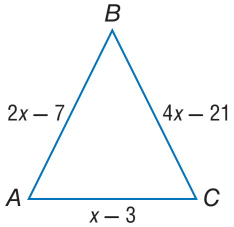
30. $\triangle ABC$ **31.** $\triangle AEF$

32. $\triangle ADF$ **33.** $\triangle ACD$

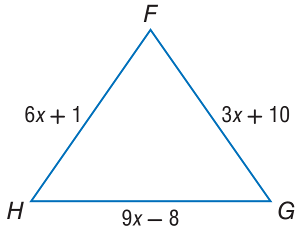
34. $\triangle AED$ **35.** $\triangle ABD$



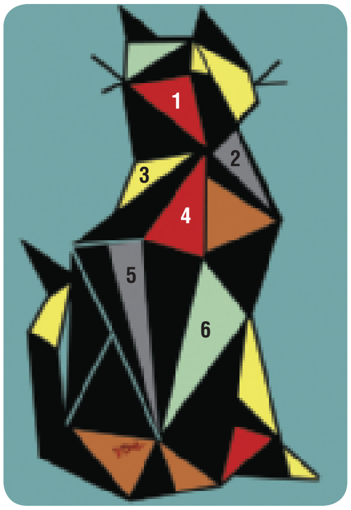
Example 5 **36.** **ALGEBRA** Find x and the length of each side if $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{BC}$.



37 **ALGEBRA** Find x and the length of each side if $\triangle FGH$ is an equilateral triangle.



38. **GRAPHIC ART** Refer to the illustration shown. Classify each numbered triangle in *Kat* by its angles and by its sides. Use the corner of a sheet of notebook paper to classify angle measures and a ruler to measure sides.



Kat, 2002, by Diana Ong, computer graphic

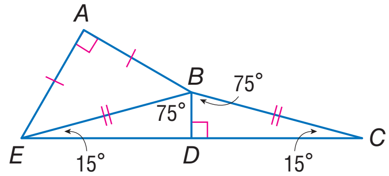
39 **KALEIDOSCOPE** Josh is building a kaleidoscope using PVC pipe, cardboard, bits of colored paper, and a 12-inch square mirror tile. The mirror tile is to be cut into strips and arranged to form an open prism with a base like that of an equilateral triangle. Make a sketch of the prism, giving its dimensions. Explain your reasoning.

CCSS PRECISION **Classify each triangle in the figure by its angles and sides.**

40. $\triangle ABE$

41. $\triangle EBC$

42. $\triangle BDC$



COORDINATE GEOMETRY **Find the measures of the sides of $\triangle XYZ$ and classify each triangle by its sides.**

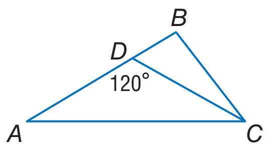
43. $X(-5, 9), Y(2, 1), Z(-8, 3)$

44. $X(7, 6), Y(5, 1), Z(9, 1)$

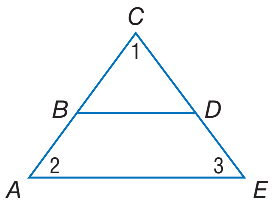
45. $X(3, -2), Y(1, -4), Z(3, -4)$

46. $X(-4, -2), Y(-3, 7), Z(4, -2)$

47. **PROOF** Write a paragraph proof to prove that $\triangle DBC$ is an acute triangle if $m\angle ADC = 120$ and $\triangle ABC$ is acute.



48. **PROOF** Write a two-column proof to prove that $\triangle BCD$ is equiangular if $\triangle ACE$ is equiangular and $\overline{BD} \parallel \overline{AE}$.



ALGEBRA **For each triangle, find x and the measure of each side.**

49. $\triangle FGH$ is an equilateral triangle with $FG = 3x - 10$, $GH = 2x + 5$, and $HF = x + 20$.

50. $\triangle JKL$ is isosceles with $\overline{JK} \cong \overline{KL}$, $JK = 4x - 1$, $KL = 2x + 5$, and $LJ = 2x - 1$.

51. $\triangle MNP$ is isosceles with $\overline{MN} \cong \overline{NP}$. MN is two less than five times x, NP is seven more than two times x, and PM is two more than three times x.

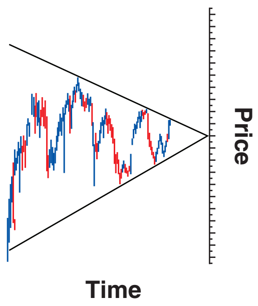
52. $\triangle RST$ is equilateral. RS is three more than four times x, ST is seven more than two times x, and TR is one more than five times x.

53. **CONSTRUCTION** Construct an equilateral triangle. Verify your construction using measurement and justify it using mathematics. (*Hint:* Use the construction for copying a segment.)

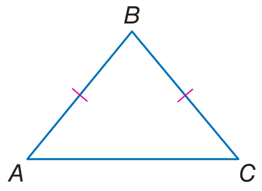
54. **STOCKS** Technical analysts use charts to identify patterns that can suggest future activity in stock prices. Symmetrical triangle charts are most useful when the fluctuation in the price of a stock is decreasing over time.

   a. Classify by its sides and angles the triangle formed if a vertical line is drawn at any point on the graph.

   b. How would the price have to fluctuate in order for the data to form an obtuse triangle? Draw an example to support your reasoning.



55. **MULTIPLE REPRESENTATIONS** In the diagram, the vertex *opposite* side $\overline{BC}$ is $\angle A$.



   a. **Geometric** Draw four isosceles triangles, including one acute, one right, and one obtuse isosceles triangle. Label the vertices opposite the congruent sides as A and C. Label the remaining vertex B. Then measure the angles of each triangle and label each angle with its measure.

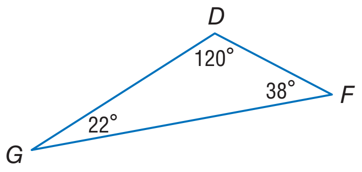
   b. **Tabular** Measure all the angles of each triangle. Organize the measures for each triangle into a table. Include a column in your table to record the sum of these measures.

   c. **Verbal** Make a conjecture about the measures of the angles that are opposite the congruent sides of an isosceles triangle. Then make a conjecture about the sum of the measures of the angles of an isosceles triangle.

   d. **Algebraic** If x is the measure of one of the angles opposite one of the congruent sides in an isosceles triangle, write expressions for the measures of each of the other two angles in the triangle. Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

56. **ERROR ANALYSIS** Elaina says that $\triangle DFG$ is obtuse. Ines disagrees, explaining that the triangle has more acute angles than obtuse angles so it must be acute. Is either of them correct? Explain your reasoning.



CCSS PRECISION Determine whether the statements below are *sometimes*, *always*, or *never* true. Explain your reasoning.

57. Equiangular triangles are also right triangles.

58. Equilateral triangles are isosceles.

59. Right triangles are equilateral.

60. **CHALLENGE** An equilateral triangle has sides that measure $5x + 3$ units and $7x - 5$ units. What is the perimeter of the triangle? Explain.

OPEN ENDED Draw an example of each type of triangle below using a protractor and a ruler. Label the sides and angles of each triangle with their measures. If not possible, explain why not.

61. scalene right
62. isosceles obtuse
63. equilateral obtuse

64. **WRITING IN MATH** Explain why classifying an equiangular triangle as an *acute* equiangular triangle is unnecessary.

Standardized Test Practice

65. Which type of triangle can serve as a counterexample to the conjecture below?

If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

A equilateral **C** right
B obtuse **D** scalene

66. ALGEBRA A baseball glove originally cost \$84.50. Kenji bought it at 40% off. How much was deducted from the original price?

F \$50.70 **H** \$33.80
G \$44.50 **J** \$32.62

67. GRIDDED RESPONSE Jorge is training for a 20-mile race. Jorge runs 7 miles on Monday, Tuesday, and Friday, and 12 miles on Wednesday and Saturday. After 6 weeks of training, Jorge will have run the equivalent of how many races?

68. SAT/ACT What is the slope of the line determined by the equation $2x + y = 5$?

A $-\frac{5}{2}$ **D** 2
B -2 **E** $\frac{5}{2}$
C -1

Spiral Review

Find the distance between each pair of parallel lines with the given equations. (Lesson 3-6)

69. $x = -2$, $x = 5$

70. $y = -6$, $y = 1$

71. $y = 2x + 3$, $y = 2x - 7$

72. $y = x + 2$, $y = x - 4$

73. FOOTBALL When striping the practice football field, Mr. Hawkins first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel? (Lesson 3-5)

Identify the hypothesis and conclusion of each conditional statement. (Lesson 2-3)

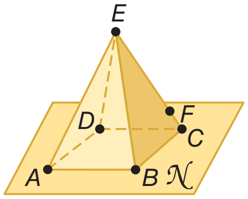
74. If three points lie on a line, then they are collinear.

75. If you are a teenager, then you are at least 13 years old.

76. If $2x + 6 = 10$, then $x = 2$.

77. If you have a driver's license, then you are at least 16 years old.

Refer to the figure at the right. (Lesson 1-1)



78. How many planes appear in this figure?

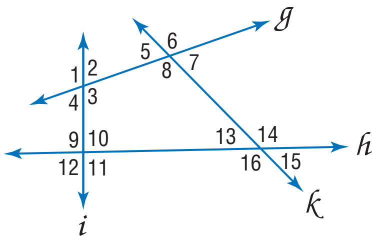
79. Name the intersection of plane AEB with plane $\mathcal{N}$.

80. Name three points that are collinear.

81. Are points D, E, C, and B coplanar?

Skills Review

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior angles*.



82. $\angle 5$ and $\angle 3$ **83.** $\angle 9$ and $\angle 4$

84. $\angle 11$ and $\angle 13$ **85.** $\angle 1$ and $\angle 11$

EXPLORE 4-2

Geometry Lab
Angles of Triangles

In this lab, you will find special relationships among the angles of a triangle.

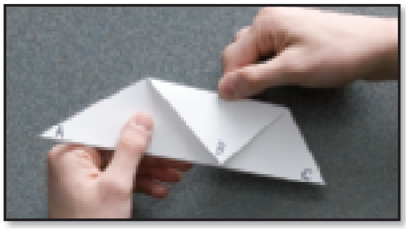
Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
Mathematical Practices 5

Activity 1 Interior Angles of a Triangle

Step 1

Draw and cut out several different triangles. Label the vertices *A*, *B*, and *C*.

Step 2



For each triangle, fold vertex *B* down so that the fold line is parallel to $\overline{AC}$. Relabel as vertex *B*.

Step 3

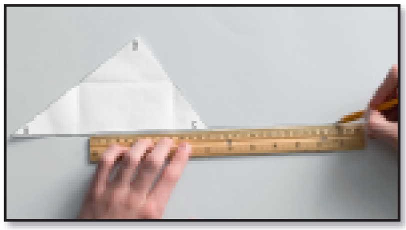
Then fold vertices *A* and *C* so that they meet vertex *B*. Relabel as vertices *A* and *C*.

Analyze the Results

1. Angles *A*, *B*, and *C* are called *interior angles* of triangle *ABC*. What type of figure do these three angles form when joined together in Step 3?
2. **Make a conjecture** about the sum of the measures of the interior angles of a triangle.

Activity 2 Exterior Angles of a Triangle

Step 1

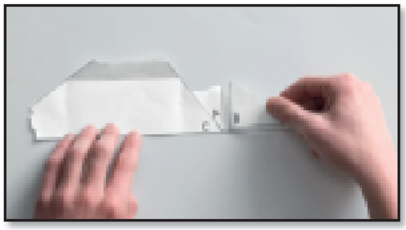


Unfold each triangle from Activity 1 and place each on a separate piece of paper. Extend $\overline{AC}$ as shown.

Step 2

For each triangle, tear off $\angle A$ and $\angle B$.

Step 3



Arrange $\angle A$ and $\angle B$ so that they fill the angle adjacent to $\angle C$ as shown.

Model and Analyze the Results

3. The angle adjacent to $\angle C$ is called an *exterior angle* of triangle *ABC*. **Make a conjecture** about the relationship among $\angle A$, $\angle B$, and the exterior angle at *C*.
4. Repeat the steps in Activity 2 for the exterior angles of $\angle A$ and $\angle B$ in each triangle.
5. **Make a conjecture** about the measure of an exterior angle and the sum of the measures of its nonadjacent interior angles.

Ed-Imaging

LESSON

4-2 Angles of Triangles

Then

- You classified triangles by their side or angle measures.

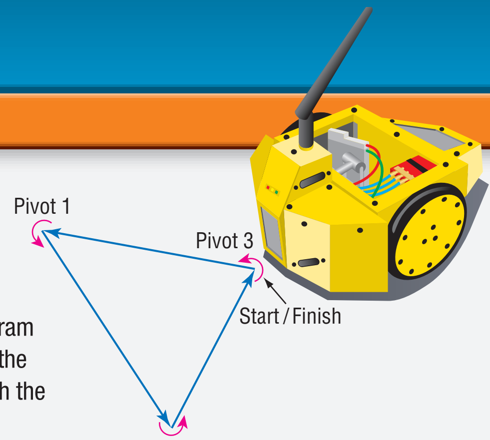
Now

1. Apply the Triangle Angle-Sum Theorem.
2. Apply Exterior Angle Theorem.

Why?

- Massachusetts Institute of Technology (MIT) sponsors the annual *Design 2.007* contest in which students design and build a robot.

One test of a robot's movements is to program it to move in a triangular path. The sum of the measures of the pivot angles through which the robot must turn will always be the same.



NewVocabulary

auxiliary line
exterior angle
remote interior angles
flow proof
corollary

Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

Mathematical Practices

1 Make sense of problems and persevere in solving them.
3 Construct viable arguments and critique the reasoning of others.

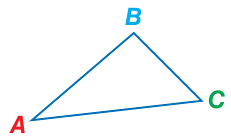
1 Triangle Angle-Sum Theorem

The Triangle Angle-Sum Theorem gives the relationship among the interior angle measures of any triangle.

Theorem 4.1 Triangle Angle-Sum Theorem

Words The sum of the measures of the angles of a triangle is 180.

Example $m\angle A + m\angle B + m\angle C = 180$

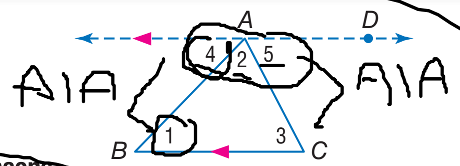


The proof of the Triangle Angle-Sum Theorem requires the use of an auxiliary line. An **auxiliary line** is an extra line or segment drawn in a figure to help analyze geometric relationships. As with any statement in a proof, you must justify any properties of an auxiliary line that you have drawn.

Proof Triangle Angle-Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$



Proof:

Statements	Reasons
1. $\triangle ABC$	1. Given
2. Draw $\overleftrightarrow{AD}$ through A parallel to $\overline{BC}$.	2. Parallel Postulate
3. $\angle 4$ and $\angle BAD$ form a linear pair.	3. Def. of a linear pair
4. $\angle 4$ and $\angle BAD$ are supplementary.	4. If 2 ∠s form a linear pair, they are supplementary.
5. $m\angle 4 + m\angle BAD = 180$	5. Def. of suppl. ∠s
6. $m\angle BAD = m\angle 2 + m\angle 5$	6. Angle Addition Postulate
7. $m\angle 4 + m\angle 2 + m\angle 5 = 180$	7. Substitution
8. $\angle 4 \cong \angle 1$, $\angle 5 \cong \angle 3$	8. Alt. Int. ∠s Theorem
9. $m\angle 4 = m\angle 1$, $m\angle 5 = m\angle 3$	9. Def. of ≅ ∠s
10. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	10. Substitution

Real-WorldLink

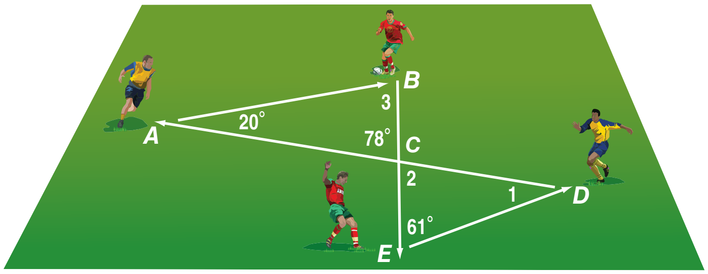
The pass-and-move soccer drill incorporates several fundamental aspects of passing. All passes in this drill are made in a triangle, which is the basis of all ball movement. Additionally, the players are forced to move immediately after passing the ball.

The Triangle Angle-Sum Theorem can be used to determine the measure of the third angle of a triangle when the other two angle measures are known.

Real-World Example 1 Use the Triangle Angle-Sum Theorem

PT

SOCCER The diagram shows the path of the ball in a passing drill created by four friends. Find the measure of each numbered angle.



Understand Examine the information given in the diagram. You know the measures of two angles of one triangle and only one measure of another. You also know that $\angle ACB$ and $\angle 2$ are vertical angles.

Plan Find $m\angle 3$ using the Triangle Angle-Sum Theorem, because the measures of two angles of $\angle ABC$ are known. Use the Vertical Angles Theorem to find $m\angle 2$. Then you will have enough information to find the measure of $\angle 1$ in $\triangle CDE$.

Solve

$m\angle 3 + m\angle BAC + m\angle ACB = 180$	Triangle Angle-Sum Theorem
$m\angle 3 + 20 + 78 = 180$	Substitution
$m\angle 3 + 98 = 180$	Simplify.
$m\angle 3 = 82$	Subtract 98 from each side.

$\angle ACB$ and $\angle 2$ are congruent vertical angles. So, $m\angle 2 = 78$.

Use $m\angle 2$ and $\angle CED$ of $\triangle CDE$ to find $m\angle 1$.

$m\angle 1 + m\angle 2 + m\angle CED = 180$	Triangle Angle-Sum Theorem
$m\angle 1 + 78 + 61 = 180$	Substitution
$m\angle 1 + 139 = 180$	Simplify.
$m\angle 1 = 41$	Subtract 139 from each side.

Check The sums of the measures of the angles of $\triangle ABC$ and $\triangle CDE$ should be 180.

$\triangle ABC$: $m\angle 3 + m\angle BAC + m\angle ACB = 82 + 20 + 78$ or 180 ✓

$\triangle CDE$: $m\angle 1 + m\angle 2 + m\angle CED = 41 + 78 + 61$ or 180 ✓

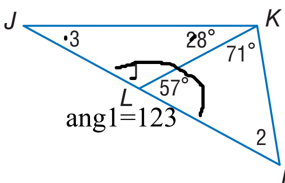
Problem-SolvingTip

CCSS Sense-Making Often a complex problem can be more easily solved if you first break it into more manageable parts. In Example 1, before you can find $m\angle 1$, you must first find $m\angle 2$.

GuidedPractice

Find the measures of each numbered angle.

1A.

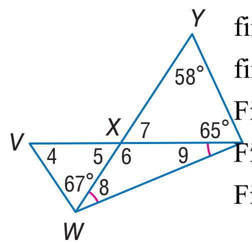


ang1=123

ang3=180-(123+28)

ang2= 180-(71+57)

1B.



find angle 7

find angle 6 (LPT)

Find ang8/9

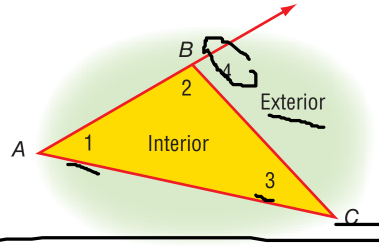
Find ang5

Find ang4

moodboard/CORBIS

2 Exterior Angle Theorem

In addition to its three interior angles, a triangle can have **exterior angles** formed by one side of the triangle and the extension of an adjacent side. Each exterior angle of a triangle has two **remote interior angles** that are not adjacent to the exterior angle.

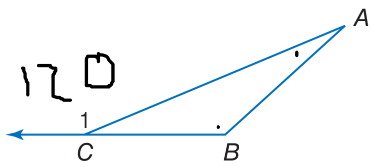


$\angle 4$ is an exterior angle of $\triangle ABC$. Its two remote interior angles are $\angle 1$ and $\angle 3$.

Theorem 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example $m\angle A + m\angle B = m\angle 1$



ReadingMath

Flowchart Proof A flow proof is sometimes called a *flowchart* proof.

A **flow proof** uses statements written in boxes and arrows to show the logical progression of an argument. The reason justifying each statement is written below the box. You can use a flow proof to prove the Exterior Angle Theorem.

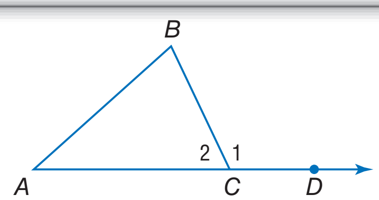
StudyTip

Flow Proofs Flow proofs can be written vertically or horizontally.

Proof Exterior Angle Theorem

Given: $\triangle ABC$

Prove: $m\angle A + m\angle B = m\angle 1$



Flow Proof:

$\triangle ABC$
Given
↓
$m\angle A + m\angle B + m\angle 2 = 180$
Triangle Angle-Sum Theorem

$\angle 2$ and $\angle 1$ form a linear pair.
Definition of a linear pair
↓
$\angle 2$ and $\angle 1$ are supplementary.
If 2 $\angle$s form a linear pair, they are supplementary.
↓
$m\angle 2 + m\angle 1 = 180$
Definition of supplementary

↘ ↙

$m\angle A + m\angle B + m\angle 2 = m\angle 2 + m\angle 1$
Substitution
↓
$m\angle A + m\angle B = m\angle 1$
Subtraction Property of Equality

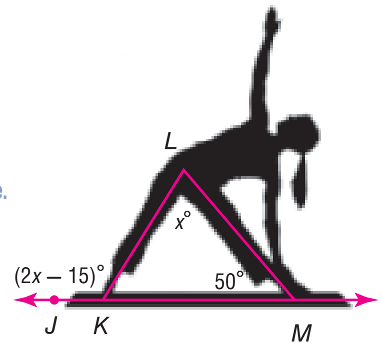
The Exterior Angle Theorem can also be used to find missing measures.

Real-World Example 2 Use the Exterior Angle Theorem

FITNESS **Find the measure of $\angle JKL$ in the Triangle Pose shown.**

$m\angle KLM + m\angle LMK = m\angle JKL$	Exterior Angle Theorem
$x + 50 = 2x - 15$	Substitution
$50 = x - 15$	Subtract x from each side.
$65 = x$	Add 15 to each side.

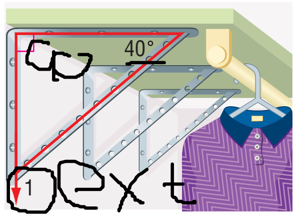
So, $m\angle JKL = 2(65) - 15$ or 115.



Guided Practice

2. **CLOSET ORGANIZING** Tanya mounts the shelving bracket shown to the wall of her closet. What is the measure of $\angle 1$, the angle that the bracket makes with the wall?

90 + 40 = 130



Real-World Career

Personal Trainer Personal trainers instruct and motivate individuals in exercise activities. They demonstrate various exercises and help clients improve their exercise techniques. Personal trainers must obtain certification in the fitness field.

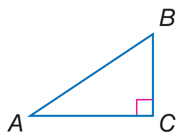
A **corollary** is a theorem with a proof that follows as a direct result of another theorem. As with a theorem, a corollary can be used as a reason in a proof. The corollaries below follow directly from the Triangle Angle-Sum Theorem.

Corollaries Triangle Angle-Sum Corollaries

4.1 The acute angles of a right triangle are complementary.

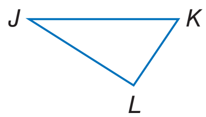
Abbreviation: *Acute ∠s of a rt. △ are comp.*

Example: If $\angle C$ is a right angle, then $\angle A$ and $\angle B$ are complementary.



4.2 There can be at most one right or obtuse angle in a triangle.

Example: If $\angle L$ is a right or an obtuse angle, then $\angle J$ and $\angle K$ must be acute angles.

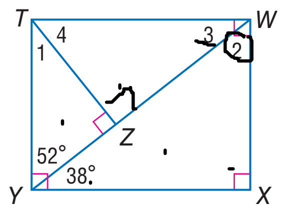


You will prove Corollaries 4.1 and 4.2 in Exercises 34 and 35.

Example 3 Find Angle Measures in Right Triangles

Find the measures of each numbered angle.

$m\angle 1 + m\angle TYZ = 90$	Acute ∠s of a rt. △ are comp.
$m\angle 1 + 52 = 90$	Substitution
$m\angle 1 = 38$	Subtract 52 from each side.



Guided Practice

3A. $\angle 2$ **3B.** $\angle 3$ **3C.** $\angle 4$

Study Tip

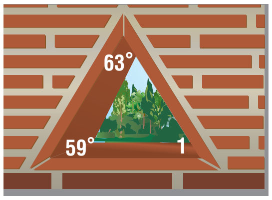
Check for Reasonableness When you are solving for the measure of one or more angles of a triangle, always check to make sure that the sum of the angle measures is 180.

Check Your Understanding

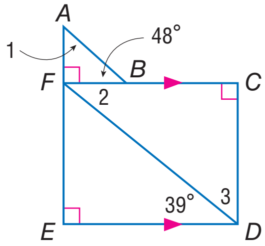
= Step-by-Step Solutions begin on page R14.

Example 1 Find the measures of each numbered angle.

1.

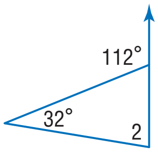


2.

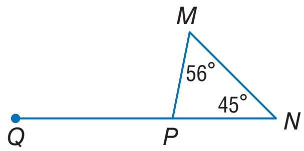


Example 2 Find each measure.

3. $m\angle 2$



4. $m\angle MPQ$



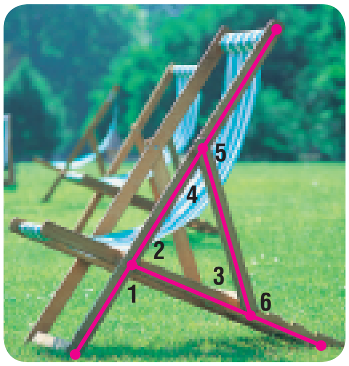
DECK CHAIRS The brace of this deck chair forms a triangle with the rest of the chair's frame as shown. If $m\angle 1 = 102$ and $m\angle 3 = 53$, find each measure.

5. $m\angle 4$

6. $m\angle 6$

7. $m\angle 2$

8. $m\angle 5$

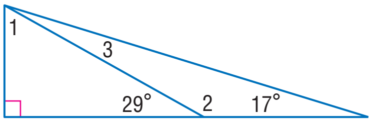


Example 3 CCSS **REGULARITY** Find each measure.

9. $m\angle 1$

10. $m\angle 3$

11. $m\angle 2$

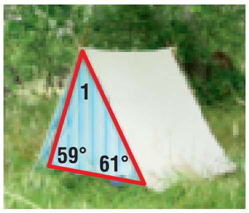


Practice and Problem Solving

Extra Practice is on page R4.

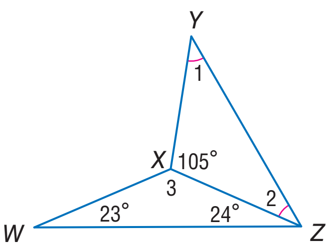
Example 1 Find the measure of each numbered angle.

12.

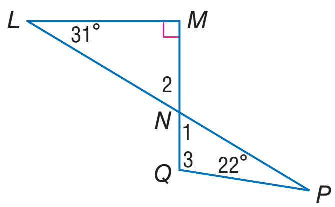


13.

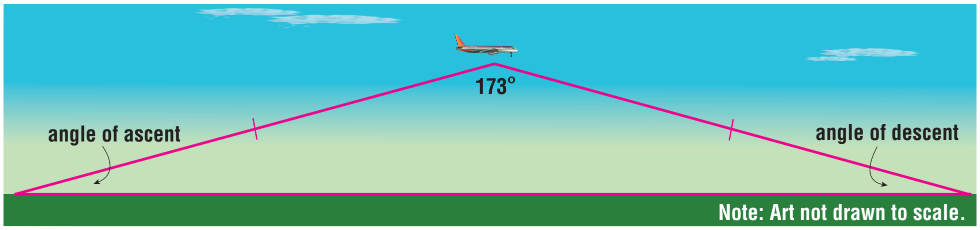
14.



15



16. AIRPLANES The path of an airplane can be modeled using two sides of a triangle as shown. The distance covered during the plane's ascent is equal to the distance covered during its descent.

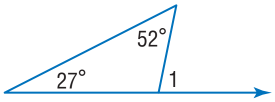


a. Classify the model using its sides and angles.

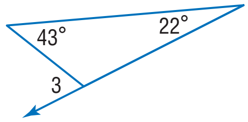
b. The angles of ascent and descent are congruent. Find their measures.

Example 2 **Find each measure.**

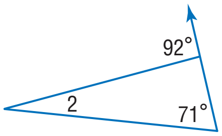
17. $m\angle 1$



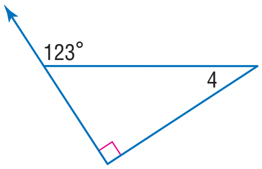
18. $m\angle 3$



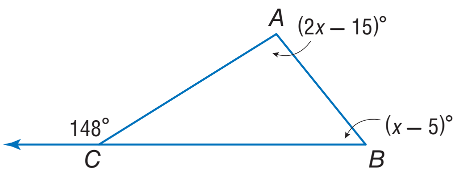
19. $m\angle 2$



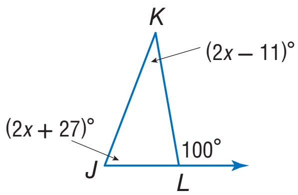
20. $m\angle 4$



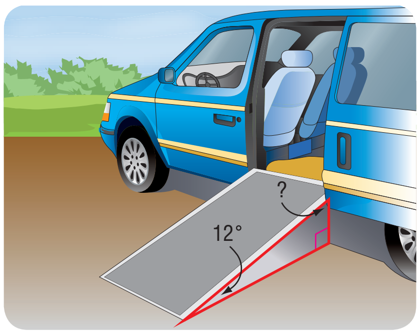
21 $m\angle ABC$



22. $m\angle JKL$



Example 3 **23. WHEELCHAIR RAMP** Suppose the wheelchair ramp shown makes a 12° angle with the ground. What is the measure of the angle the ramp makes with the van door?



CCSS REGULARITY **Find each measure.**

24. $m\angle 1$

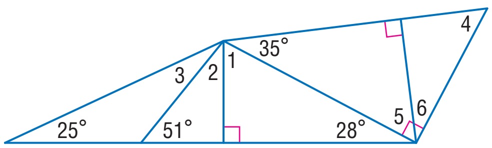
25. $m\angle 2$

26. $m\angle 3$

27. $m\angle 4$

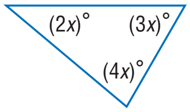
28. $m\angle 5$

29. $m\angle 6$

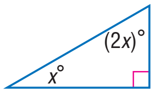


ALGEBRA Find the value of x. Then find the measure of each angle.

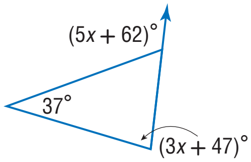
30.



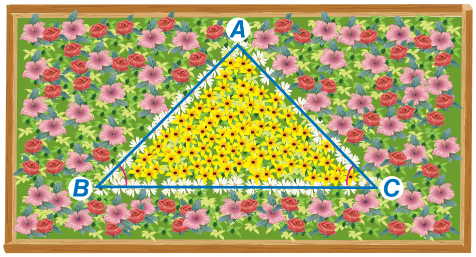
31.



32.



33 **GARDENING** A landscaper is forming an isosceles triangle in a flowerbed using chrysanthemums. She wants $m\angle A$ to be three times the measure of $\angle B$ and $\angle C$. What should the measure of each angle be?



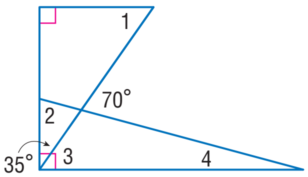
PROOF Write the specified type of proof.

34. flow proof of Corollary 4.1

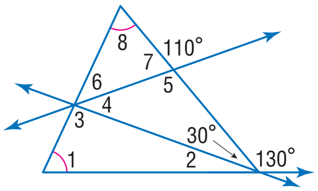
35. paragraph proof of Corollary 4.2

CCSS REGULARITY Find the measure of each numbered angle.

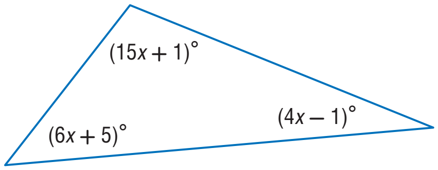
36.



37.



38. **ALGEBRA** Classify the triangle shown by its angles. Explain your reasoning.



39. **ALGEBRA** The measure of the larger acute angle in a right triangle is two degrees less than three times the measure of the smaller acute angle. Find the measure of each angle.

40. Determine whether the following statement is *true* or *false*. If false, give a counterexample. If true, give an argument to support your conclusion.

If the sum of two acute angles of a triangle is greater than 90, then the triangle is acute.

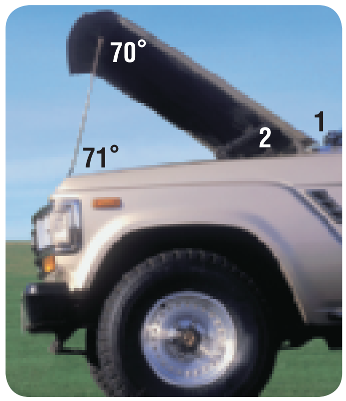
41. **ALGEBRA** In $\triangle XYZ$, $m\angle X = 157$, $m\angle Y = y$, and $m\angle Z = z$. Write an inequality to describe the possible measures of $\angle Z$. Explain your reasoning.

42. **CARS** Refer to the photo at the right.

a. Find $m\angle 1$ and $m\angle 2$.

b. If the support for the hood were shorter than the one shown, how would $m\angle 1$ change? Explain.

c. If the support for the hood were shorter than the one shown, how would $m\angle 2$ change? Explain.



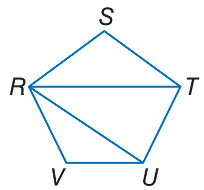
Jan Stromme/Photonica/Getty Images

PROOF **Write the specified type of proof.**

43 two-column proof

Given: *RSTUV* is a pentagon.

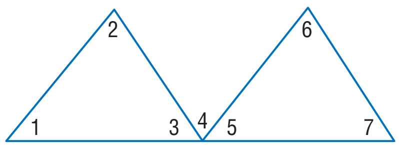
Prove: $m\angle S + m\angle STU + m\angle TUV + m\angle V + m\angle VRS = 540$



44. flow proof

Given: $\angle 3 \cong \angle 5$

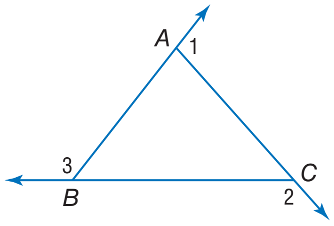
Prove: $m\angle 1 + m\angle 2 = m\angle 6 + m\angle 7$



45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the sum of the measures of the exterior angles of a triangle.

a. Geometric Draw five different triangles, extending the sides and labeling the angles as shown. Be sure to include at least one obtuse, one right, and one acute triangle.

b. Tabular Measure the exterior angles of each triangle. Record the measures for each triangle and the sum of these measures in a table.



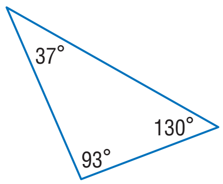
c. Verbal Make a conjecture about the sum of the exterior angles of a triangle. State your conjecture using words.

d. Algebraic State the conjecture you wrote in part **c** algebraically.

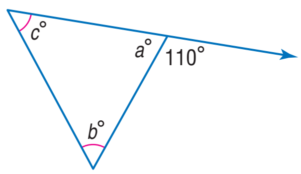
e. Analytical Write a paragraph proof of your conjecture.

H.O.T. Problems Use Higher-Order Thinking Skills

46. **CCSS CRITIQUE** Curtis measured and labeled the angles of the triangle as shown. Arnoldo says that at least one of his measures is incorrect. Explain in at least two different ways how Arnoldo knows that this is true.

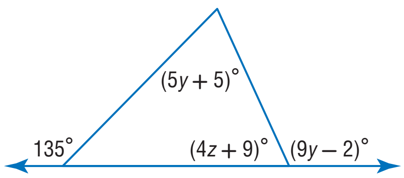


47. **WRITING IN MATH** Explain how you would find the missing measures in the figure shown.



48. **OPEN ENDED** Construct a right triangle and measure one of the acute angles. Find the measure of the second acute angle using calculation and explain your method. Confirm your result using a protractor.

49. **CHALLENGE** Find the values of y and z in the figure at the right.



50. **REASONING** If an exterior angle adjacent to $\angle A$ is acute, is $\triangle ABC$ acute, right, obtuse, or can its classification not be determined? Explain your reasoning.

51. **WRITING IN MATH** Explain why a triangle cannot have an obtuse, acute, and a right exterior angle.

Standardized Test Practice

52. PROBABILITY Mr. Glover owns a video store and wants to survey his customers to find what type of movies he should buy. Which of the following options would be the best way for Mr. Glover to get accurate survey results?

A surveying customers who come in from 9 P.M. until 10 P.M.

B surveying customers who come in on the weekend

C surveying the male customers

D surveying at different times of the week and day

53. SHORT RESPONSE Two angles of a triangle have measures of 35° and 80°. Find the values of the exterior angle measures of the triangle.

54. ALGEBRA Which equation is equivalent to $7x - 3(2 - 5x) = 8x$?

F $2x - 6 = 8$

G $22x - 6 = 8x$

H $-8x - 6 = 8x$

J $22x + 6 = 8x$

55. SAT/ACT Joey has 4 more video games than Solana and half as many as Melissa. If together they have 24 video games, how many does Melissa have?

A 7

B 9

C 12

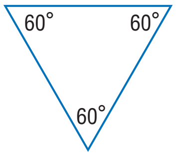
D 13

E 14

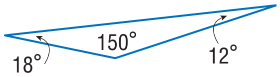
Spiral Review

Classify each triangle as *acute, equiangular, obtuse,* or *right*. (Lesson 4-1)

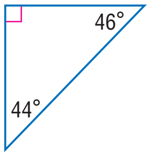
56.



57.



58.



COORDINATE GEOMETRY **Find the distance from P to ℓ.** (Lesson 3-6)

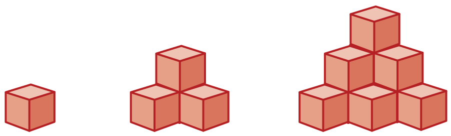
59. Line ℓ contains points $(0, -2)$ and $(1, 3)$. Point P has coordinates $(-4, 4)$.

60. Line ℓ contains points $(-3, 0)$ and $(3, 0)$. Point P has coordinates $(4, 3)$.

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence. (Lesson 2-1)

61.

62.



Skills Review

State the property that justifies each statement.

63. If $\frac{x}{2} = 7$, then $x = 14$.

64. If $x = 5$ and $b = 5$, then $x = b$.

65. If $XY - AB = WZ - AB$, then $XY = WZ$.

66. If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, $m\angle A = m\angle C$.

67. If $m\angle 1 + m\angle 2 = 90$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 90$.

LESSON

4-3 Congruent Triangles

Then

- You identified and used congruent angles.

Now

1. Name and use corresponding parts of congruent polygons.
2. Prove triangles congruent using the definition of congruence.

Why?

- As an antitheft device, many manufacturers make car stereos with removable faceplates. The shape and size of the faceplate and of the space where it fits must be exactly the same for the faceplate to properly attach to the car's dashboard.

NewVocabulary

congruent
congruent polygons
corresponding parts

Common Core State Standards

Content Standards

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

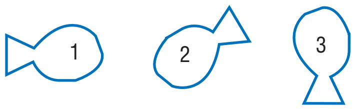
6 Attend to precision.
3 Construct viable arguments and critique the reasoning of others.

1 Congruence and Corresponding Parts

If two geometric figures have exactly the same shape and size, they are **congruent**.

≅

= Equal ~ Similar
Same size Same shape

Congruent	Not Congruent
	
While positioned differently, Figures 1, 2, and 3 are exactly the same shape and size.	Figures 4 and 5 are exactly the same shape but not the same size. Figures 5 and 6 are the same size but not exactly the same shape.

In two **congruent polygons**, all of the parts of one polygon are congruent to the **corresponding parts** or matching parts of the other polygon. These corresponding parts include *corresponding angles* and *corresponding sides*.

KeyConcept Definition of Congruent Polygons

Words Two polygons are congruent if and only if their corresponding parts are congruent.

Example Corresponding Angles

$\angle A \cong \angle H$ $\quad \angle B \cong \angle J$ $\quad \angle C \cong \angle K$

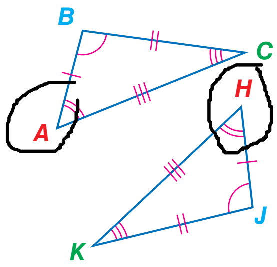
Corresponding Sides

$\overline{AB} \cong \overline{HJ}$ $\quad \overline{BC} \cong \overline{JK}$ $\quad \overline{AC} \cong \overline{HK}$

Congruence Statement

$\triangle ABC \cong \triangle HJK$

Model



Other congruence statements for the triangles above exist. Valid congruence statements for congruent polygons list corresponding vertices in the same order.

Valid Statement	Not a Valid Statement
$\triangle BCA \cong \triangle JKH$	$\triangle ABC \cong \triangle HKJ$

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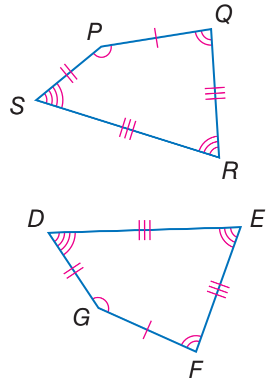
Math History Link

Johann Carl Friedrich Gauss (1777–1855) Gauss developed the congruence symbol to show that two sides of an equation were the same even if they weren't equal. He made many advances in math and physics, including a proof of the fundamental theorem of algebra.

Source: The Granger Collection, New York

Example 1 Identify Corresponding Congruent Parts

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.



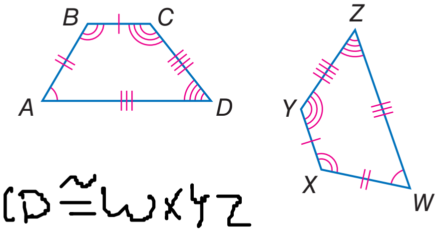
Angles: $\angle P \cong \angle G$, $\angle Q \cong \angle F$, $\angle R \cong \angle E$, $\angle S \cong \angle D$

Sides: $\overline{PQ} \cong \overline{GF}$, $\overline{QR} \cong \overline{FE}$, $\overline{RS} \cong \overline{ED}$, $\overline{SP} \cong \overline{DG}$

All corresponding parts of the two polygons are congruent. Therefore, polygon $PQRS \cong$ polygon $GFED$.

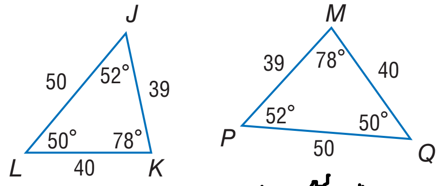
GuidedPractice

1A.



ABCD ≅ WXYZ

1B.



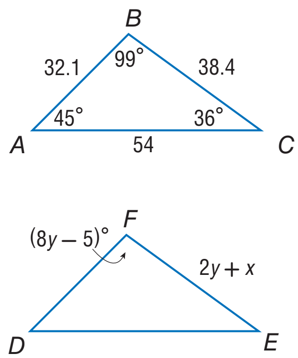
→ △LJK ≅ △QPM

∠L ≅ ∠Q (CPCTC)

The phrase "if and only if" in the congruent polygon definition means that both the conditional and its converse are true. So, if two polygons are congruent, then their corresponding parts are congruent. For triangles, we say *Corresponding parts of congruent triangles are congruent*, or CPCTC.

Example 2 Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle ABC \cong \triangle DFE$. Find the values of x and y.



$\angle F \cong \angle B$	CPCTC
$m\angle F = m\angle B$	Definition of congruence
$8y - 5 = 99$	Substitution
$8y = 104$	Add 5 to each side.
$y = 13$	Divide each side by 8.
$\overline{FE} \cong \overline{BC}$	CPCTC
$FE = BC$	Definition of congruence
$2y + x = 38.4$	Substitution
$2(13) + x = 38.4$	Substitution
$26 + x = 38.4$	Simplify.
$x = 12.4$	Subtract 26 from each side.

StudyTip

Using a Congruence Statement You can use a congruence statement to help you correctly identify corresponding sides.

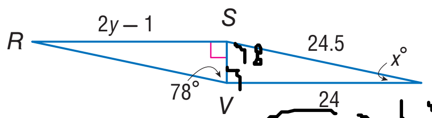
$\triangle ABC \cong \triangle DFE$

$\overline{BC} \cong \overline{FE}$

2y − 1 = 24 → 2y = 25
+1 +1
y = 12.5

GuidedPractice

2. In the diagram, $\triangle RSV \cong \triangle TVS$. Find the values of x and y.



90
78
168
12

y=12

akg-images

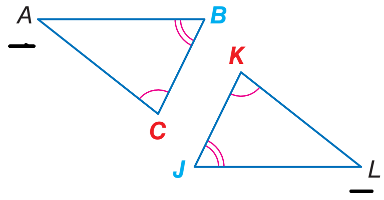
2 Prove Triangles Congruent

The Triangle Angle-Sum Theorem you learned in Lesson 4-2 leads to another theorem about the angles in two triangles.

Theorem 4.3 Third Angles Theorem

Words: If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Example: If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$.



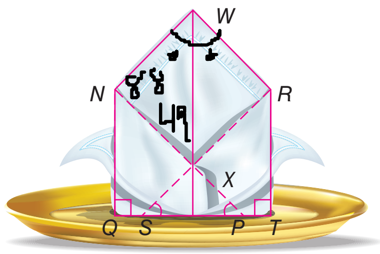
You will prove this theorem in Exercise 21.

Real-WorldLink

Using some basic skills with napkin folding can add an elegant touch to any party. Many of the folds use triangles.

Real-World Example 3 Use the Third Angles Theorem

PARTY PLANNING **The planners of the Senior Banquet decide to fold the dinner napkins using the Triangle Pocket Fold so that they can place a small gift in the pocket. If $\angle NPQ \cong \angle RST$, and $m\angle NPQ = 40$, find $m\angle SRT$.**



$\angle NPQ \cong \angle RST$, and since all right angles are congruent, $\angle NQP \cong \angle RTS$. So by the Third Angles Theorem, $\angle QNP \cong \angle SRT$. By the definition of congruence, $m\angle QNP = m\angle TRS$.

$m\angle QNP + m\angle NPQ = 90$	The acute angles of a right triangle are complementary.
$m\angle QNP + 40 = 90$	Substitution
$m\angle QNP = 50$	Subtract 40 from each side.

By substitution, $m\angle SRT = m\angle QNP$ or 50.

GuidedPractice

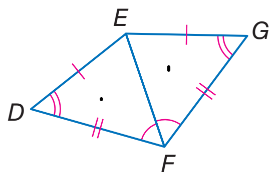
3. In the diagram above, if $\angle WNX \cong \angle WRX$, $\overline{WX}$ bisects $\angle NXR$, $m\angle WNX = 88$, and $m\angle NXW = 49$, find $m\angle NWR$. Explain your reasoning.

Example 4 Prove That Two Triangles are Congruent

Write a two-column proof.

Given: $\overline{DE} \cong \overline{GE}$, $\overline{DF} \cong \overline{GF}$, $\angle D \cong \angle G$, $\angle DFE \cong \angle GFE$

Prove: $\triangle DEF \cong \triangle GEF$



Proof:

Statements	Reasons
1. $\overline{DE} \cong \overline{GE}$, $\overline{DF} \cong \overline{GF}$	**1.** Given
2. $\overline{EF} \cong \overline{EF}$	**2.** Reflexive Property of Congruence
3. $\angle D \cong \angle G$, $\angle DFE \cong \angle GFE$	**3.** Given
4. $\angle DEF \cong \angle GEF$	**4.** Third Angles Theorem
5. $\triangle DEF \cong \triangle GEF$	**5.** Definition of Congruent Polygons

StudyTip

Reflexive Property When two triangles share a common side, use the Reflexive Property of Congruence to establish that the common side is congruent to itself.

S	R
1) $\angle J \cong \angle P$, $\overline{JK} \cong \overline{PM}$, $\overline{JL} \cong \overline{PL}$, L bisects $\overline{KM}$	1) Given
2) $\angle 1 \cong \angle 2$	2) VAT
3) $\overline{KL} \cong \overline{LM}$	3) Def of bisector
4) $\triangle JLK \cong \triangle PLM$	4) Def of $\cong$ poly's

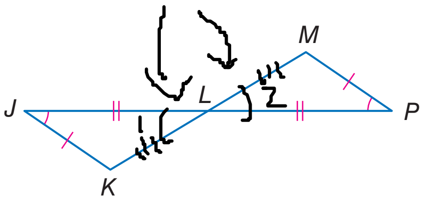
GuidedPractice

4. Write a two column proof.

Given: $\angle J \cong \angle P$, $\overline{JK} \cong \overline{PM}$, $\overline{JL} \cong \overline{PL}$, and L bisects $\overline{KM}$.

Prove: $\triangle JLK \cong \triangle PLM$

VAT



Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

Theorem 4.4 Properties of Triangle Congruence

Reflexive Property of Triangle Congruence

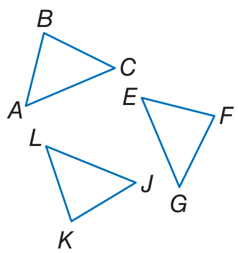
$\triangle ABC \cong \triangle ABC$

Symmetric Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Transitive Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



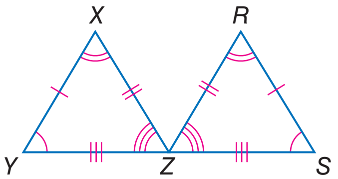
You will prove the reflexive, symmetric, and transitive parts of Theorem 4.4 in Exercises 27, 22, and 26, respectively.

Check Your Understanding

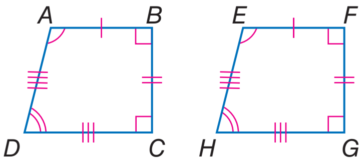
= Step-by-Step Solutions begin on page R14.

Example 1 **Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.**

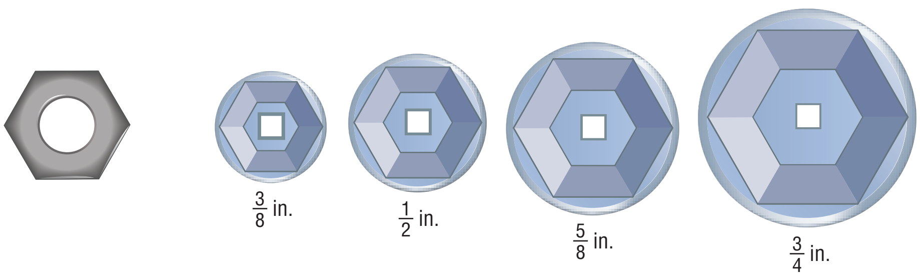
1.



2.



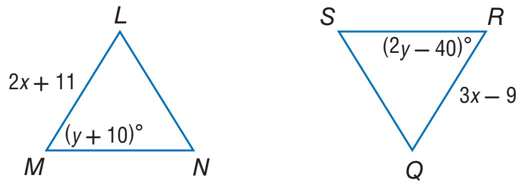
3. TOOLS Sareeta is changing the tire on her bike and the nut securing the tire looks like the one shown. Which of the sockets below should she use with her wrench to remove the tire? Explain your reasoning.



Example 2 In the figure, $\triangle LMN \cong \triangle QRS$.

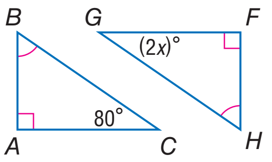
4. Find x.

5. Find y.

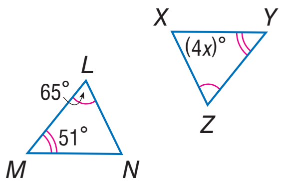


Example 3 CCSS **REGULARITY** Find x. Explain your reasoning.

6.



7.

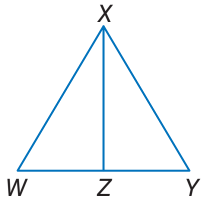


Example 4

8. **PROOF** Write a paragraph proof.

Given: $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$, $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$

Prove: $\triangle WXZ \cong \triangle YXZ$

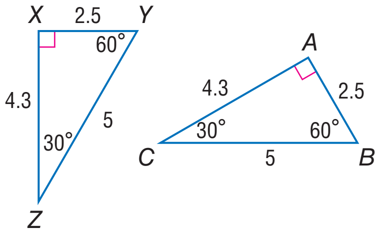


Practice and Problem Solving

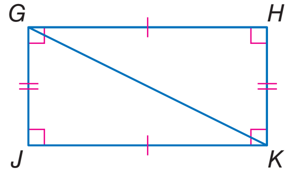
Extra Practice is on page R4.

Example 1 Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

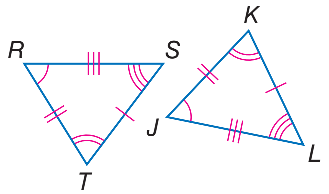
9.



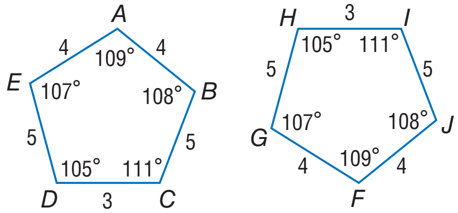
10.



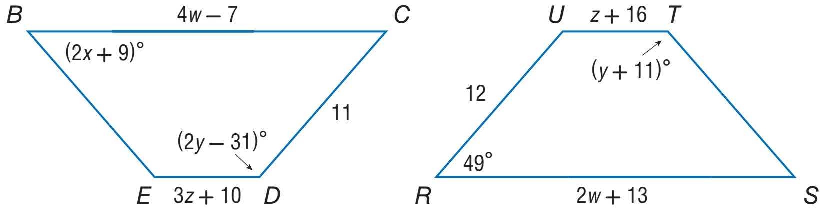
11.



12.



Example 2 Polygon $BCDE \cong$ polygon $RSTU$. Find each value.



13. x

14. y

15. z

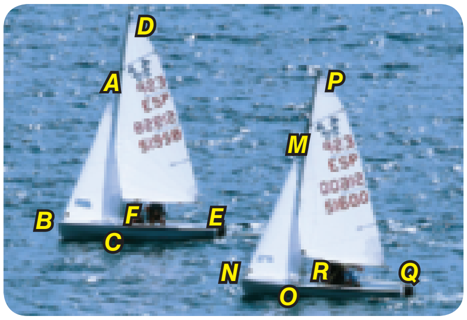
16. w

17. **SAILING** To ensure that sailboat races are fair, the boats and their sails are required to be the same size and shape.

a. Write a congruence statement relating the triangles in the photo.

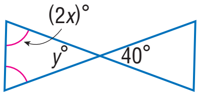
b. Name six pairs of congruent segments.

c. Name six pairs of congruent angles.

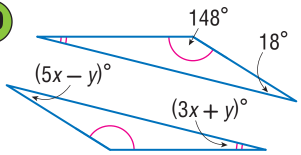


Example 3 **Find x and y.**

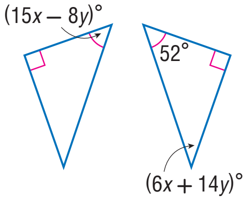
18.



19.



20.



Example 4

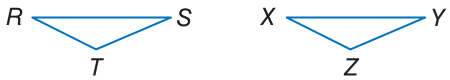
21. **PROOF** Write a two-column proof of Theorem 4.3.

22. **PROOF** Put the statements used to prove the statement below in the correct order. Provide the reasons for each statement.

Congruence of triangles is symmetric. (Theorem 4.4)

Given: $\triangle RST \cong \triangle XYZ$

Prove: $\triangle XYZ \cong \triangle RST$



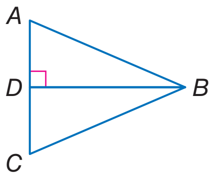
Proof:

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$	$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$	$\triangle RST \cong \triangle XYZ$	$\triangle XYZ \cong \triangle RST$
?	?	?	?

CCSS ARGUMENTS **Write a two-column proof.**

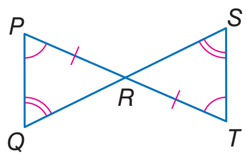
23. **Given:** $\overline{BD}$ bisects $\angle B$.
$\overline{BD} \perp \overline{AC}$

Prove: $\angle A \cong \angle C$

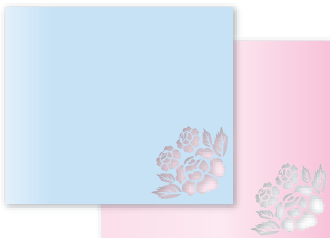


24. **Given:** $\angle P \cong \angle T$, $\angle S \cong \angle Q$
$\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$,
$\overline{RT} \cong \overline{RS}$
$\overline{PQ} \cong \overline{TS}$

Prove: $\triangle PRQ \cong \triangle TRS$



25. **SCRAPBOOKING** Lanie is using a flower-shaped corner decoration punch for a scrapbook she is working on. If she punches the corners of two pages as shown, what property guarantees that the punched designs are congruent? Explain.



Enrique Algarra/age fotostock

PROOF **Write the specified type of proof of the indicated part of Theorem 4.4.**

26. Congruence of triangles is transitive. (paragraph proof)

27. Congruence of triangles is reflexive. (flow proof)

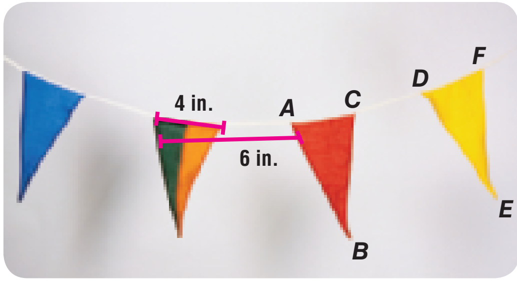
ALGEBRA **Draw and label a figure to represent the congruent triangles. Then find x and y.**

28. $\triangle ABC \cong \triangle DEF$, $AB = 7$, $BC = 9$, $AC = 11 + x$, $DF = 3x - 13$, and $DE = 2y - 5$

29. $\triangle LMN \cong \triangle RST$, $m\angle L = 49$, $m\angle M = 10y$, $m\angle S = 70$, and $m\angle T = 4x + 9$

30. $\triangle JKL \cong \triangle MNP$, $JK = 12$, $LJ = 5$, $PM = 2x - 3$, $m\angle L = 67$, $m\angle K = y + 4$ and $m\angle N = 2y - 15$

31 **PENNANTS** Scott is in charge of roping off an area of 100 square feet for the band to use during a pep rally. He is using a string of pennants that are congruent isosceles triangles.

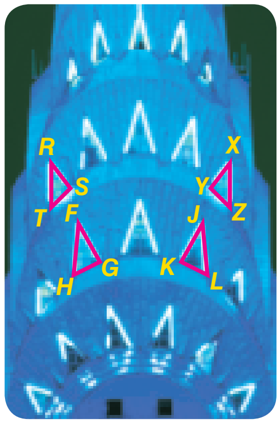


a. List seven pairs of congruent segments in the photo.

b. If the area he ropes off is a square, how long will the pennant string need to be?

c. How many pennants will be on the string?

32. **CCSS SENSE-MAKING** In the photo of New York City's Chrysler Building at the right, $\overline{TS} \cong \overline{ZY}$, $\overline{XY} \cong \overline{RS}$, $\overline{TR} \cong \overline{ZX}$, $\angle X \cong \angle R$, $\angle T \cong \angle Z$, $\angle Y \cong \angle S$, and $\triangle HGF \cong \triangle LKJ$.



a. Which triangle, if any, is congruent to $\triangle YXZ$? Explain your reasoning.

b. Which side(s) are congruent to $\overline{JL}$? Explain your reasoning.

c. Which angle(s) are congruent to $\angle G$? Explain your reasoning.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the statement *The areas of congruent triangles are equal.*

a. Verbal Write a conditional statement to represent the relationship between the areas of a pair of congruent triangles.

b. Verbal Write the converse of your conditional statement. Is the converse *true* or *false*? Explain your reasoning.

c. Geometric If possible, draw two equilateral triangles that have the same area but are not congruent. If not possible, explain why not.

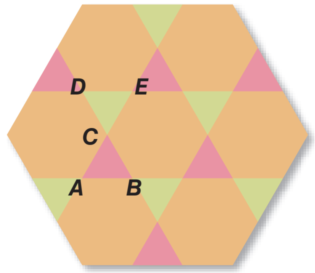
d. Geometric If possible, draw two rectangles that have the same area but are not congruent. If not possible, explain why not.

e. Geometric If possible, draw two squares that have the same area but are not congruent. If not possible, explain why not.

f. Verbal For which polygons will the following conditional and its converse both be true? Explain your reasoning.

If a pair of __________ *are congruent, then they have the same area.*

34. PATTERNS The pattern shown is created using regular polygons.



a. What two polygons are used to create the pattern?

b. Name a pair of congruent triangles.

c. Name a pair of corresponding angles.

d. If $CB = 2$ inches, what is AE? Explain.

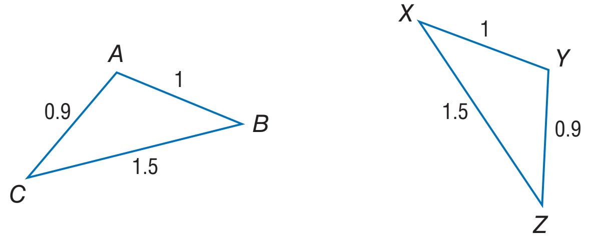
e. What is the measure of $\angle D$? Explain.

35. FITNESS A fitness instructor is starting a new aerobics class using fitness hoops. She wants to confirm that all of the hoops are the same size. What measure(s) can she use to prove that all of the hoops are congruent? Explain your reasoning.

H.O.T. Problems Use Higher-Order Thinking Skills

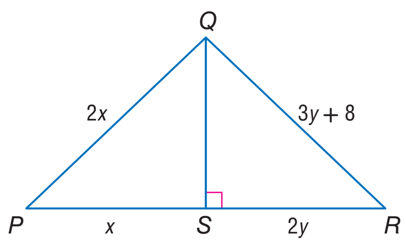
36. WRITING IN MATH Explain why the order of the vertices is important when naming congruent triangles. Give an example to support your answer.

37. ERROR ANALYSIS Jasmine and Will are evaluating the congruent figures below. Jasmine says that $\triangle CAB \cong \triangle ZYX$ and Will says that $\triangle ABC \cong \triangle YXZ$. Is either of them correct? Explain.



38. WRITE A QUESTION A classmate is using the Third Angles Theorem to show that if 2 corresponding pairs of the angles of two triangles are congruent, then the third pair is also congruent. Write a question to help him decide if he can use the same strategy for quadrilaterals.

39. CHALLENGE Find x and y if $\triangle PQS \cong \triangle RQS$.

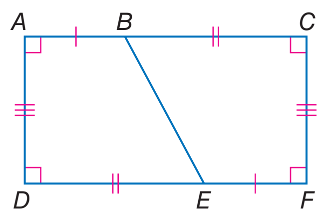


CCSS ARGUMENTS Determine whether each statement is *true* or *false*. If false, give a counterexample. If true, explain your reasoning.

40. Two triangles with two pairs of congruent corresponding angles and three pairs of congruent corresponding sides are congruent.

41. Two triangles with three pairs of corresponding congruent angles are congruent.

42. CHALLENGE Write a paragraph proof to prove polygon $ABED \cong$ polygon $FEBC$.

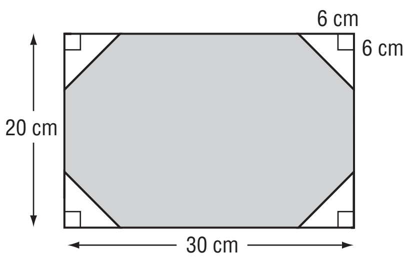


43. WRITING IN MATH Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

Equilateral triangles are congruent.

Standardized Test Practice

44. Barrington cut four congruent triangles off the corners of a rectangle to make an octagon as shown below. What is the area of the octagon?



A 456 cm^2 **C** 552 cm^2
B 528 cm^2 **D** 564 cm^2

45. GRIDDED RESPONSE Triangle ABC is congruent to $\triangle HIJ$. The vertices of $\triangle ABC$ are $A(-1, 2)$, $B(0, 3)$ and $C(2, -2)$. What is the measure of side $\overline{HJ}$?

46. ALGEBRA Which is a factor of $x^2 + 19x - 42$?

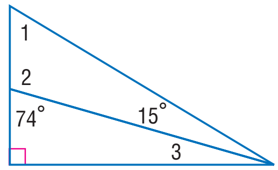
F $x + 14$ **H** $x - 2$
G $x + 2$ **J** $x - 14$

47. SAT/ACT Mitsu travels a certain distance at 30 miles per hour and returns the same route at 65 miles per hour. What is his average speed in miles per hour for the round trip?

A 32.5 **D** 47.5
B 35.0 **E** 55.3
C 41.0

Spiral Review

Find each measure in the triangle at the right. (Lesson 4-2)



48. $m\angle 2$ **49.** $m\angle 1$ **50.** $m\angle 3$

COORDINATE GEOMETRY **Find the measures of the sides of $\triangle JKL$ and classify each triangle by the measures of its sides.** (Lesson 4-1)

51. $J(-7, 10)$, $K(15, 0)$, $L(-2, -1)$

52. $J(9, 9)$, $K(12, 14)$, $L(14, 6)$

53. $J(4, 6)$, $K(4, 11)$, $L(9, 6)$

54. $J(16, 14)$, $K(7, 6)$, $L(-5, -14)$

Determine whether each statement is *always*, *sometimes*, or *never* true. (Lesson 1-5)

55. Two angles that form a linear pair are supplementary.

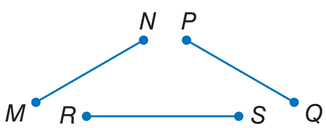
56. If two angles are supplementary, then one of the angles is obtuse.

57. CARPENTRY A carpenter must cut two pieces of wood at angles so that they fit together to form the corner of a picture frame. What type of angles must he use to make sure that a 90° corner results? (Lesson 1-5)

Skills Review

58. Copy and complete the proof.

Given: $\overline{MN} \cong \overline{PQ}$, $\overline{PQ} \cong \overline{RS}$
Prove: $\overline{MN} \cong \overline{RS}$
Proof:



Statements	Reasons
a. ___?___	**a.** Given
b. $MN = PQ$, $PQ = RS$	**b.** ___?___
c. ___?___	**c.** ___?___
d. $\overline{MN} \cong \overline{RS}$	**d.** Definition of congruent segments

LESSON 4-4

Proving Triangles Congruent—SSS, SAS

Then

- You proved triangles congruent using the definition of congruence.

Now

- **1** Use the SSS Postulate to test for triangle congruence.
- **2** Use the SAS Postulate to test for triangle congruence.

Why?

- An A-frame sandwich board is a convenient way to display information. Not only does it fold flat for easy storage, but with each sidearm locked into place, the frame is extremely sturdy. With the sidearms the same length and positioned the same distance from the top on either side, the open frame forms two congruent triangles.

NewVocabulary
included angle

Common Core State Standards

Content Standards

G.CO.10 Prove theorems about triangles.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

3 Construct viable arguments and critique the reasoning of others.

1 Make sense of problems and persevere in solving them.

1 SSS Postulate

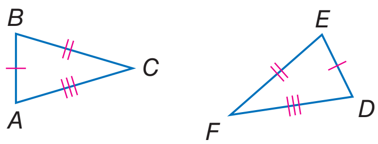
In Lesson 4-3, you proved that two triangles were congruent by showing that all six pairs of corresponding parts were congruent. It is possible to prove two triangles congruent using fewer pairs.

The sandwich board demonstrates that if two triangles have the same three side lengths, then they are congruent. This is expressed in the postulate below.

Postulate 4.1 Side-Side-Side (SSS) Congruence

If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example If Side $\overline{AB} \cong \overline{DE}$,
Side $\overline{BC} \cong \overline{EF}$, and
Side $\overline{AC} \cong \overline{DF}$,
then $\triangle ABC \cong \triangle DEF$.



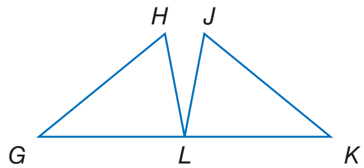
PT

Example 1 Use SSS to Prove Triangles Congruent

Write a flow proof.

Given: $\overline{GH} \cong \overline{KJ}$, $\overline{HL} \cong \overline{JL}$, and L is the midpoint of $\overline{GK}$.

Prove: $\triangle GHL \cong \triangle KJL$



Flow Proof:

$\overline{GH} \cong \overline{KJ}$ — Given

$\overline{HL} \cong \overline{JL}$ — Given

L is the midpoint of $\overline{GK}$. — Given → $\overline{GL} \cong \overline{KL}$ — Midpoint Theorem

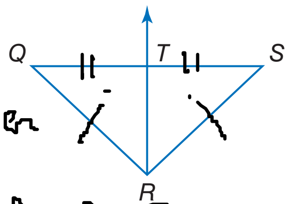
→ $\triangle GHL \cong \triangle KJL$ — SSS

GuidedPractice

1. Write a flow proof.

Given: 1) $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$. $\overline{RT}$ bisects $\overline{QS}$ at point T.

Prove: ~~$\triangle QRT \cong \triangle SRT$~~



1) Given

2) Def of Isos.

3) Def of bisect

S 2) $\overline{QR} \cong \overline{SR}$

S 3) $\overline{QT} \cong \overline{ST}$

S 4) TR ≅ TR   4) reflexive prop ≅
5) △QRT ≅ △SRT   5) SSS △ ≅

Standardized Test Example 2 SSS on the Coordinate Plane

EXTENDED RESPONSE **Triangle *ABC* has vertices *A*(1, 1), *B*(0, 3), and *C*(2, 5). Triangle *EFG* has vertices *E*(1, −1), *F*(2, −5), and *G*(4, −4).**

a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

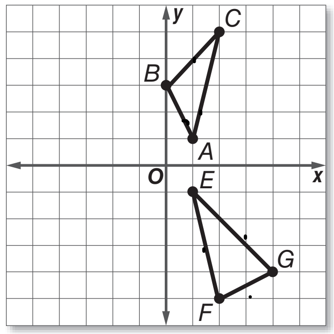
c. Write a logical argument using coordinate geometry to support the conjecture you made in part b.

Read the Test Item

You are asked to do three things in this problem. In part **a**, you are to graph $\triangle ABC$ and $\triangle EFG$ on the same coordinate plane. In part **b**, you should make a conjecture that $\triangle ABC \cong \triangle EFG$ or $\triangle ABC \ncong \triangle EFG$ based on your graph. Finally, in part **c**, you are asked to prove your conjecture.

Solve the Test Item

a.



b. From the graph, it appears that the triangles do not have the same shape, so we can conjecture that they are not congruent.

c. Use the Distance Formula to show that not all corresponding sides have the same measure.

$$AB = \sqrt{(0-1)^2 + (3-1)^2} = \sqrt{1+4} \text{ or } \sqrt{5}$$

$$EF = \sqrt{(2-1)^2 + [-5-(-1)]^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$BC = \sqrt{(2-0)^2 + (5-3)^2} = \sqrt{4+4} \text{ or } \sqrt{8}$$

$$FG = \sqrt{(4-2)^2 + [-4-(-5)]^2} = \sqrt{4+1} \text{ or } \sqrt{5}$$

$$AC = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1+16} \text{ or } \sqrt{17}$$

$$EG = \sqrt{(4-1)^2 + [-4-(-1)]^2} = \sqrt{9+9} \text{ or } \sqrt{18}$$

While $AB = FG$ and $AC = EF$, $BC \neq EG$. Since SSS congruence is not met, $\triangle ABC \ncong \triangle EFG$.

> **Test-Taking Tip**
> **CCSS Tools** When you are solving problems using the coordinate plane, remember to use tools like the Distance, Midpoint, and Slope Formulas to solve problems and to check your solutions.

> **Reading Math**
> **Symbols** $\triangle ABC \ncong \triangle EFG$ is read as *triangle ABC is not congruent to triangle EFG.*

Guided Practice

2. Triangle *JKL* has vertices *J*(2, 5), *K*(1, 1), and *L*(5, 2). Triangle *NPQ* has vertices *N*(−3, 0), *P*(−7, 1), and *Q*(−4, 4).

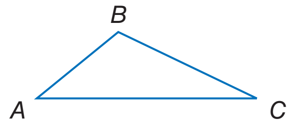
a. Graph both triangles on the same coordinate plane.

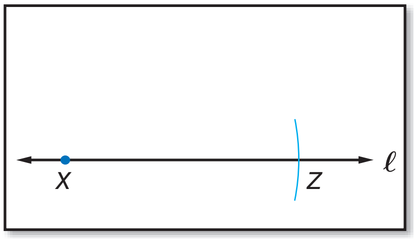
b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

c. Write a logical argument using coordinate geometry to support the conjecture you made in part **b**.

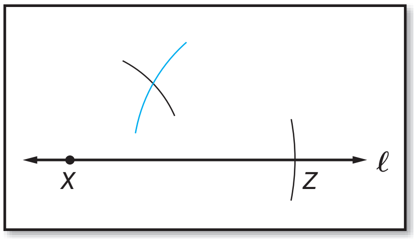
Construction Congruent Triangles Using Sides

Draw a triangle and label it $\triangle ABC$. Then use the SSS Postulate to construct $\triangle XYZ \cong \triangle ABC$.

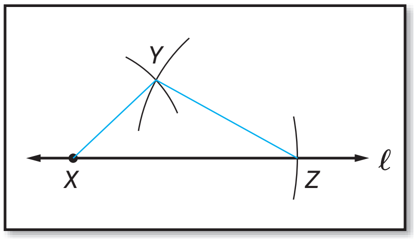




Step 1 Draw point X on a line ℓ. Then construct $\overline{XZ} \cong \overline{AC}$ on line ℓ



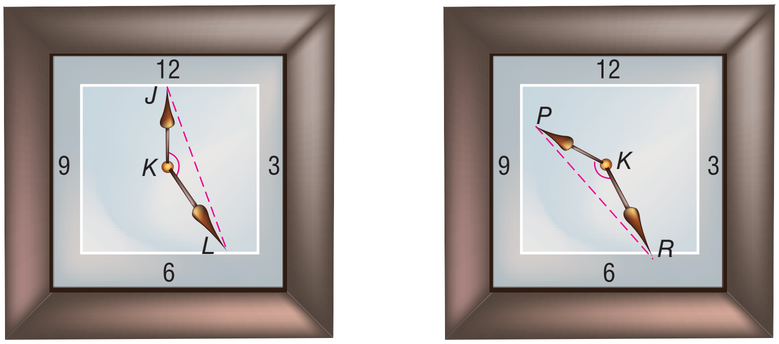
Step 2 Construct one arc with radius AB centered at point X and another arc with radius BC centered at point Z.



Step 3 Label the point of intersection of the two arcs Y. Draw $\overline{XY}$ and $\overline{ZY}$ to form $\triangle XYZ$.

2 SAS Postulate

The angle formed by two adjacent sides of a polygon is called an **included angle**. Consider included angle JKL formed by the hands on the first clock shown below. Any time the hands form an angle with the same measure, the distance between the ends of the hands $\overline{JL}$ and $\overline{PR}$ will be the same.



$\triangle PKR \cong \triangle JKL$

Any two triangles formed using the same side lengths and included angle measure will be congruent. This illustrates the following postulate.

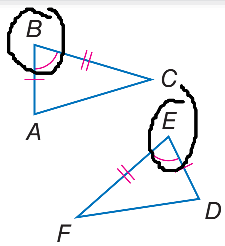
StudyTip

Side-Side-Angle The measures of two sides and a nonincluded angle are not sufficient to prove two triangles congruent.

Postulate 4.2 Side-Angle-Side (SAS) Congruence

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

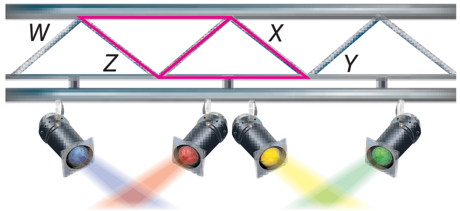
Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Angle $\angle B \cong \angle E$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Real-World Example 3 Use SAS to Prove Triangles are Congruent

PT

LIGHTING The scaffolding for stage lighting shown appears to be made up of congruent triangles. If $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{ZY}$, write a two-column proof to prove that $\triangle WXZ \cong \triangle YZX$.



Proof:

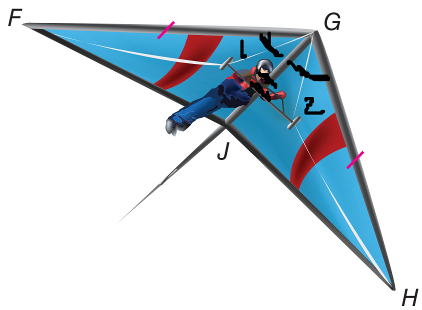
Statements	Reasons
1. $\overline{WX} \cong \overline{YZ}$	**1.** Given
2. $\overline{WX} \parallel \overline{ZY}$	**2.** Given
3. $\angle WXZ \cong \angle XZY$	**3.** Alternate Interior Angle Theorem
4. $\overline{XZ} \cong \overline{ZX}$	**4.** Reflexive Property of Congruence
5. $\triangle WXZ \cong \triangle YZX$	**5.** SAS

Real-WorldCareer

Lighting Technicians In the motion picture industry, gaffers, or lighting technicians, place the lighting required for a film. Gaffers make sure the angles the lights form are in the correct positions. They may have college or technical school degrees, or they may have completed a formal training program.

GuidedPractice

3. **EXTREME SPORTS** The wings of the hang glider shown appear to be congruent triangles. If $\overline{FG} \cong \overline{GH}$ and $\overline{JG}$ bisects $\angle FGH$, prove that $\triangle FGJ \cong \triangle HGJ$.

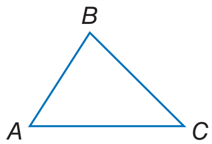


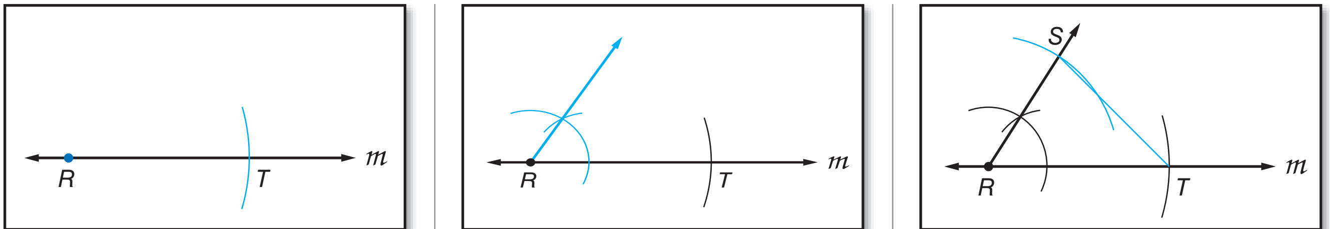
S	R
S1) FG ≅ GH	1) Given
2) JG bisects ∠FGH	2) Given
3) ∠1 ≅ ∠2	3) Def of bisector
S4) JG ≅ JG	4) Reflexive
5) △FGJ ≅ △HGJ	5) SAS △ ≅

You can also construct congruent triangles given two sides and the included angle.

Construction Congruent Triangles Using Two Sides and the Included Angle

Draw a triangle and label it $\triangle ABC$. Then use the SAS Postulate to construct $\triangle RST \cong \triangle ABC$.





Step 1 Draw point R on a line m. Then construct $\overline{RT} \cong \overline{AC}$ on line m.

Step 2 Construct $\angle R \cong \angle A$ using $\overline{RT}$ as a side of the angle and point R.

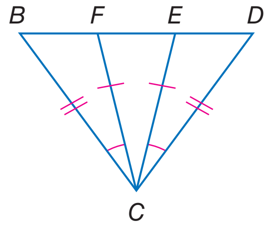
Step 3 Construct $\overline{RS} \cong \overline{AB}$. Then draw $\overline{ST}$ to form $\triangle RST$.

Example 4 Use SAS or SSS in Proofs

Write a paragraph proof.

Given: $\overline{BC} \cong \overline{DC}$, $\angle BCF \cong \angle DCE$, $\overline{FC} \cong \overline{EC}$

Prove: $\angle CFD \cong \angle CEB$

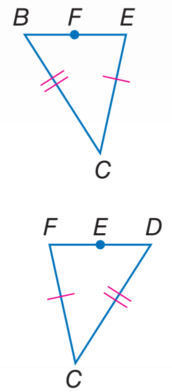


Proof:

Since $\overline{BC} \cong \overline{DC}$, $\angle BCF \cong \angle DCE$, and $\overline{FC} \cong \overline{EC}$, then $\triangle BCF \cong \triangle DCE$ by SAS. By CPCTC, $\angle CFB \cong \angle CED$. $\angle CFD$ forms a linear pair with $\angle CFB$, and $\angle CEB$ forms a linear pair with $\angle CED$. By the Congruent Supplements Theorem, $\angle CFD$ is supplementary to $\angle CFB$ and $\angle CEB$ is supplementary to $\angle CED$. Since angles supplementary to the same angle or congruent angles are congruent, $\angle CFD \cong \angle CEB$.

StudyTip

Overlapping Figures When triangles overlap, it can be helpful to draw each triangle separately and label the congruent parts. In Example 4, the figure could have been separated as shown.

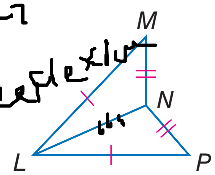


GuidedPractice

4. **Write a two-column proof.**

Given: $\overline{MN} \cong \overline{PN}$, $\overline{LM} \cong \overline{LP}$

Prove: $\angle LNM \cong \angle LNP$



S	R
1) $\overline{MN} \cong \overline{PN}$, $\overline{LM} \cong \overline{LP}$	1) G
2) $\overline{LN} \cong \overline{LN}$	2) Reflexive
3) $\triangle LNM \cong \triangle LNP$	3) SSS $\triangle \cong$
4) $\angle LNM \cong \angle LNP$	4) CPCTC

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

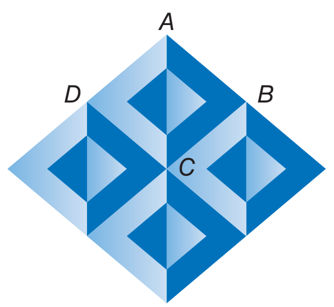
Example 1

1. **OPTICAL ILLUSION** The figure shown is a pattern formed using four large congruent squares and four small congruent squares.

a. How many different-sized triangles are used to create the illusion?

b. Use the Side-Side-Side Congruence Postulate to prove that $\triangle ABC \cong \triangle CDA$.

c. What is the relationship between $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$? Explain your reasoning.



Example 2

2. **EXTENDED RESPONSE** Triangle ABC has vertices $A(-3, -5)$, $B(-1, -1)$, and $C(-1, -5)$. Triangle XYZ has vertices $X(5, -5)$, $Y(3, -1)$, and $Z(3, -5)$.

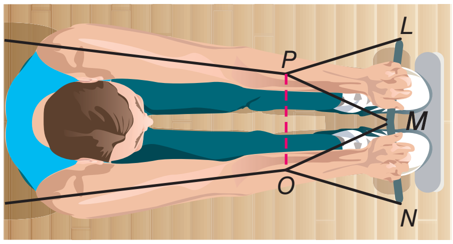
a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

c. Write a logical argument using coordinate geometry to support your conjecture.

Example 3

3. **EXERCISE** In the exercise diagram, if $\overline{LP} \cong \overline{NO}$, $\angle LPM \cong \angle NOM$, and $\triangle MOP$ is equilateral, write a paragraph proof to show that $\triangle LMP \cong \triangle NMO$.

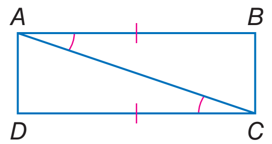


Example 4

4. **Write a two-column proof.**

Given: $\overline{BA} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$

Prove: $\overline{BC} \cong \overline{DA}$



Practice and Problem Solving

Extra Practice is on page R4.

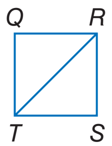
Example 1

PROOF **Write the specified type of proof.**

5. paragraph proof

Given: $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$

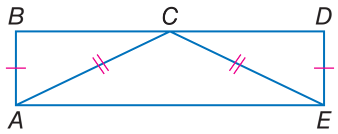
Prove: $\triangle QRT \cong \triangle SRT$



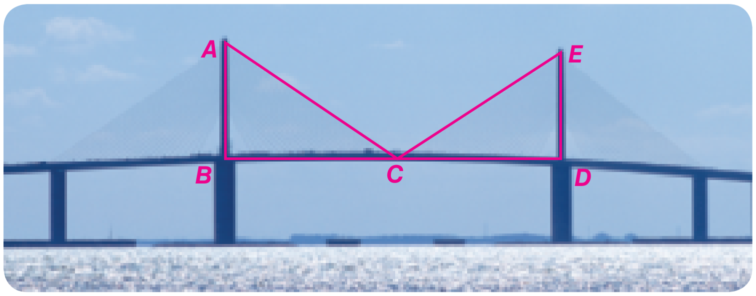
6. two-column proof

Given: $\overline{AB} \cong \overline{ED}$, $\overline{CA} \cong \overline{CE}$; $\overline{AC}$ bisects $\overline{BD}$.

Prove: $\triangle ABC \cong \triangle EDC$



7. **BRIDGES** The Sunshine Skyway Bridge in Florida is the world's longest cable-stayed bridge, spanning 4.1 miles of Tampa Bay. It is supported using steel cables suspended from two concrete supports. If the supports are the same height above the roadway and perpendicular to the roadway, and the topmost cables meet at a point midway between the supports, prove that the two triangles shown in the photo are congruent.



Example 2

CCSS **SENSE-MAKING** **Determine whether $\triangle MNO \cong \triangle QRS$. Explain.**

8. $M(2, 5)$, $N(5, 2)$, $O(1, 1)$, $Q(-4, 4)$, $R(-7, 1)$, $S(-3, 0)$

9. $M(0, -1)$, $N(-1, -4)$, $O(-4, -3)$, $Q(3, -3)$, $R(4, -4)$, $S(3, 3)$

10. $M(0, -3)$, $N(1, 4)$, $O(3, 1)$, $Q(4, -1)$, $R(6, 1)$, $S(9, -1)$

11. $M(4, 7)$, $N(5, 4)$, $O(2, 3)$, $Q(2, 5)$, $R(3, 2)$, $S(0, 1)$

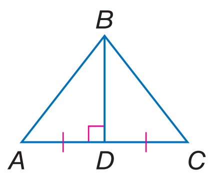
Example 3

PROOF **Write the specified type of proof.**

12. two-column proof

Given: $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\overline{AC}$.

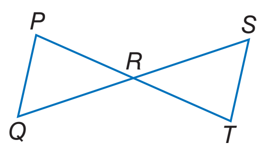
Prove: $\triangle ABD \cong \triangle CBD$



13. paragraph proof

Given: R is the midpoint of $\overline{QS}$ and $\overline{PT}$.

Prove: $\triangle PRQ \cong \triangle TRS$

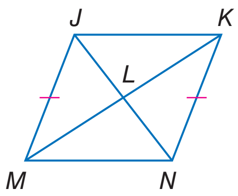


Example 4 **PROOF** **Write the specified type of proof.**

14. flow proof

Given: $\overline{JM} \cong \overline{NK}$; L is the midpoint of $\overline{JN}$ and $\overline{KM}$.

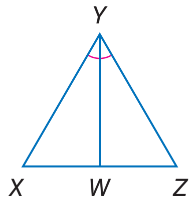
Prove: $\angle MJL \cong \angle KNL$



15. paragraph proof

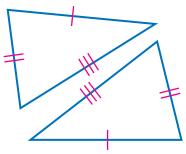
Given: $\triangle XYZ$ is equilateral. $\overline{WY}$ bisects $\angle XYZ$.

Prove: $\overline{XW} \cong \overline{ZW}$



CCSS ARGUMENTS **Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.**

16.



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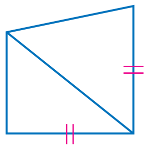

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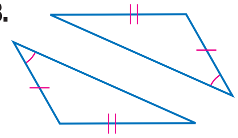

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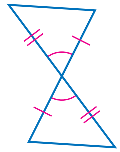


20. **SIGNS** Refer to the diagram at the right.

a. Identify the three-dimensional figure represented by the wet floor sign.

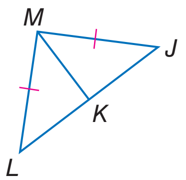
b. If $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$, prove that $\triangle ACB \cong \triangle ACD$.

c. Why do the triangles not look congruent in the diagram?

PROOF **Write a flow proof.**

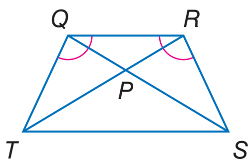
21. **Given:** $\overline{MJ} \cong \overline{ML}$; K is the midpoint of $\overline{JL}$.

Prove: $\triangle MJK \cong \triangle MLK$



22. **Given:** $\triangle TPQ \cong \triangle SPR$ $\angle TQR \cong \angle SRQ$

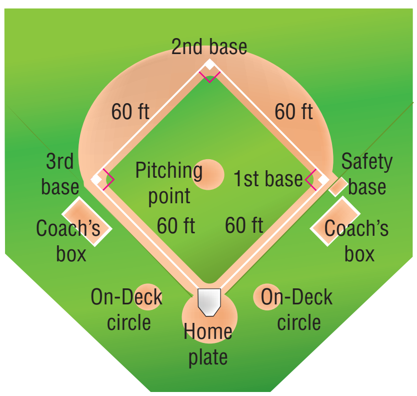
Prove: $\triangle TQR \cong \triangle SRQ$



23. **SOFTBALL** Use the diagram of a fast-pitch softball diamond shown. Let F = first base, S = second base, T = third base, P = pitching point, and R = home plate.

a. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.

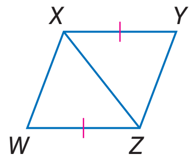
b. Write a two-column proof to prove that the angle formed between second base, home plate, and third base is the same as the angle formed between second base, home plate, and first base.



PROOF **Write a two-column proof.**

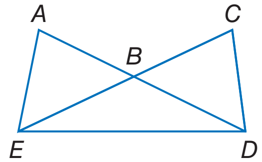
24. **Given:** $\overline{YX} \cong \overline{WZ}$, $\overline{YX} \parallel \overline{ZW}$

Prove: $\triangle YXZ \cong \triangle WZX$



25. **Given:** $\triangle EAB \cong \triangle DCB$

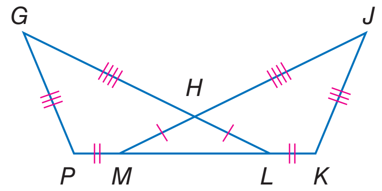
Prove: $\triangle EAD \cong \triangle DCE$



26. **CCSS ARGUMENTS** Write a paragraph proof.

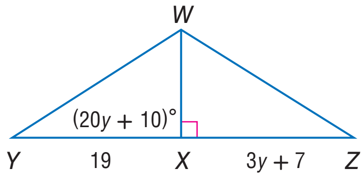
Given: $\overline{HL} \cong \overline{HM}$, $\overline{PM} \cong \overline{KL}$, $\overline{PG} \cong \overline{KJ}$, $\overline{GH} \cong \overline{JH}$

Prove: $\angle G \cong \angle J$

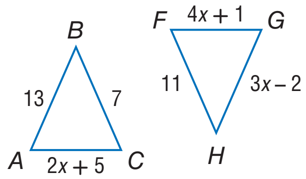


ALGEBRA **Find the value of the variable that yields congruent triangles. Explain.**

27 $\triangle WXY \cong \triangle WXZ$

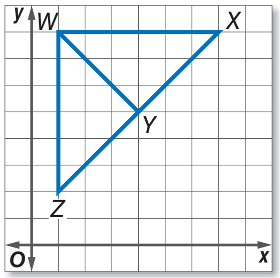


28. $\triangle ABC \cong \triangle FGH$



H.O.T. Problems Use Higher-Order Thinking Skills

29. **CHALLENGE** **Refer to the graph shown.**



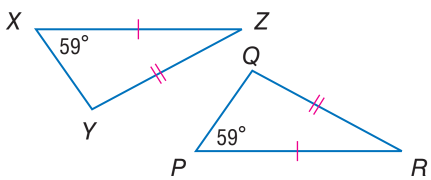
a. Describe two methods you could use to prove that $\triangle WYZ$ is congruent to $\triangle WYX$. You may not use a ruler or a protractor. Which method do you think is more efficient? Explain.

b. Are $\triangle WYZ$ and $\triangle WYX$ congruent? Explain your reasoning.

30. **REASONING** Determine whether the following statement is *true* or *false*. If true, explain your reasoning. If *false*, provide a counterexample.

If the congruent sides in one isosceles triangle have the same measure as the congruent sides in another isosceles triangle, then the triangles are congruent.

31. **ERROR ANALYSIS** Bonnie says that $\triangle PQR \cong \triangle XYZ$ by SAS. Shada disagrees. She says that there is not enough information to prove that the two triangles are congruent. Is either of them correct? Explain.



32. **OPEN ENDED** Use a straightedge to draw obtuse triangle ABC. Then construct $\triangle XYZ$ so that it is congruent to $\triangle ABC$ using either SSS or SAS. Justify your construction mathematically and verify it using measurement.

33. **WRITING IN MATH** Two pairs of corresponding sides of two right triangles are congruent. Are the triangles congruent? Explain your reasoning.

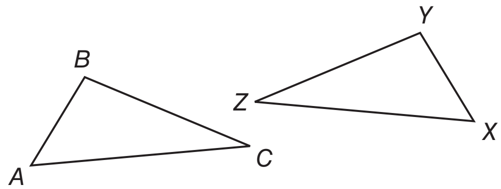
Standardized Test Practice

34. ALGEBRA The Ross Family drove 300 miles to visit their grandparents. Mrs. Ross drove 70 miles per hour for 65% of the trip and 35 miles per hour or less for 20% of the trip that was left. Assuming that Mrs. Ross never went over 70 miles per hour, how many miles did she travel at a speed between 35 and 70 miles per hour?

A 195 **C** 21

B 84 **D** 18

35. In the figure, $\angle C \cong \angle Z$ and $\overline{AC} \cong \overline{XZ}$.



What additional information could be used to prove that $\triangle ABC \cong \triangle XYZ$?

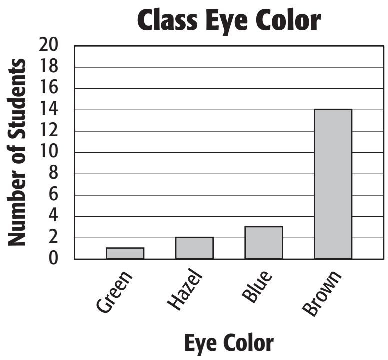
F $\overline{BC} \cong \overline{YZ}$

G $\overline{AB} \cong \overline{XY}$

H $\overline{BC} \cong \overline{XZ}$

J $\overline{XZ} \cong \overline{XY}$

36. EXTENDED RESPONSE The graph below shows the eye colors of all of the students in a class. What is the probability that a student chosen at random from this class will have blue eyes? Explain your reasoning.



37. SAT/ACT If $4a + 6b = 6$ and $-2a + b = -7$, what is the value of a?

A -2

B -1

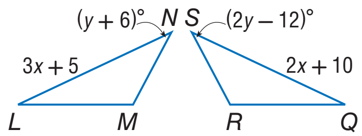
C 2

D 3

E 4

Spiral Review

In the diagram, $\triangle LMN \cong \triangle QRS$. (Lesson 4-3)



38. Find x.

39. Find y.

40. ASTRONOMY The Big Dipper is a part of the larger constellation Ursa Major. Three of the brighter stars in the constellation form $\triangle RSA$. If $m\angle R = 41$ and $m\angle S = 109$, find $m\angle A$. (Lesson 4-2)

Write an equation in slope-intercept form for each line. (Lesson 3-4)

41. $(-5, -3)$ and $(10, -6)$

42. $(4, -1)$ and $(-2, -1)$

43. $(-4, -1)$ and $(-8, -5)$

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample. (Lesson 2-3)

44. If $x^2 = 25$, then $x = 5$.

45. If you are 16, you are a junior in high school.

Skills Review

State the property that justifies each statement.

46. $AB = AB$

47. If $EF = GH$ and $GH = JK$, then $EF = JK$.

48. If $a^2 = b^2 - c^2$, then $b^2 - c^2 = a^2$.

49. If $XY + 20 = YW$ and $XY + 20 = DT$, then $YW = DT$.

EXTEND 4-4 Geometry Lab
Proving Constructions

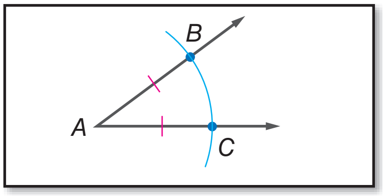
When you perform a construction using a straightedge and compass, you assume that segments constructed using the same compass setting are congruent. You can use this information, along with definitions, postulates, and theorems to prove constructions.

CCSS Common Core State Standards
Content Standards
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Mathematical Practices 3, 5

Activity

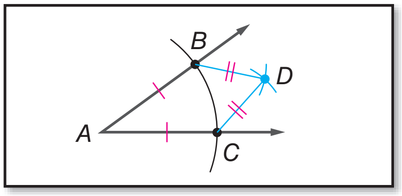
Follow the steps below to bisect an angle. Then prove the construction.

Step 1



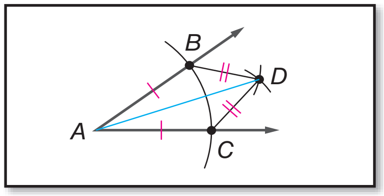
Draw any angle with vertex A. Place the compass point at A and draw an arc that intersects both sides of $\angle A$. Label the points B and C. Mark the congruent segments.

Step 2



With the compass point at B, draw an arc in the interior of $\angle A$. With the same radius, draw an arc from C intersecting the first arc at D. Draw the segments $\overline{BD}$ and $\overline{CD}$. Mark the congruent segments.

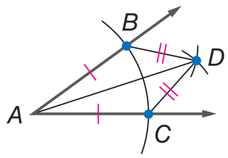
Step 3



Draw $\overline{AD}$.

Given: Description of steps and diagram of construction

Prove: $\overline{AD}$ bisects $\angle BAC$.



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	**1.** The same compass setting was used from point A to construct points B and C.
2. $\overline{BD} \cong \overline{CD}$	**2.** The same compass setting was used from points B and C to construct point D.
3. $\overline{AD} \cong \overline{AD}$	**3.** Reflexive Property
4. $\triangle ABD \cong \triangle ACD$	**4.** SSS Postulate
5. $\angle BAD \cong \angle CAD$	**5.** CPCTC
6. $\overline{AD}$ bisects $\angle BAC$.	**6.** Definition of angle bisector

Exercises

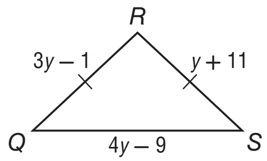
1. Construct a line parallel to a given line through a given point. Write a two-column proof of your construction.
2. Construct an equilateral triangle. Write a paragraph proof of your construction.
3. **CHALLENGE** Construct the bisector of a segment that is also perpendicular to the segment and write a two-column proof of your construction. (*Hint:* You will need to use more than one pair of congruent triangles.).

CHAPTER 4 Mid-Chapter Quiz

Lessons 4-1 through 4-4

1. **COORDINATE GEOMETRY** Classify $\triangle ABC$ with vertices $A(-2, -1)$, $B(-1, 3)$, and $C(2, 0)$ as *scalene*, *equilateral*, or *isosceles*. (Lesson 4-1)

2. **MULTIPLE CHOICE** Which of the following are the measures of the sides of isosceles triangle QRS? (Lesson 4-1)



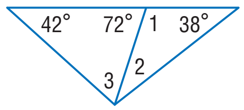
A 17, 17, 15 **C** 14, 15, 14

B 15, 15, 16 **D** 14, 14, 16

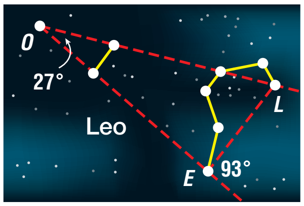
3. **ALGEBRA** Find x and the length of each side if $\triangle WXY$ is an equilateral triangle with sides $\overline{WX} = 6x - 12$, $\overline{XY} = 2x + 10$, and $\overline{WY} = 4x - 1$. (Lesson 4-1)

Find the measure of each angle indicated. (Lesson 4-2)

4. $m\angle 1$
5. $m\angle 2$
6. $m\angle 3$

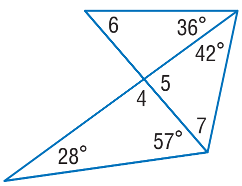


7. **ASTRONOMY** Leo is a constellation that represents a lion. Three of the brighter stars in the constellation form $\triangle LEO$. If the angles have measures as shown in the figure, find $m\angle OLE$. (Lesson 4-2)

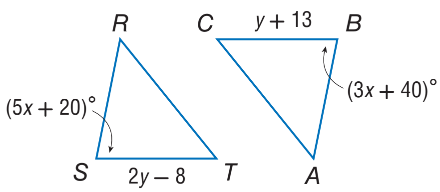


Find the measure of each numbered angle. (Lesson 4-2)

8. $m\angle 4$
9. $m\angle 5$
10. $m\angle 6$
11. $m\angle 7$



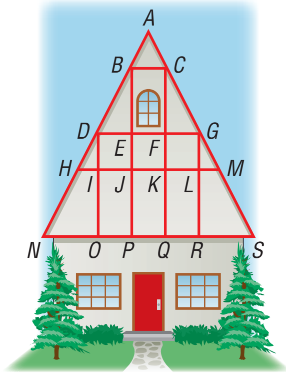
In the diagram, $\triangle RST \cong \triangle ABC$. (Lesson 4-3)



12. Find x.

13. Find y.

14. **ARCHITECTURE** The diagram shows an A-frame house with various points labeled. Assume that segments and angles that appear to be congruent in the diagram are congruent. Indicate which triangles are congruent. (Lesson 4-3)

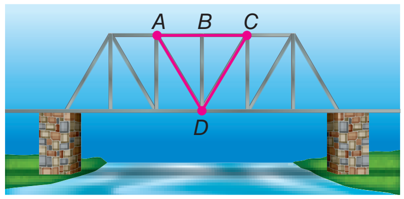


15. **MULTIPLE CHOICE** Determine which statement is true given that $\triangle CBX \cong \triangle SML$. (Lesson 4-3)

F $\overline{MO} \cong \overline{SL}$ **H** $\angle X \cong \angle S$

G $\overline{XC} \cong \overline{ML}$ **J** $\angle XCB \cong \angle LSM$

16. **BRIDGES** A bridge truss is shown in the diagram below, where $\overline{AC} \perp \overline{BD}$ and B is the midpoint of $\overline{AC}$. What method can be used to prove that $\triangle ABD \cong \triangle CBD$? (Lesson 4-4)



Determine whether $\triangle PQR \cong \triangle XYZ$. (Lesson 4-4)

17. $P(3, -5)$, $Q(11, 0)$, $R(1, 6)$, $X(5, 1)$, $Y(13, 6)$, $Z(3, 12)$

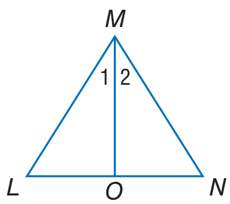
18. $P(-3, -3)$, $Q(-5, 1)$, $R(-2, 6)$, $X(2, -6)$, $Y(3, 3)$, $Z(5, -1)$

19. $P(8, 1)$, $Q(-7, -15)$, $R(9, -6)$, $X(5, 11)$, $Y(-10, -5)$, $Z(6, 4)$

20. **Write a two-column proof.** (Lesson 4-4)

Given: $\triangle LMN$ is isos. with $\overline{LM} \cong \overline{NM}$, and $\overline{MO}$ bisects $\angle LMN$.

Prove: $\triangle MLO \cong \triangle MNO$



LESSON 4-5

Proving Triangles Congruent—ASA, AAS

SSS ✓
SAS ✓
ASA ✓
AAS ✓
~~AAA~~
~~SSA~~

: : Then

- You proved triangles congruent using SSS and SAS.

AAA

: : Now

1. Use the ASA Postulate to test for congruence.
2. Use the AAS Theorem to test for congruence.

S S A S A

HL

: : Why?

- Competitive sweep rowing, also called *crew*, involves two or more people who sit facing the stern of the boat, with each rower pulling one oar. In high school competitions, a race, called a *regatta*, usually requires a body of water that is more than 1500 meters long. Congruent triangles can be used to measure distances that are not easily measured directly, like the length of a regatta course.

NewVocabulary
included side

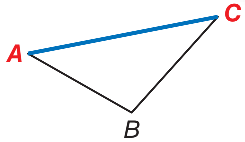
CCSS Common Core State Standards

Content Standards
G.CO.10 Prove theorems about triangles.
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
5 Use appropriate tools strategically.

Ben Blankenburg/CORBIS

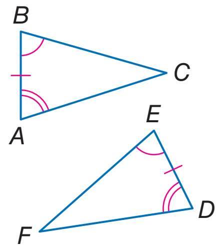
1 ASA Postulate An **included side** is the side located between two consecutive angles of a polygon. In $\triangle ABC$ at the right, $\overline{AC}$ is the included side between $\angle A$ and $\angle C$.



Postulate 4.3 Angle-Side-Angle (ASA) Congruence

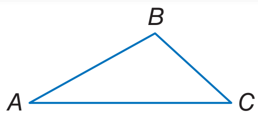
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example If **A**ngle $\angle A \cong \angle D$,
Side $\overline{AB} \cong \overline{DE}$, and
Angle $\angle B \cong \angle E$,
then $\triangle ABC \cong \triangle DEF$.



Construction Congruent Triangles Using Two Angles and Included Side

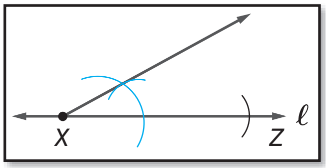
Draw a triangle and label it $\triangle ABC$. Then use the ASA Postulate to construct $\triangle XYZ \cong \triangle ABC$.



Step 1

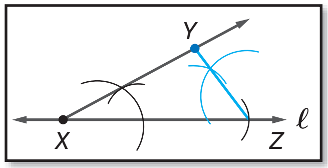
Draw a line ℓ and select a point X. Construct $\overline{XZ}$ such that $\overline{XZ} \cong \overline{AC}$.

Step 2



Construct an angle congruent to $\angle A$ at X using $\overleftrightarrow{XZ}$ as a side of the angle.

Step 3



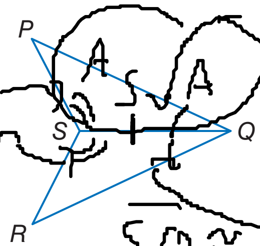
Construct an angle congruent to $\angle C$ at Z using $\overleftrightarrow{XZ}$ as a side of the angle. Label the point where the new sides of the angles meet as Y.

Example 1 Use ASA to Prove Triangles Congruent

Write a two-column proof.

Given: $\overline{QS}$ bisects $\angle PQR$;
$\angle PSQ \cong \angle RSQ$.

Prove: $\triangle PQS \cong \triangle RQS$



Given

A S A

Def of bis.

$\overline{SQ} \cong \overline{SQ}$

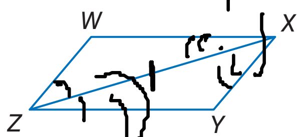
Proof:

Statements	Reasons
1. $\overline{QS}$ bisects $\angle PQR$; $\angle PSQ \cong \angle RSQ$.	1. Given
2. $\angle PQS \cong \angle RQS$	2. Definition of Angle Bisector
3. $\overline{QS} \cong \overline{QS}$	3. Reflexive Property of Congruence
4. $\triangle PQS \cong \triangle RQS$	4. ASA

Def of bis

GuidedPractice

1. Write a flow proof.
Given: $\overline{ZX}$ bisects $\angle WZY$; $\overline{XZ}$ bisects $\angle YXW$.
Prove: $\triangle WXZ \cong \triangle XZY$



$\overline{ZX} \cong \overline{ZX}$

RPOC

Def of bis.

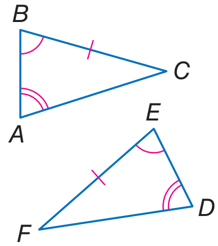
2 AAS Theorem

The congruence of two angles and a nonincluded side are also sufficient to prove two triangles congruent. This congruence relationship is a theorem because it can be proved using the Third Angles Theorem.

Theorem 4.5 Angle-Angle-Side (AAS) Congruence

If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

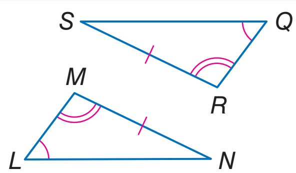
Example If **A**ngle $\angle A \cong \angle D$,
Angle $\angle B \cong \angle E$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Proof Angle-Angle-Side Theorem

Given: $\angle L \cong \angle Q$, $\angle M \cong \angle R$, $\overline{MN} \cong \overline{RS}$

Prove: $\triangle LMN \cong \triangle QRS$



Proof:

$\angle L \cong \angle Q$ — Given

$\angle M \cong \angle R$ — Given

$\overline{MN} \cong \overline{RS}$ — Given

$\angle N \cong \angle S$ — Third Angles Theorem

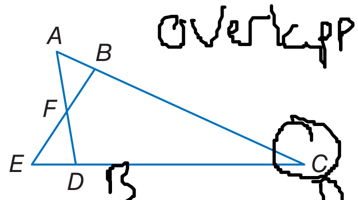
$\triangle LMN \cong \triangle QRS$ — AAS

Example 2 Use AAS to Prove Triangles Congruent

Write a two-column proof.

Given: $\angle DAC \cong \angle BEC$
$\overline{DC} \cong \overline{BC}$

Prove: $\triangle ACD \cong \triangle ECB$



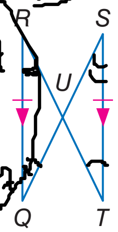
∠C ≅ ∠C
overlapping

Proof: We are given that $\angle DAC \cong \angle BEC$ and $\overline{DC} \cong \overline{BC}$. $\angle C \cong \angle C$ by the Reflexive Property. By AAS, $\triangle ACD \cong \triangle ECB$.

GuidedPractice

2. Write a flow proof.
Given: $\overline{RQ} \cong \overline{ST}$ and $\overline{RQ} \parallel \overline{ST}$
Prove: $\triangle RUQ \cong \triangle TUS$

AIA



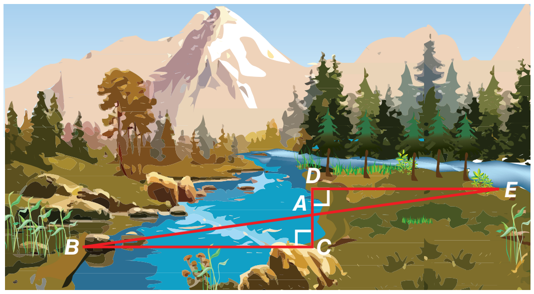
flipped
turned

∠R ≅ ∠T
∠Q ≅ ∠S

You can use congruent triangles to measure distances that are difficult to measure directly.

Real-World Example 3 Apply Triangle Congruence

COMMUNITY SERVICE **Jeremias is working with a community service group to build a bridge across a creek at a local park. The bridge will span the creek between points *C* and *B*. Jeremias located a fixed point *D* to use as a reference point so that the segments have the relationships shown. *A* is the midpoint of $\overline{CD}$ and *DE* is 15 feet. How long does the bridge need to be?**



In order to determine the length of $\overline{CB}$, we must first prove that the two triangles Jeremias has created are congruent.

- Since $\overline{CD}$ is perpendicular to both $\overline{CB}$ and $\overline{DE}$, the segments form right angles as shown on the diagram.
- All right angles are congruent, so $\angle BCA \cong \angle EDA$.
- Point A is the midpoint of $\overline{CD}$, so $\overline{CA} \cong \overline{AD}$.
- $\angle BAC$ and $\angle EAD$ are vertical angles, so they are congruent.

Therefore, by ASA, $\triangle BAC \cong \triangle EAD$.

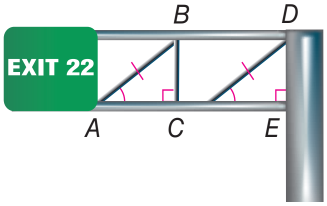
Since $\triangle BAC \cong \triangle EAD$, $\overline{DE} \cong \overline{CB}$ by CPCTC. Since the measure of $\overline{DE}$ is 15 feet, the measure of $\overline{CB}$ is also 15 feet. Therefore, the bridge needs to be 15 feet long.

StudyTip

Angle-Angle-Angle In Example 3, $\angle B$ and $\angle E$ are congruent by the Third Angles Theorem. Congruence of all three corresponding angles is not sufficient, however, to prove two triangles congruent.

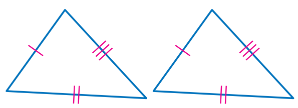
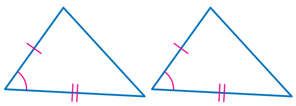
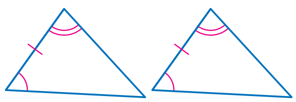
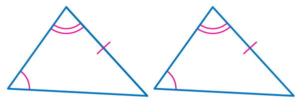
GuidedPractice

3. In the sign scaffold shown at the right, $\overline{BC} \perp \overline{AC}$ and $\overline{DE} \perp \overline{CE}$. $\angle BAC \cong \angle DCE$, and $\overline{AB} \cong \overline{CD}$. Write a paragraph proof to show that $\overline{BC} \cong \overline{DE}$.



You have learned several methods for proving triangle congruence.

ConceptSummary Proving Triangles Congruent

SSS	SAS	ASA	AAS
			
Three pairs of corresponding sides are congruent.	Two pairs of corresponding sides and their included angles are congruent.	Two pairs of corresponding angles and their included sides are congruent.	Two pairs of corresponding angles and the corresponding nonincluded sides are congruent.

Check Your Understanding

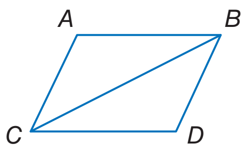
= Step-by-Step Solutions begin on page R14.

Example 1 **PROOF** **Write the specified type of proof.**

1. two-column proof

Given: $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$.

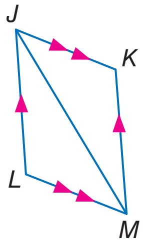
Prove: $\triangle ABC \cong \triangle DBC$



2. flow proof

Given: $\overline{JK} \parallel \overline{LM}$, $\overline{JL} \parallel \overline{KM}$

Prove: $\triangle JML \cong \triangle MJK$

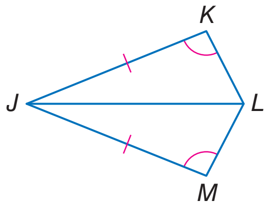


Example 2

3. paragraph proof

Given: $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, $\overline{JL}$ bisects $\angle KLM$.

Prove: $\triangle JKL \cong \triangle JML$

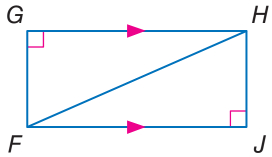


4. two-column proof

Given: $\overline{GH} \parallel \overline{FJ}$

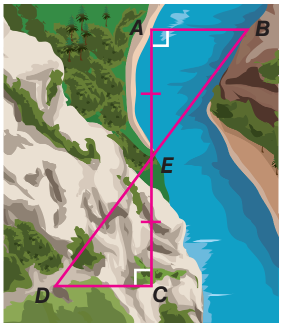
$m\angle G = m\angle J = 90$

Prove: $\triangle HJF \cong \triangle FGH$



Example 3

5. **BRIDGE BUILDING** A surveyor needs to find the distance from point A to point B across a canyon. She places a stake at A, and a coworker places a stake at B on the other side of the canyon. The surveyor then locates C on the same side of the canyon as A such that $\overline{CA} \perp \overline{AB}$. A fourth stake is placed at E, the midpoint of $\overline{CA}$. Finally, a stake is placed at D such that $\overline{CD} \perp \overline{CA}$ and D, E, and B are sited as lying along the same line.



a. Explain how the surveyor can use the triangles formed to find AB.

b. If $AC = 1300$ meters, $DC = 550$ meters, and $DE = 851.5$ meters, what is AB? Explain your reasoning.

Practice and Problem Solving

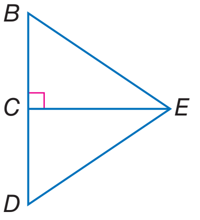
Extra Practice is on page R4.

Example 1

PROOF **Write a paragraph proof.**

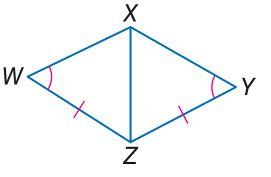
6. **Given:** $\overline{CE}$ bisects $\angle BED$; $\angle BCE$ and $\angle ECD$ are right angles.

Prove: $\triangle ECB \cong \triangle ECD$

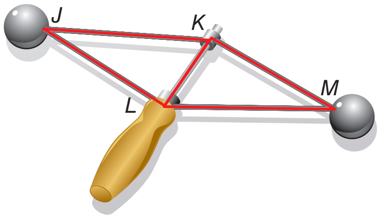


7. **Given:** $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ}$ bisects $\angle WZY$.

Prove: $\triangle XWZ \cong \triangle XYZ$



8. **TOYS** The object of the toy shown is to make the two spheres meet and strike each other repeatedly on one side of the wand and then again on the other side. If $\angle JKL \cong \angle MLK$ and $\angle JLK \cong \angle MKL$, prove that $\overline{JK} \cong \overline{ML}$.

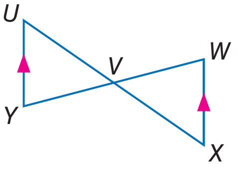


Example 2

PROOF **Write a two-column proof.**

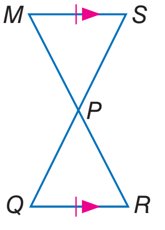
9. **Given:** V is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$.

Prove: $\triangle UVY \cong \triangle XVW$



10. **Given:** $\overline{MS} \cong \overline{RQ}$, $\overline{MS} \parallel \overline{RQ}$

Prove: $\triangle MSP \cong \triangle RQP$

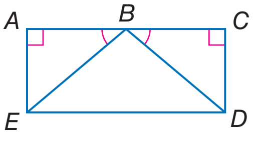


11. **CCSS ARGUMENTS** Write a flow proof.

Given: $\angle A$ and $\angle C$ are right angles.

$\angle ABE \cong \angle CBD$, $\overline{AE} \cong \overline{CD}$

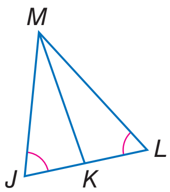
Prove: $\overline{BE} \cong \overline{BD}$



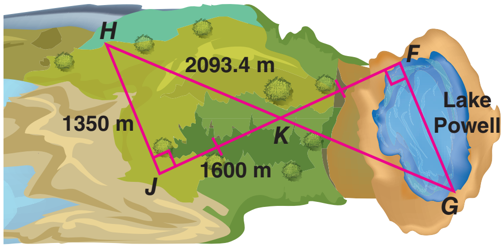
12. **PROOF** Write a flow proof.

Given: $\overline{KM}$ bisects $\angle JML$; $\angle J \cong \angle L$.

Prove: $\overline{JM} \cong \overline{LM}$



Example 3 **13.** **CCSS MODELING** A high school wants to hold a 1500-meter regatta on Lake Powell but is unsure if the lake is long enough. To measure the distance across the lake, the crew members locate the vertices of the triangles below and find the measures of the lengths of $\triangle HJK$ as shown below.

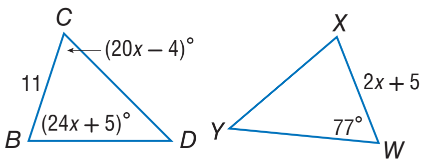


a. Explain how the crew team can use the triangles formed to estimate the distance FG across the lake.

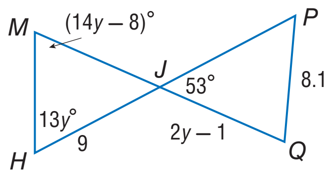
b. Using the measures given, is the lake long enough for the team to use as the location for their regatta? Explain your reasoning.

ALGEBRA **Find the value of the variable that yields congruent triangles.**

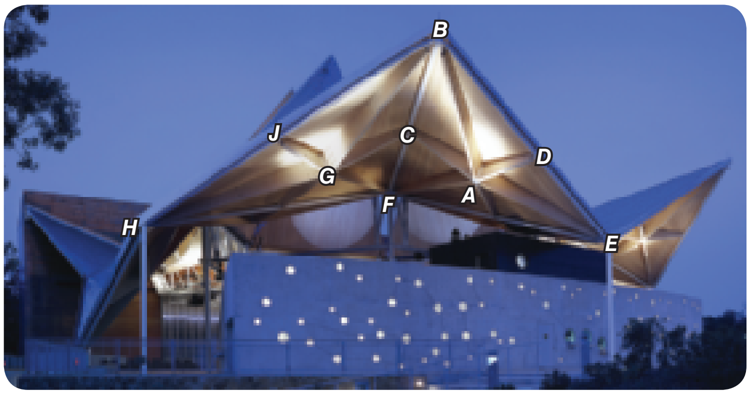
14. $\triangle BCD \cong \triangle WXY$



15 $\triangle MHJ \cong \triangle PQJ$



16. **THEATER DESIGN** The trusses of the roof of the outdoor theater shown below appear to be several different pairs of congruent triangles. Assume that trusses that appear to lie on the same line actually lie on the same line.



a. If $\overline{AB}$ bisects $\angle CBD$ and $\angle CAD$, prove that $\triangle ABC \cong \triangle ABD$.

b. If $\triangle ABC \cong \triangle ABD$ and $\angle FCA \cong \angle EDA$, prove that $\triangle CAF \cong \triangle DAE$.

c. If $\overline{HB} \cong \overline{EB}$, $\angle BHG \cong \angle BEA$, $\angle HGJ \cong \angle EAD$, and $\angle JGB \cong \angle DAB$, prove that $\triangle BHG \cong \triangle BEA$.

Greg Murphey

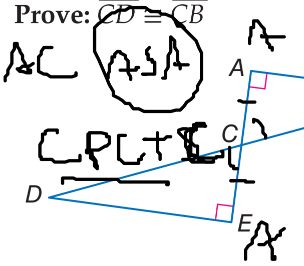
PROOF **Write a paragraph proof.**

17. **Given:** $\overline{AE} \perp \overline{DE}$, $\overline{EA} \perp \overline{AB}$, C is the midpoint of $\overline{AE}$.

Prove: $\overline{CD} \cong \overline{CB}$

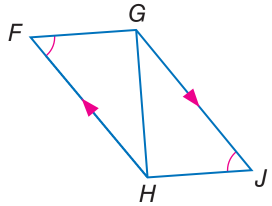
$\triangle DCE \cong \triangle BAC$ ASA

$\overline{CD} \cong \overline{CB}$ CPCTC



18. **Given:** $\angle F \cong \angle J$, $\overline{FH} \parallel \overline{GJ}$

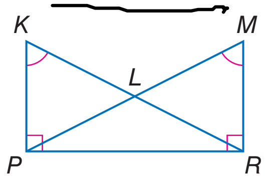
Prove: $\overline{FH} \cong \overline{JG}$



PROOF **Write a two-column proof.**

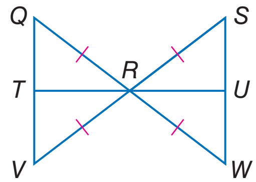
19. **Given:** $\angle K \cong \angle M$, $\overline{KP} \perp \overline{PR}$, $\overline{MR} \perp \overline{PR}$

Prove: $\angle KPL \cong \angle MRL$

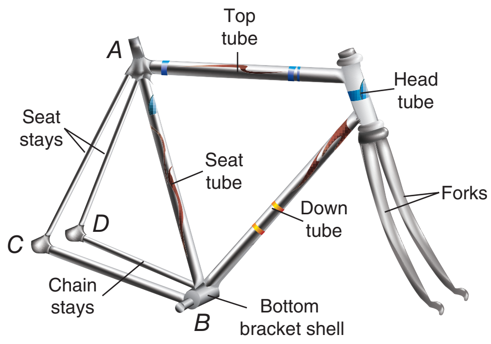


20. **Given:** $\overline{QR} \cong \overline{SR} \cong \overline{WR} \cong \overline{VR}$

Prove: $\overline{QT} \cong \overline{WU}$



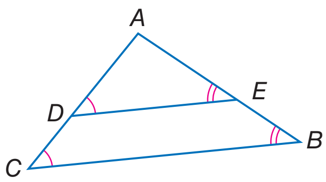
21 **FITNESS** The seat tube of a bicycle forms a triangle with each seat and chain stay as shown. If each seat stay makes a 44° angle with its corresponding chain stay and each chain stay makes a 68° angle with the seat tube, show that the two seat stays are the same length.



H.O.T. Problems Use Higher-Order Thinking Skills

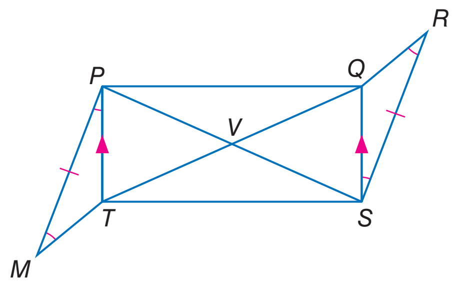
22. **OPEN ENDED** Draw and label two triangles that could be proved congruent by ASA.

23. **CCSS CRITIQUE** Tyrone says it is not possible to show that $\triangle ADE \cong \triangle ACB$. Lorenzo disagrees, explaining that since $\angle ADE \cong \angle ACB$, and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ADE \cong \triangle ACB$. Is either of them correct? Explain.



24. **REASONING** Find a counterexample to show why SSA (Side-Side-Angle) cannot be used to prove the congruence of two triangles.

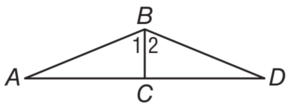
25. **CHALLENGE** Using the information given in the diagram, write a flow proof to show that $\triangle PVQ \cong \triangle SVT$.



26. **WRITING IN MATH** How do you know what method (SSS, SAS, etc.) to use when proving triangle congruence? Use a chart to explain your reasoning.

Standardized Test Practice

27. Given: $\overline{BC}$ is perpendicular to $\overline{AD}$; $\angle 1 \cong \angle 2$.



Which theorem or postulate could be used to prove $\triangle ABC \cong \triangle DBC$?

A AAS
B ASA
C SAS
D SSS

28. SHORT RESPONSE Write an expression that can be used to find the values of $s(n)$ in the table.

n	-8	-4	-1	0	1
$s(n)$	1.00	2.00	2.75	3.00	3.25

29. ALGEBRA If -7 is multiplied by a number greater than 1, which of the following describes the result?

F a number greater than 7
G a number between -7 and 7
H a number greater than -7
J a number less than -7

30. SAT/ACT $\sqrt{121 + 104} = ?$

A 15
B 21
C 25
D 125
E 225

Spiral Review

Determine whether $\triangle ABC \cong \triangle XYZ$. Explain. (Lesson 4-4)

31. $A(6, 4)$, $B(1, -6)$, $C(-9, 5)$,
$X(0, 7)$, $Y(5, -3)$, $Z(15, 8)$

32. $A(0, 5)$, $B(0, 0)$, $C(-2, 0)$,
$X(4, 8)$, $Y(4, 3)$, $Z(6, 3)$

33. ALGEBRA If $\triangle RST \cong \triangle JKL$, $RS = 7$, $ST = 5$, $RT = 9 + x$, $JL = 2x - 10$, and $JK = 4y - 5$, draw and label a figure to represent the congruent triangles. Then find x and y. (Lesson 4-3)

34. FINANCIAL LITERACY Maxine charges \$5 to paint a mailbox and \$4 per hour to mow a lawn. Write an equation to represent the amount of money Maxine can earn from a homeowner who has his or her mailbox painted and lawn mowed. (Lesson 3-4)

Copy and complete each truth table. (Lesson 2-2)

35.

p	q	$\sim p$	$\sim p \vee q$
F	T		
T	T		
F	F		
T	F		

36.

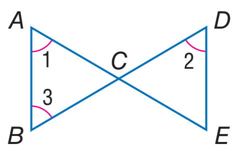
p	q	$\sim q$	$\sim q \wedge p$
F		F	
T		T	
T		F	
F		T	

Skills Review

PROOF **Write a two-column proof for each of the following.**

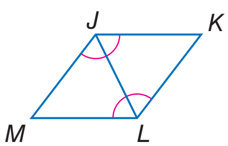
37. Given: $\angle 2 \cong \angle 1$
$\angle 1 \cong \angle 3$

Prove: $\overline{AB} \parallel \overline{DE}$



38. Given: $\angle MJK \cong \angle KLM$
$\angle LMJ$ and $\angle KLM$ are supplementary.

Prove: $\overline{KJ} \parallel \overline{LM}$



EXTEND 4-5

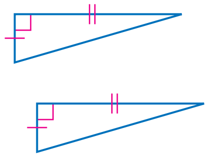
Geometry Lab
Congruence in Right Triangles

In Lessons 4-4 and 4-5, you learned theorems and postulates to prove triangles congruent. How do these theorems and postulates apply to right triangles?

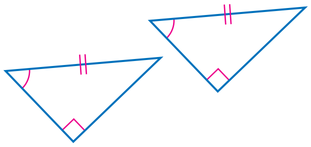
CCSS Common Core State Standards
Content Standards
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Mathematical Practices 5

Study each pair of right triangles.

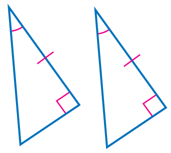
a.



b.



c.



Analyze

1. Is each pair of triangles congruent? If so, which congruence theorem or postulate applies?

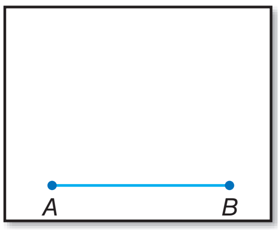
2. Rewrite the congruence rules from Exercise 1 using *leg*, (L), or *hypotenuse*, (H), to replace *side*. Omit the *A* for any right angle since we know that all right triangles contain a right angle and all right angles are congruent.

3. **MAKE A CONJECTURE** If you know that the corresponding legs of two right triangles are congruent, what other information do you need to declare the triangles congruent? Explain.

In Lesson 4-5, you learned that SSA is not a valid test for determining triangle congruence. Can SSA be used to prove right triangles congruent?

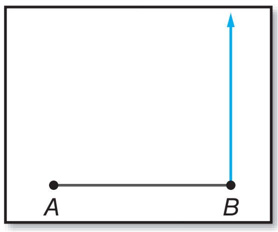
Activity SSA and Right Triangles

Step 1



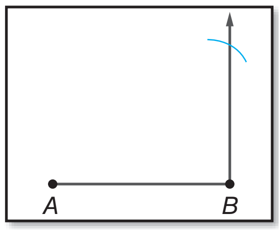
Draw $\overline{AB}$ so that $AB = 6$ centimeters.

Step 2



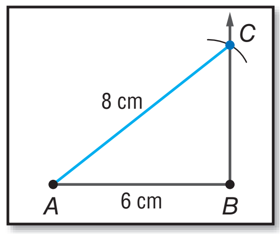
Use a protractor to draw a ray from *B* that is perpendicular to $\overline{AB}$.

Step 3



Open your compass to a width of 8 centimeters. Place the point at *A* and draw an arc to intersect the ray.

Step 4



Label the intersection *C* and draw $\overline{AC}$ to complete $\triangle ABC$.

Analyze

4. Does the model yield a unique triangle?

5. Can you use the lengths of the hypotenuse and a leg to show right triangles are congruent?

6. **Make a conjecture** about the case of SSA that exists for right triangles.

(continued on the next page)

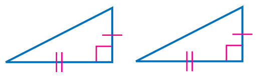
Geometry Lab
Congruence in Right Triangles *Continued*

Your work on the previous page provides evidence for four ways to prove right triangles congruent.

Theorem Right Triangle Congruence

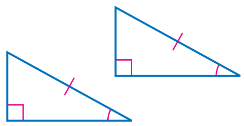
Theorem 4.6 Leg-Leg Congruence
If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.

Abbreviation *LL*



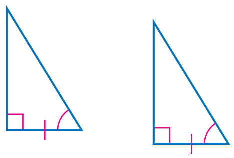
Theorem 4.7 Hypotenuse-Angle Congruence
If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.

Abbreviation *HA*



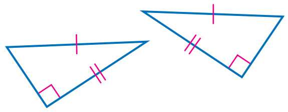
Theorem 4.8 Leg-Angle Congruence
If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.

Abbreviation *LA*



Theorem 4.9 Hypotenuse-Leg Congruence
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

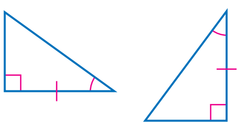
Abbreviation *HL*



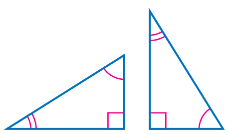
Exercises

Determine whether each pair of triangles is congruent. If yes, tell which postulate or theorem applies.

7.



8.



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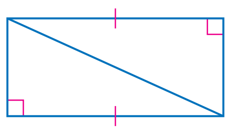


PROOF **Write a proof for each of the following.**

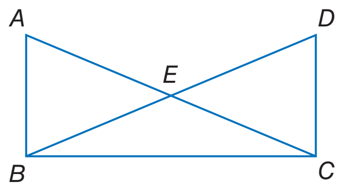
10. Theorem 4.6

11. Theorem 4.7

12. Theorem 4.8 (*Hint*: There are two possible cases.)

13. Theorem 4.9 (*Hint*: Use the Pythagorean Theorem.)

Use the figure at the right.



14. Given: $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$
$\overline{AC} \cong \overline{BD}$

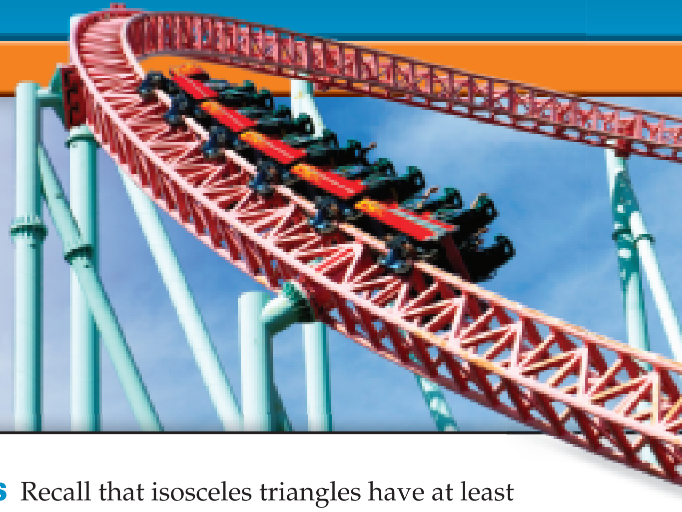
Prove: $\overline{AB} \cong \overline{DC}$

15. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \perp \overline{BC}$
E is the midpoint of $\overline{AC}$ and $\overline{BD}$.

Prove: $\overline{AC} \cong \overline{DB}$

LESSON 4-6

Isosceles and Equilateral Triangles



Then	Now	Why?
You identified isosceles and equilateral triangles.	1 Use properties of isosceles triangles. 2 Use properties of equilateral triangles.	The tracks on the roller coaster have triangular reinforcements between the tracks for support and stability. The triangle supports in the photo are isosceles triangles.

NewVocabulary
legs of an isosceles triangle
vertex angle
base angles

Common Core State Standards

Content Standards
G.CO.10 Prove theorems about triangles.
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

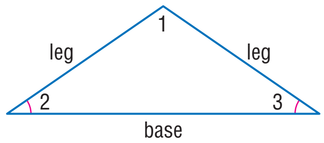
Mathematical Practices
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.

1 Properties of Isosceles Triangles

Recall that isosceles triangles have at least two congruent sides. The parts of an isosceles triangle have special names.

The two congruent sides are called the **legs of an isosceles triangle**, and the angle with sides that are the legs is called the **vertex angle**. The side of the triangle opposite the vertex angle is called the *base*. The two angles formed by the base and the congruent sides are called the **base angles**.

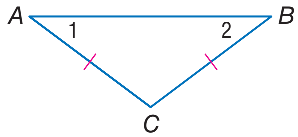
$\angle 1$ is the vertex angle.
$\angle 2$ and $\angle 3$ are the base angles.



Theorems Isosceles Triangle

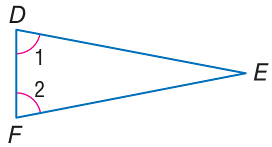
4.10 Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.



4.11 Converse of Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\overline{FE} \cong \overline{DE}$.

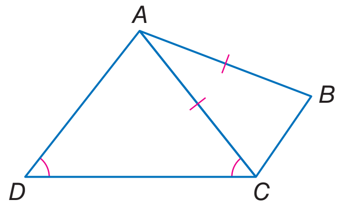


You will prove Theorem 4.11 in Exercise 37.

Example 1 Congruent Segments and Angles

a. Name two unmarked congruent angles.
$\angle ACB$ is opposite $\overline{AB}$ and $\angle B$ is opposite $\overline{AC}$, so $\angle ACB \cong \angle B$.

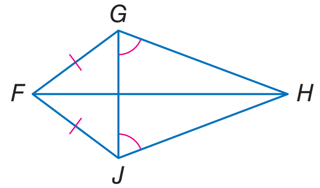
b. Name two unmarked congruent segments.
$\overline{AD}$ is opposite $\angle ACD$ and $\overline{AC}$ is opposite $\angle D$, so $\overline{AD} \cong \overline{AC}$.



GuidedPractice

1A. Name two unmarked congruent angles.

1B. Name two unmarked congruent segments.

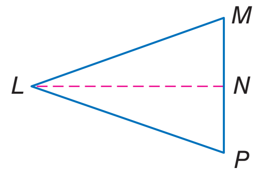


To prove the Isosceles Triangle Theorem, draw an auxiliary line and use the two triangles formed.

Proof Isosceles Triangle Theorem

Given: $\triangle LMP$; $\overline{LM} \cong \overline{LP}$

Prove: $\angle M \cong \angle P$



Proof:

Statements	Reasons
1. Let N be the midpoint of $\overline{MP}$.	**1.** Every segment has exactly one midpoint.
2. Draw an auxiliary segment $\overline{LN}$.	**2.** Two points determine a line.
3. $\overline{MN} \cong \overline{PN}$	**3.** Midpoint Theorem
4. $\overline{LN} \cong \overline{LN}$	**4.** Reflexive Property of Congruence
5. $\overline{LM} \cong \overline{LP}$	**5.** Given
6. $\triangle LMN \cong \triangle LPN$	**6.** SSS
7. $\angle M \cong \angle P$	**7.** CPCTC

2 Properties of Equilateral Triangles

The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

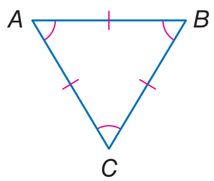
ReviewVocabulary

equilateral triangle a triangle with three congruent sides

Corollaries Equilateral Triangle

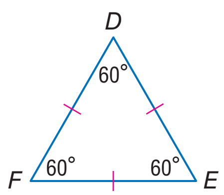
4.3 A triangle is equilateral if and only if it is equiangular.

Example If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.



4.4 Each angle of an equilateral triangle measures 60.

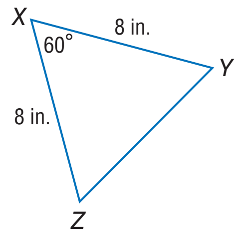
Example If $\overline{DE} \cong \overline{EF} \cong \overline{FE}$, then $m\angle A = m\angle B = m\angle C = 60$.



You will prove Corollaries 4.3 and 4.4 in Exercises 35 and 36.

Example 2 Find Missing Measures

Find each measure.



a. $m\angle Y$

Since $XY = XZ$, $\overline{XY} \cong \overline{XZ}$. By the Isosceles Triangle Theorem, base angles Z and Y are congruent, so $m\angle Z = m\angle Y$. Use the Triangle Sum Theorem to write and solve an equation to find $m\angle Y$.

$m\angle X + m\angle Y + m\angle Z = 180$	Triangle Sum Theorem
$60 + m\angle Y + m\angle Y = 180$	$m\angle X = 60$, $m\angle Z = m\angle Y$
$60 + 2(m\angle Y) = 180$	Simplify.
$2(m\angle Y) = 120$	Subtract 60 from each side.
$m\angle Y = 60$	Divide each side by 2.

b. YZ

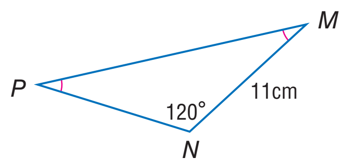
$m\angle Z = m\angle Y$, so $m\angle Z = 60$ by substitution. Since $m\angle X = 60$, all three angles measure 60, so the triangle is equiangular. Because an equiangular triangle is also equilateral, $XY = XZ = ZY$. Since $XY = 8$ inches, $YZ = 8$ inches by substitution.

Study Tip

Isosceles Triangles As you discovered in Example 2, any isosceles triangle that has one 60° angle must be an equilateral triangle.

Guided Practice

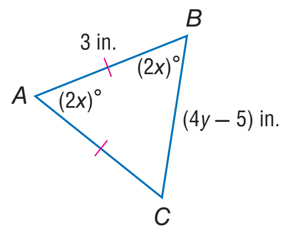
2A. $m\angle M$ **2B.** PN



You can use the properties of equilateral triangles and algebra to find missing values.

Example 3 Find Missing Values

ALGEBRA **Find the value of each variable.**



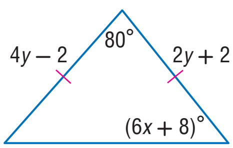
Since $\angle B = \angle A$, $\overline{AC} \cong \overline{BC}$ by the Converse of the Isosceles Triangle Theorem. All of the sides of the triangle are congruent, so the triangle is equilateral. Each angle of an equilateral triangle measures 60°, so $2x = 60$ and $x = 30$.

The triangle is equilateral, so all of the sides are congruent, and the lengths of all of the sides are equal.

$AB = BC$	Definition of equilateral triangle
$3 = 4y - 5$	Substitution
$8 = 4y$	Add 5 to each side.
$2 = y$	Divide each side by 4.

Guided Practice

3. Find the value of each variable.



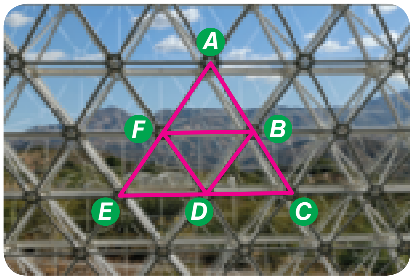
Real-WorldLink

Biosphere II is the largest totally enclosed ecosystem ever built, covering 3.14 acres in Oracle, Arizona. The controlled-environment facility is 91 feet at its highest point, and it has 6500 windows that enclose a volume of 7.2 million cubic feet.

Source: University of Arizona

Real-World Example 4 Apply Triangle Congruence

ENVIRONMENT **Refer to the photo of Biosphere II at the right. $\triangle ACE$ is an equilateral triangle. F is the midpoint of $\overline{AE}$, D is the midpoint of $\overline{EC}$, and B is the midpoint of $\overline{CA}$. Prove that $\triangle FBD$ is also equilateral.**



Given: $\triangle ACE$ is equilateral. F is the midpoint of $\overline{AE}$, D is the midpoint of $\overline{EC}$, and B is the midpoint of $\overline{CA}$.

Prove: $\triangle FBD$ is equilateral.

Proof:

Statements	Reasons
1. $\triangle ACE$ is equilateral.	**1.** Given
2. F is the midpoint of AE, D is the midpoint of EC, and B is the midpoint of CA.	**2.** Given
3. $m\angle A = 60$, $m\angle C = 60$, $m\angle E = 60$	**3.** Each angle of an equilateral triangle measures 60.
4. $\angle A \cong \angle C \cong \angle E$	**4.** Definition of congruence and substitution
5. $\overline{AE} \cong \overline{EC} \cong \overline{CA}$	**5.** Definition of equilateral triangle
6. $AE = EC = CA$	**6.** Definition of congruence
7. $\overline{AF} \cong \overline{FE}$, $\overline{ED} \cong \overline{DC}$, $\overline{CB} \cong \overline{BA}$	**7.** Midpoint Theorem
8. $AF = FE$, $ED = DC$, $CB = BA$	**8.** Definition of congruence
9. $AF + FE = AE$, $ED + DC = EC$, $CB + BA = CA$	**9.** Segment Addition Postulate
10. $AF + AF = AE$, $FE + FE = AE$, $ED + ED = EC$, $DC + DC = EC$, $CB + CB = CA$, $BA + BA = CA$	**10.** Substitution
11. $2AF = AE$, $2FE = AE$, $2ED = EC$, $2DC = EC$, $2CB = CA$, $2BA = CA$	**11.** Addition Property
12. $2AF = AE$, $2FE = AE$, $2ED = AE$, $2DC = AE$, $2CB = AE$, $2BA = AE$	**12.** Substitution Property
13. $2AF = 2ED = 2CB$, $2FE = 2DC = 2BA$	**13.** Transitive Property
14. $AF = ED = CB$, $FE = DC = BA$	**14.** Division Property
15. $\overline{AF} \cong \overline{ED} \cong \overline{CB}$, $\overline{FE} \cong \overline{DC} \cong \overline{BA}$	**15.** Definition of congruence
16. $\triangle AFB \cong \triangle EDF \cong \triangle CBD$	**16.** SAS
17. $\overline{DF} \cong \overline{FB} \cong \overline{BD}$	**17.** CPCTC
18. $\triangle FBD$ is equilateral.	**18.** Definition of equilateral triangle

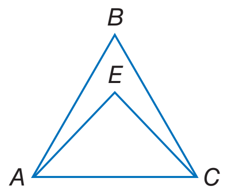
GuidedPractice

4. Given that $\triangle ACE$ is equilateral, $\overline{FB} \parallel \overline{EC}$, $\overline{FD} \parallel \overline{BC}$, $\overline{BD} \parallel \overline{EF}$, and D is the midpoint of $\overline{EC}$, prove that $\triangle FED \cong \triangle BDC$.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Example 1 **Refer to the figure at the right.**

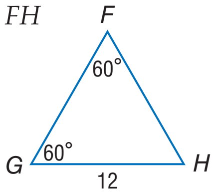


1. If $\overline{AB} \cong \overline{CB}$, name two congruent angles.

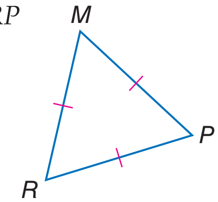
2. If $\angle EAC \cong \angle ECA$, name two congruent segments.

Example 2 **Find each measure.**

3. FH

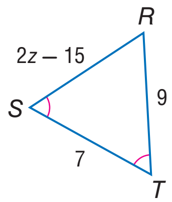


4. $m\angle MRP$

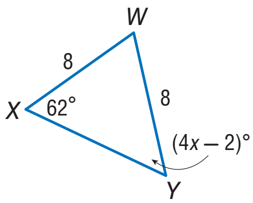


Example 3 CCSS **SENSE-MAKING** **Find the value of each variable.**

5.



6.

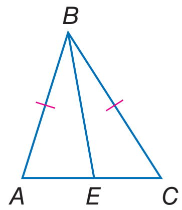


Example 4

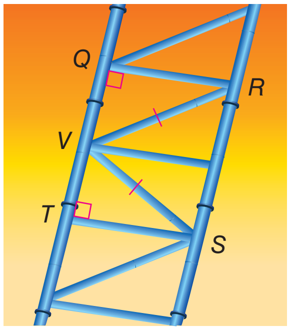
7. **PROOF** **Write a two-column proof.**

Given: $\triangle ABC$ is isosceles; $\overline{EB}$ bisects $\angle ABC$.

Prove: $\triangle ABE \cong \triangle CBE$



8. **ROLLER COASTERS** The roller coaster track shown in the photo on page 285 appears to be composed of congruent triangles. A portion of the track is shown.



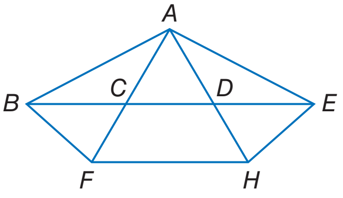
a. If $\overline{QR}$ and $\overline{ST}$ are perpendicular to $\overline{QT}$, $\triangle VSR$ is isosceles with base $\overline{SR}$, and $\overline{QT} \parallel \overline{SR}$, prove that $\triangle RQV \cong \triangle STV$.

b. If $VR = 2.5$ meters and $QR = 2$ meters, find the distance between $\overline{QR}$ and $\overline{ST}$. Explain your reasoning.

Practice and Problem Solving

Extra Practice is on page R4.

Example 1 **Refer to the figure at the right.**



9. If $\overline{AB} \cong \overline{AE}$, name two congruent angles.

10. If $\angle ABF \cong \angle AFB$, name two congruent segments.

11. If $\overline{CA} \cong \overline{DA}$, name two congruent angles.

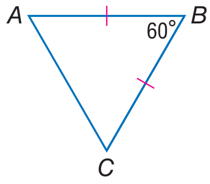
12. If $\angle DAE \cong \angle DEA$, name two congruent segments.

13. If $\angle BCF \cong \angle BFC$, name two congruent segments.

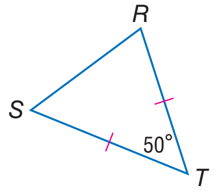
14. If $\overline{FA} \cong \overline{AH}$, name two congruent angles.

Example 2 **Find each measure.**

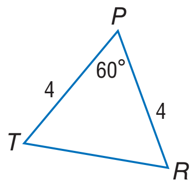
15. $m\angle BAC$



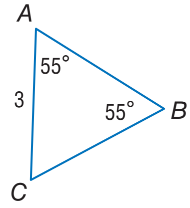
16. $m\angle SRT$



17. TR

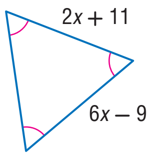


18. CB



Example 3 **CCSS REGULARITY** **Find the value of each variable.**

19



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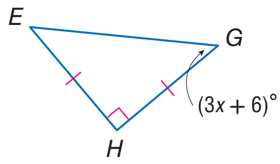

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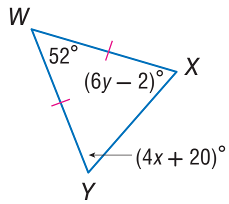

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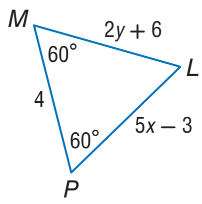
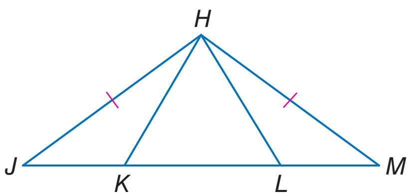


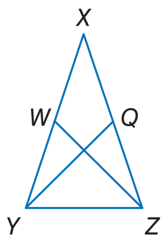
Example 4 **PROOF** **Write a paragraph proof.**

23. Given: $\triangle HJM$ is isosceles, and $\triangle HKL$ is equilateral. $\angle JKH$ and $\angle HKL$ are supplementary and $\angle HLK$ and $\angle MLH$ are supplementary.

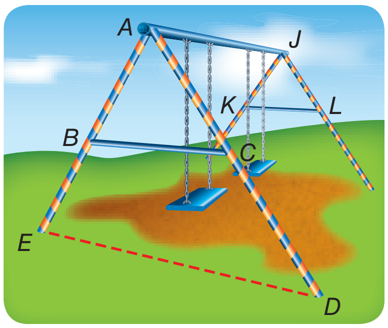
Prove: $\angle JHK \cong \angle MHL$



24. Given: $\overline{XY} \cong \overline{XZ}$
W is the midpoint of $\overline{XY}$.
Q is the midpoint of $\overline{XZ}$.
Prove: $\overline{WZ} \cong \overline{QY}$



25. BABYSITTING While babysitting her neighbor's children, Elisa observes that the supports on either side of a park swing set form two sets of triangles. Using a jump rope to measure, Elisa is able to determine that $\overline{AB} \cong \overline{AC}$, but $\overline{BC} \ncong \overline{AB}$.



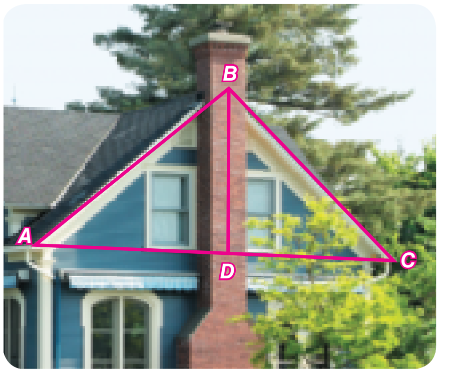
a. Elisa estimates $m\angle BAC$ to be 50. Based on this estimate, what is $m\angle ABC$? Explain.

b. If $\overline{BE} \cong \overline{CD}$, show that $\triangle AED$ is isosceles.

c. If $\overline{BC} \parallel \overline{ED}$ and $\overline{ED} \cong \overline{AD}$, show that $\triangle AED$ is equilateral.

d. If $\triangle JKL$ is isosceles, what is the minimum information needed to prove that $\triangle ABC \cong \triangle JLK$? Explain your reasoning.

26. **CHIMNEYS** In the picture, $\overline{BD} \perp \overline{AC}$ and $\triangle ABC$ is an isosceles triangle with base $\overline{AC}$. Show that the chimney of the house, represented by $\overline{BD}$, bisects the angle formed by the sloped sides of the roof, $\angle ABC$.



27. **CONSTRUCTION** Construct three different isosceles right triangles. Explain your method. Then verify your constructions using measurement and mathematics.

28. **PROOF** Based on your construction in Exercise 27, make and prove a conjecture about the relationship between the base angles of an isosceles right triangle.

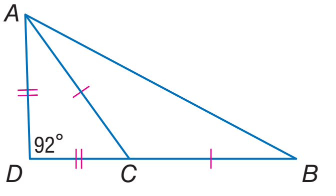
CCSS REGULARITY **Find each measure.**

29. $m\angle CAD$

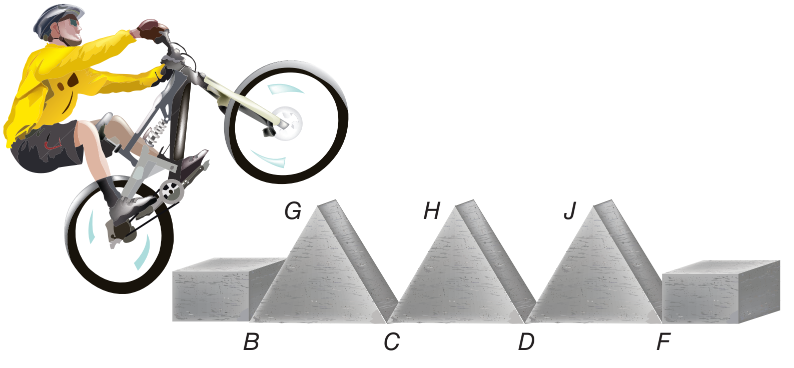
30. $m\angle ACD$

31. $m\angle ACB$

32. $m\angle ABC$

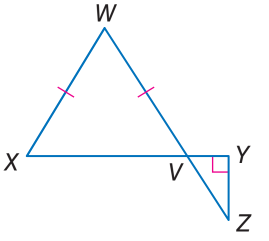


33. **FITNESS** In the diagram, the rider will use his bike to hop across the tops of each of the concrete solids shown. If each triangle is isosceles with vertex angles G, H, and J, and $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$, show that the distance from B to F is three times the distance from D to F.



34. **Given:** $\triangle XWV$ is isosceles; $\overline{ZY} \perp \overline{YV}$.

Prove: $\angle X$ and $\angle YZV$ are complementary.



PROOF **Write a two-column proof of each corollary or theorem.**

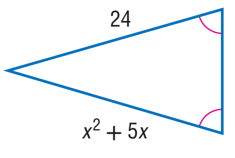
35. Corollary 4.3

36. Corollary 4.4

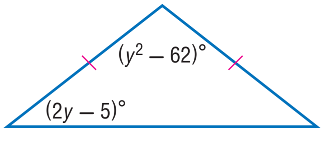
37. Theorem 4.11

Find the value of each variable.

38.



39.



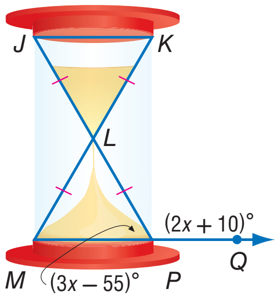
GAMES **Use the diagram of a game timer shown to find each measure.**

40. $m\angle LPM$

41. $m\angle LMP$

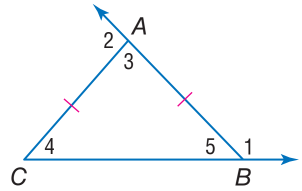
42. $m\angle JLK$

43. $m\angle JKL$



44. **MULTIPLE REPRESENTATIONS** In this problem, you will explore possible measures of the interior angles of an isosceles triangle given the measure of one exterior angle.

a. Geometric Use a ruler and a protractor to draw three different isosceles triangles, extending one of the sides adjacent to the vertex angle and to one of the base angles, and labeling as shown.



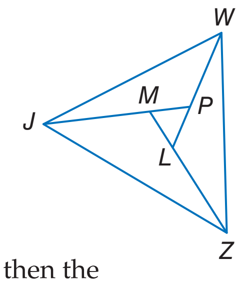
b. Tabular Use a protractor to measure and record $m\angle 1$ for each triangle. Use $m\angle 1$ to calculate the measures of $\angle 3$, $\angle 4$, and $\angle 5$. Then find and record $m\angle 2$ and use it to calculate these same measures. Organize your results in two tables.

c. Verbal Explain how you used $m\angle 1$ to find the measures of $\angle 3$, $\angle 4$, and $\angle 5$. Then explain how you used $m\angle 2$ to find these same measures.

d. Algebraic If $m\angle 1 = x$, write an expression for the measures of $\angle 3$, $\angle 4$, and $\angle 5$. Likewise, if $m\angle 2 = x$, write an expression for these same angle measures.

H.O.T. Problems Use Higher-Order Thinking Skills

45. **CHALLENGE** In the figure at the right, if $\triangle WJZ$ is equilateral and $\angle ZWP \cong \angle WJM \cong \angle JZL$, prove that $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$.

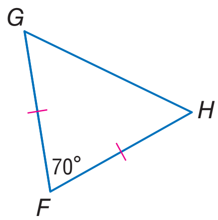


CCSS PRECISION **Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain.**

46. If the measure of the vertex angle of an isosceles triangle is an integer, then the measure of each base angle is an integer.

47. If the measures of the base angles of an isosceles triangle are integers, then the measure of its vertex angle is odd.

48. **ERROR ANALYSIS** Alexis and Miguela are finding $m\angle G$ in the figure shown. Alexis says that $m\angle G = 35$, while Miguela says that $m\angle G = 60$. Is either of them correct? Explain your reasoning.



49. **OPEN ENDED** If possible, draw an isosceles triangle with base angles that are obtuse. If it is not possible, explain why not.

50. **REASONING** In isosceles $\triangle ABC$, $m\angle B = 90$. Draw the triangle. Indicate the congruent sides and label each angle with its measure.

51. **WRITING IN MATH** How can triangle classifications help you prove triangle congruence?

Standardized Test Practice

52. ALGEBRA What quantity should be added to both sides of this equation to complete the square?

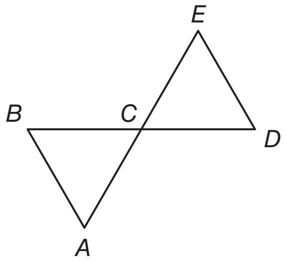
$$x^2 - 10x = 3$$

A -25 **C** 5

B -5 **D** 25

53. SHORT RESPONSE In a school of 375 students, 150 students play sports and 70 students are involved in the community service club. 30 students play sports and are involved in the community service club. How many students are *not* involved in either sports or the community service club?

54. In the figure $\overline{AE}$ and $\overline{BD}$ bisect each other at point C.



Which additional piece of information would be enough to prove that $\overline{DE} \cong \overline{DC}$?

F $\angle A \cong \angle BCA$ **H** $\angle ACB \cong \angle EDC$

G $\angle B \cong \angle D$ **J** $\angle A \cong \angle B$

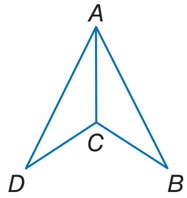
55. SAT/ACT If $x = -3$, then $4x^2 - 7x + 5 =$

A 2 **C** 20 **E** 62

B 14 **D** 42

Spiral Review

56. If $m\angle ADC = 35$, $m\angle ABC = 35$, $m\angle DAC = 26$, and $m\angle BAC = 26$, determine whether $\triangle ADC \cong \triangle ABC$. (Lesson 4-5)

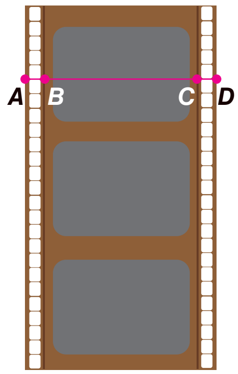


Determine whether $\triangle STU \cong \triangle XYZ$. Explain. (Lesson 4-4)

57. $S(0, 5)$, $T(0, 0)$, $U(1, 1)$, $X(4, 8)$, $Y(4, 3)$, $Z(6, 3)$

58. $S(2, 2)$, $T(4, 6)$, $U(3, 1)$, $X(-2, -2)$, $Y(-4, 6)$, $Z(-3, 1)$

59. PHOTOGRAPHY Film is fed through a traditional camera by gears that catch the perforation in the film. The distance from A to C is the same as the distance from B to D. Show that the two perforated strips are the same width. (Lesson 2-7)



State the property that justifies each statement. (Lesson 2-6)

60. If $x(y + z) = a$, then $xy + xz = a$.

61. If $n - 17 = 39$, then $n = 56$.

62. If $m\angle P + m\angle Q = 110$ and $m\angle R = 110$, then $m\angle P + m\angle Q = m\angle R$.

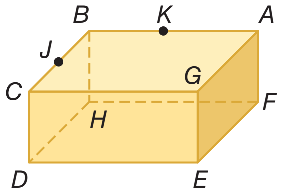
63. If $cv = md$ and $md = 15$, then $cv = 15$.

Refer to the figure at the right. (Lesson 1-1)

64. How many planes appear in this figure?

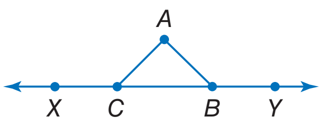
65. Name three points that are collinear.

66. Are points A, C, D, and J coplanar?



Skills Review

67. PROOF If $\angle ACB \cong \angle ABC$, then $\angle XCA \cong \angle YBA$.



EXPLORE 4-7

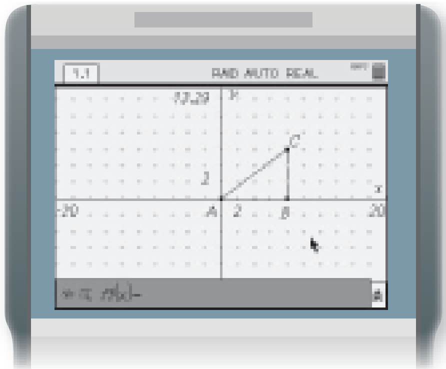
Graphing Technology Lab
Congruence Transformations

You can use TI-Nspire technology to perform *transformations* on triangles in the coordinate plane and test for congruence.

Common Core State Standards
Content Standards
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
Mathematical Practices 5

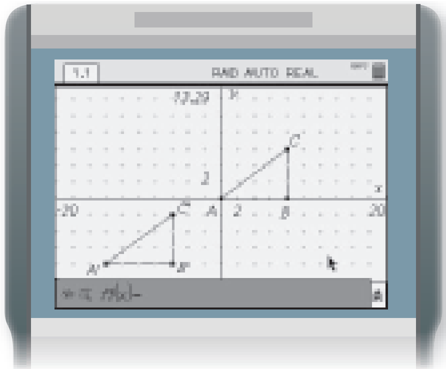
Activity 1 Translate a Triangle and Test for Congruence

Step 1 Open a new **Graphs** page. Select **Show Grid** from the **View** menu. Use the **Window/Zoom** menu to adjust the window size.

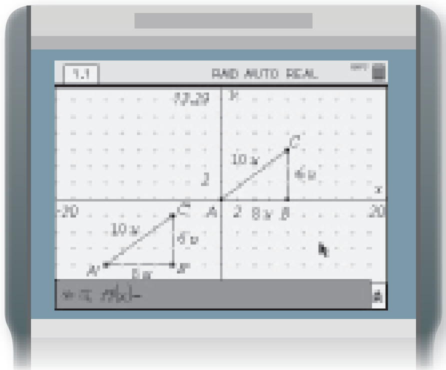


Step 2 Select **Triangle** from the **Shapes** menu and draw a right triangle with legs measuring 6 units and 8 units as shown by placing the first point at (0, 0), the second point at (8, 0), and the third point at (8, 6). Use the **Text** tool under the **Actions** menu to label the vertices of the triangle as A, B, and C.

Step 3 Select **Translation** from the **Transformation** menu. Then select $\triangle ABC$ and point A. Translate or *slide* the right triangle 8 units down and 14 units left. Label the corresponding vertices of the image as A', B', and C'.



Step 4 To verify that $\triangle A'B'C'$ is congruent to $\triangle ABC$, select **Length** from the **Measurement** menu. Then select any two endpoints and press the **ENTER** key to determine the length of the segment. Repeat this for each segment of each triangle.



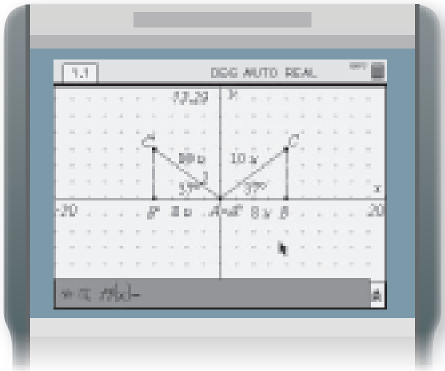
In addition to measuring lengths, the TI-Nspire can also be used to measure angles. This will allow you to use other tests for triangle congruence that involve angle measure.

Activity 2 Reflect a Triangle and Test for Congruence

Step 1 Open a new **Graphs** page, show the grid, and redraw $\triangle ABC$ from Activity 1.

Step 2 Select **Reflection** from the **Transformation** menu. Then select $\triangle ABC$ and then the y-axis to reflect or *flip* $\triangle ABC$ in the y-axis. Label the corresponding vertices of the image as A', B', and C'.

Step 3 Use the **Angle** tool from the **Measurement** menu to find $m\angle A$ and $m\angle A'$. Use the **Length** tool from the **Measurement** menu to find AB, $A'B'$, AC, and $A'C'$.



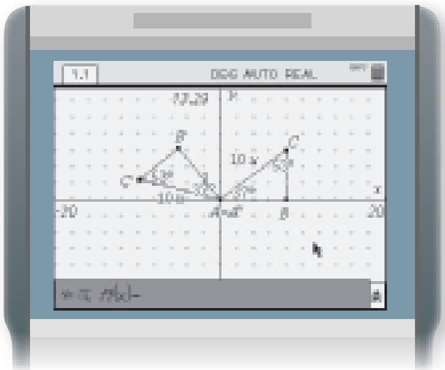
To rotate a figure about the origin using TI-Nspire technology, use the Rotation tool to select the figure, then the point (0, 0), then draw an angle of rotation.

Activity 3 Rotate a Triangle and Test for Congruence

Step 1 Open a new **Graphs** page, show the grid, and redraw $\triangle ABC$ from Activity 1.

Step 2 Select **Rotation** from the **Transformation** menu. Then select $\triangle ABC$, select the origin, and type in a number for the angle of rotation.

Step 3 Use the **Angle** tool from the **Measurement** menu to find $m\angle A$, $m\angle A'$, $m\angle C$, and $m\angle C'$. Use the **Length** tool from the **Measurement** menu to find AC and $A'C'$.



Analyze the Results

Determine whether $\triangle ABC$ and $\triangle A'B'C'$ are congruent. Explain your reasoning.

1. Activity 1

2. Activity 2

3. Activity 3

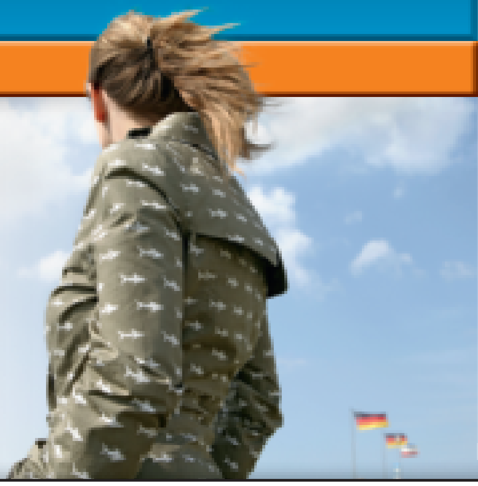
4. Explain why $\triangle A'B'C'$ in Activity 3 does not appear to be congruent to $\triangle ABC$.

5. **MAKE A CONJECTURE** Repeat Activities 1–3 using a different triangle XYZ. Analyze your results and compare them to those found in Exercises 1–3. Make a conjecture as to the relationship between a triangle and its transformed image under a translation, reflection, or a rotation.

6. Do the measurements and observations you made in Activities 1–3 constitute a proof of the conjecture you made in Exercise 5? Explain.

LESSON 4-7

Congruence Transformations



Then

- You proved whether two triangles were congruent.

Now

1. Identify reflections, translations, and rotations.
2. Verify congruence after a congruence transformation.

Why?

- The fashion industry often uses prints that display patterns. Many of these patterns are created by taking one figure and sliding it to create another figure in a different location, flipping the figure to create a mirror image of the original, or turning the original figure to create a new one.

New Vocabulary

transformation
preimage
image
congruence transformation
isometry
reflection
translation
rotation

Common Core State Standards

Content Standards

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Mathematical Practices

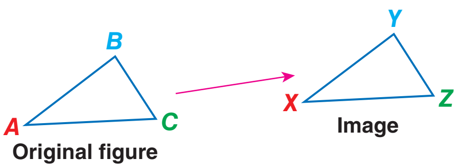
1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

1 Identify Congruence Transformations

A **transformation** is an operation that maps an original geometric figure, the **preimage**, onto a new figure called the **image**. A transformation can change the position, size, or shape of a figure.

A transformation can be noted using an arrow. The transformation statement $\triangle ABC \rightarrow \triangle XYZ$ tells you that A is mapped to X, B is mapped to Y, and C is mapped to Z.

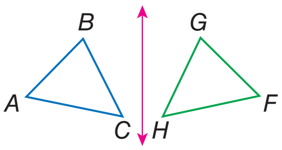


A **congruence transformation**, also called a *rigid transformation* or an **isometry**, is one in which the position of the image may differ from that of the preimage, but the two figures remain congruent. The three main types of congruence transformations are shown below.

KeyConcept Reflections, Translations, and Rotations

A **reflection** or *flip* is a transformation over a line called the *line of reflection.* Each point of the preimage and its image are the same distance from the line of reflection.

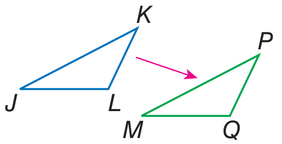
Example



$\triangle ABC \longrightarrow \triangle FGH$

A **translation** or *slide* is a transformation that moves all points of the original figure the same distance in the same direction.

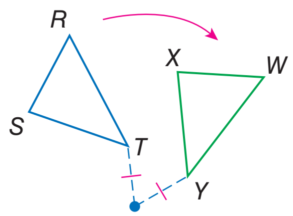
Example



$\triangle JKL \longrightarrow \triangle MPQ$

A **rotation** or *turn* is a transformation around a fixed point called the *center of rotation*, through a specific angle, and in a specific direction. Each point of the original figure and its image are the same distance from the center.

Example



$\triangle RST \longrightarrow \triangle WXY$

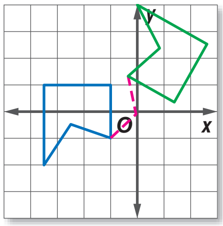
StudyTip

Transformations Not all transformations preserve congruence. Only transformations that do not change the size or shape of the figure are congruence transformations. You will learn more about transformations in Chapter 9.

Example 1 Identify Congruence Transformations

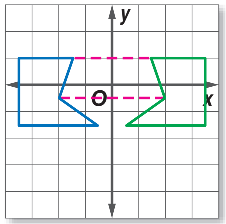
Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

a.



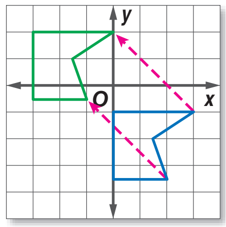
Each vertex and its image are the same distance from the origin. The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.

b.



Each vertex and its image are the same distance from the *y*-axis. This is a reflection.

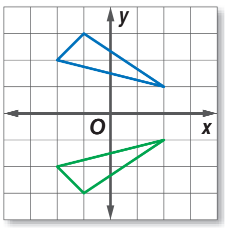
c.



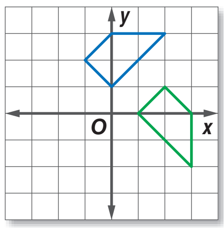
Each vertex and its image are in the same position, just 3 units left and 3 units up. This is a translation.

GuidedPractice

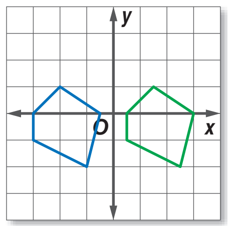
1A.



1B.



1C.



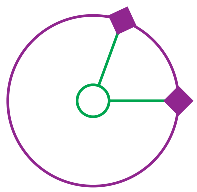
Some real-world motions or objects can be represented by transformations.

Real-WorldLink

The game shown above involves a weight attached to a ring that you can place around your ankle. As the rope passes in front of your other foot, you skip over it.

Real-World Example 2 Identify a Real-World Transformation

GAMES Refer to the information at the left. Identify the type of congruence transformation shown in the diagram as a *reflection, translation,* or *rotation.*

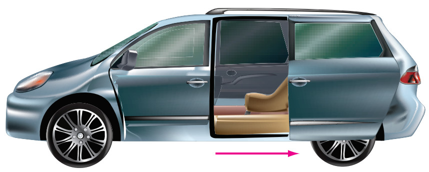


The position of the weight at different times is an example of a rotation. The center of rotation is the person's ankle.

GuidedPractice

Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

2A.



2B.

2 Verify Congruence

You can verify that reflections, translations, and rotations of triangles produce congruent triangles using SSS.

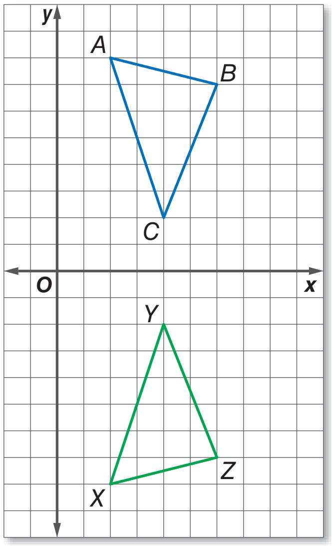
Example 3 Verify Congruence after a Transformation

Triangle XZY with vertices $X(2, -8)$, $Z(6, -7)$, and $Y(4, -2)$ is a transformation of $\triangle ABC$ with vertices $A(2, 8)$, $B(6, 7)$, and $C(4, 2)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Understand You are asked to identify the type of transformation—reflection, translation, or rotation. Then, you need to show that the two figures are congruent.

Plan Use the Distance Formula to find the measure of each side. Then show that the two triangles are congruent by SSS.

Solve Graph each figure. The transformation appears to be a reflection over the x-axis. Find the measures of the sides of each triangle.



$$AB = \sqrt{(6-2)^2 + (7-8)^2} \text{ or } \sqrt{17}$$

$$BC = \sqrt{(6-4)^2 + (7-2)^2} \text{ or } \sqrt{29}$$

$$AC = \sqrt{(4-2)^2 + (2-8)^2} \text{ or } \sqrt{40}$$

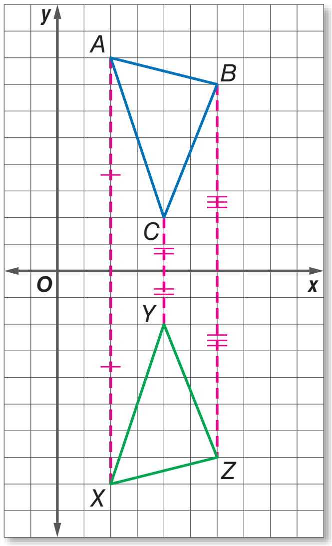
$$XZ = \sqrt{(6-2)^2 + [-7-(-8)]^2} \text{ or } \sqrt{17}$$

$$ZY = \sqrt{(6-4)^2 + [-7-(-2)]^2} \text{ or } \sqrt{29}$$

$$XY = \sqrt{(2-4)^2 + [-8-(-2)]^2} \text{ or } \sqrt{40}$$

Since $AB = XZ$, $BC = ZY$, and $AC = XY$, $\overline{AB} \cong \overline{XZ}$, $\overline{BC} \cong \overline{ZY}$, and $\overline{AC} \cong \overline{XY}$. By SSS, $\triangle ABC \cong \triangle XZY$.

Check Use the definition of a reflection. Use a ruler to measure and compare the segments connecting each vertex and its image to the line of symmetry. These segments are congruent, so the triangles are congruent. ✔



StudyTip

Isometry While an isometry preserves congruence, a *direct isometry* also preserves orientation or order of lettering. An *indirect* or *opposite isometry* changes this order, such as from clockwise to counterclockwise. The reflection shown in Example 3 is an example of an indirect isometry.

GuidedPractice

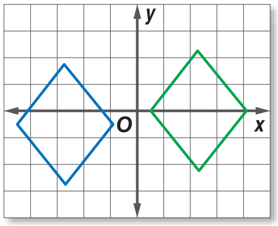
3. Triangle JKL with vertices $J(-2, 2)$, $K(-8, 5)$, and $L(-4, 6)$ is a transformation of $\triangle PQR$ with vertices $P(2, -2)$, $Q(8, -5)$, and $R(4, -6)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Check Your Understanding

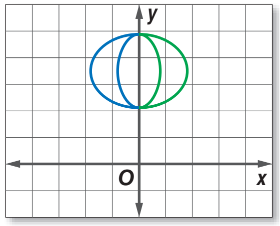
= Step-by-Step Solutions begin on page R14.

Example 1 Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

1.



2.



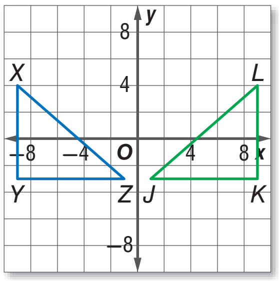
Example 2

3.

4.

Example 3 **COORDINATE GEOMETRY** Identify each transformation and verify that it is a congruence transformation.

5.



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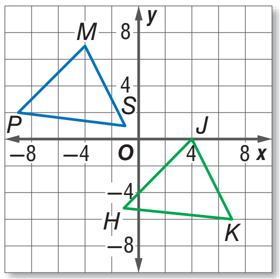
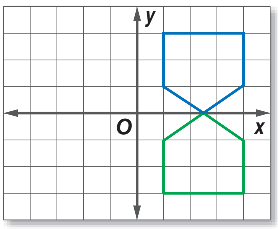


Practice and Problem Solving

Extra Practice is on page R4.

Example 1 CCSS **STRUCTURE** Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation.*

7.



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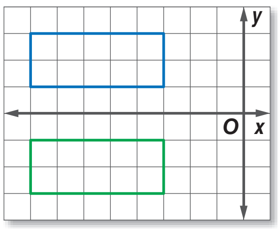

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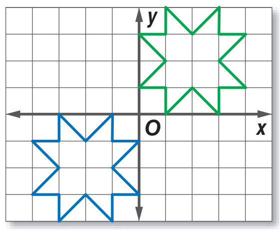

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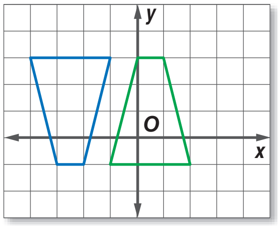

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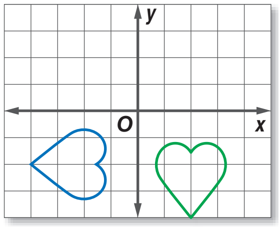

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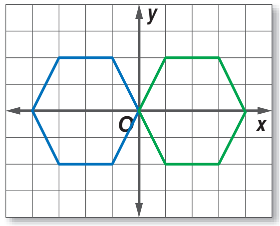
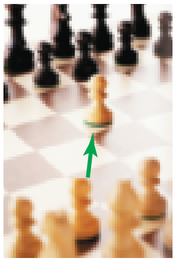


Example 2 **Identify the type of congruence transformation shown in each picture as a *reflection, translation,* or *rotation.***

13. 

14. 

15. 

16. 

Example 3 **COORDINATE GEOMETRY** **Graph each pair of triangles with the given vertices. Then, identify the transformation, and verify that it is a congruence transformation.**

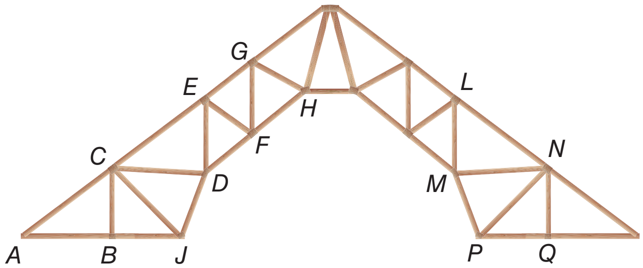
17. $M(-7, -1), P(-7, -7), R(-1, -4)$; $T(7, -1), V(7, -7), S(1, -4)$

18. $A(3, 9), B(3, 7), C(7, 7)$; $S(3, 5), T(3, 3), R(7, 3)$

19. $A(-4, 5), B(0, 2), C(-4, 2)$; $X(-5, -4), Y(-2, 0), Z(-2, -4)$

20. $A(2, 2), B(4, 7), C(6, 2)$; $D(2, -2), F(4, -7), G(6, -2)$

CONSTRUCTION **Identify the type of congruence transformation performed on each given triangle to generate the other triangle in the truss with matching left and right sides shown below.**



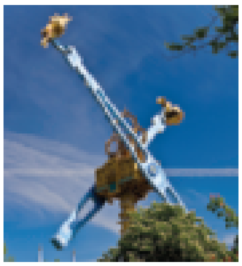
21. $\triangle NMP$ to $\triangle CJD$

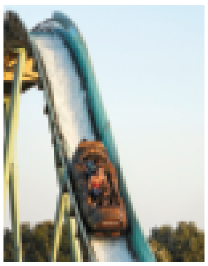
22. $\triangle EFD$ to $\triangle GHF$

23. $\triangle CBJ$ to $\triangle NQP$

AMUSEMENT RIDES **Identify the type of congruence transformation shown in each picture as a *reflection, translation,* or *rotation.***

24. 

25. 

26. 

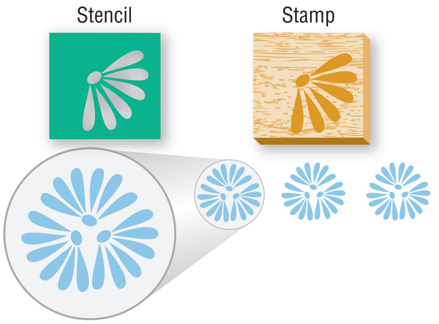
27. **SCHOOL** Identify the transformations that are used to open a combination lock on a locker. If appropriate, identify the line of symmetry or center of rotation.

28. CCSS **STRUCTURE** Determine which capital letters of the alphabet have vertical and/or horizontal lines of reflection.

29 **DECORATING** Tionne is redecorating her bedroom. She can use stencils or a stamp to create the design shown.

a. If Tionne used the stencil, what type of transformation was used to produce each flower in the design?

b. What type of transformation was used if she used the stamp to produce each flower in the design?



30. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the ordered pairs of a figure and its translated image.

a. **Geometric** Draw congruent rectangles *ABCD* and *WXYZ* on a coordinate plane.

b. **Verbal** How do you get from a vertex on *ABCD* to the corresponding vertex on *WXYZ* using only horizontal and vertical movement?

c. **Tabular** Copy the table shown. Use your rectangles to fill in the *x*-coordinates, the *y*-coordinates, and the unknown value in the transformation column.

Rectangle *ABCD*	Transformation	Rectangle *WXYZ*
A(?, ?)	$(x_1 + ?, y_1 + ?)$	*W*(?, ?)
B(?, ?)	$(x_1 + ?, y_1 + ?)$	*X*(?, ?)
C(?, ?)	$(x_1 + ?, y_1 + ?)$	*Y*(?, ?)
D(?, ?)	$(x_1 + ?, y_1 + ?)$	*Z*(?, ?)

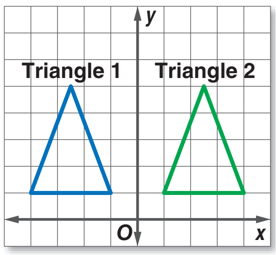
d. **Algebraic** Function notation $(x, y) \rightarrow (x + a, y + b)$, where a and b are real numbers, represents a mapping from one set of coordinates onto another. Complete the following notation that represents the rule for the translation $ABCD \rightarrow WXYZ$: $(x, y) \rightarrow (x + a, y + b)$.

H.O.T. Problems Use Higher-Order Thinking Skills

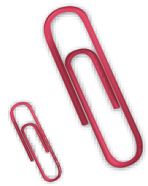
31. **CHALLENGE** Use the diagram at the right.

a. Identify two transformations of Triangle 1 that can result in Triangle 2.

b. What must be true of the triangles in order for more than one transformation on a preimage to result in the same image? Explain your reasoning.



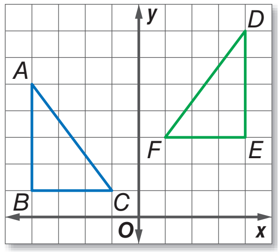
32. **CCSS REASONING** A *dilation* is another type of transformation. In the diagram, a small paper clip has been dilated to produce a larger paper clip. Explain why dilations are not a congruence transformation.



OPEN ENDED **Describe a real-world example of each of the following, other than those given in this lesson.**

33. reflection **34.** translation **35.** rotation

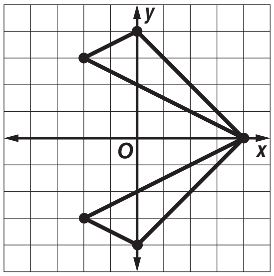
36. **WRITING IN MATH** In the diagram at the right $\triangle DEF$ is called a *glide reflection* of $\triangle ABC$. Based on the diagram, define a glide reflection. Is a glide reflection a congruence transformation? Include a definition of congruence transformation in your response. Explain your reasoning.



Standardized Test Practice

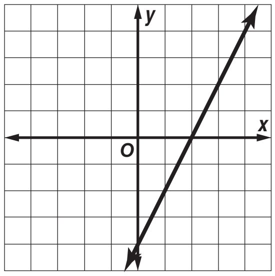
37. SHORT RESPONSE Cindy is shopping for a new desk chair at a store where the desk chairs are 50% off. She also has a coupon for 50% off any one item. Cindy thinks that she can now get the desk chair for free. Is this true? If not, what will be the percent off she will receive with both the sale and the coupon?

38. Identify the congruence transformation shown.



A dilation
B reflection
C rotation
D translation

39. Look at the graph below. What is the slope of the line shown?



F -2
G -1
H 1
J 2

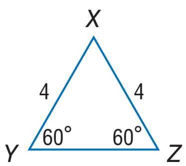
40. SAT/ACT What is the y-intercept of the line determined by the equation $3x - 4 = 12y - 3$?

A -12
B $-\frac{1}{12}$
C $\frac{1}{12}$
D $\frac{1}{4}$
E 12

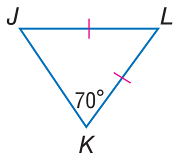
Spiral Review

Find each measure. (Lesson 4-6)

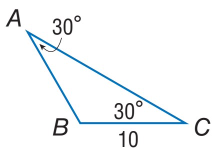
41. YZ



42. $m\angle JLK$



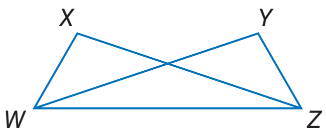
43. AB



44. PROOF Write a paragraph proof. (Lesson 4-5)

Given: $\angle YWZ \cong \angle XZW$ and $\angle YZW \cong \angle XWZ$

Prove: $\triangle WXZ \cong \triangle ZYW$



45. ROLLER COASTERS The sign in front of the Electric Storm roller coaster states that all riders must be at least 54 inches tall to ride. If Andy is 5 feet 8 inches tall, can he ride the Electric Storm? Which law of logic leads you to this conclusion? (Lesson 2-4)

Skills Review

Find the coordinates of the midpoint of a segment with the given endpoints.

46. $A(10, -12), C(5, -6)$

47. $A(13, 14), C(3, 5)$

48. $A(-28, 8), C(-10, 2)$

49. $A(-12, 2), C(-3, 5)$

50. $A(0, 0), C(3, -4)$

51. $A(2, 14), C(0, 5)$

LESSON 4-8 Triangles and Coordinate Proof

Then

- You used coordinate geometry to prove triangle congruence.

Now

1. Position and label triangles for use in coordinate proofs.
2. Write coordinate proofs.

Why?

- A global positioning system (GPS) receives transmissions from satellites that allow the exact location of a car to be determined. The information can be used with navigation software to provide driving directions.

NewVocabulary
coordinate proof

Common Core State Standards

Content Standards
G.CO.10 Prove theorems about triangles.
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

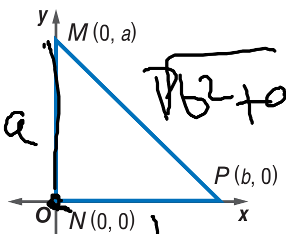
Mathematical Practices
3 Construct viable arguments and critique the reasoning of others.
2 Reason abstractly and quantitatively.

1 Position and Label Triangles

As with global positioning systems, knowing the coordinates of a figure in a coordinate plane allows you to explore its properties and draw conclusions about it. **Coordinate proofs** use figures in the coordinate plane and algebra to prove geometric concepts. The first step in a coordinate proof is placing the figure on the coordinate plane.

Example 1 Position and Label a Triangle

Position and label right triangle *MNP* on the coordinate plane so that leg $\overline{MN}$ is *a* units long and leg $\overline{NP}$ is *b* units long.



$\sqrt{b^2+a^2}$

a

b

- The length(s) of the side(s) that are along the axes will be easier to determine than the length(s) of side(s) that are not along an axis. Since this is a right triangle, two sides can be located on an axis.
- Placing the right angle of the triangle, ∠*N*, at the origin will allow the two legs to be along the *x*- and *y*-axes.

$\sqrt{(x_2-x_1)^2+(y_2-x_1)^2}$

- Position the triangle in the first quadrant.

$(b-0)^2+(0-a)^2=$

- Since *M* is on the *y*-axis, its *x*-coordinate is 0. Its *y*-coordinate is *a* because the leg is *a* units long.

$\sqrt{b^2+a^2} \neq b+a$

- Since *P* is on the *x*-axis, its *y*-coordinate is 0. Its *x*-coordinate is *b* because the leg is *b* units long.

GuidedPractice

1. Position and label isosceles triangle *JKL* on the coordinate plane so that its base $\overline{JL}$ is *a* units long, vertex *K* is on the *y*-axis, and the height of the triangle is *b* units.

KeyConcept Placing Triangles on Coordinate Plane

Step 1 Use the origin as a vertex or center of the triangle.

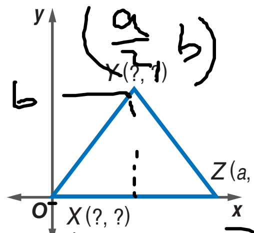
Step 2 Place at least one side of a triangle on an axis.

Step 3 Keep the triangle within the first quadrant if possible.

Step 4 Use coordinates that make computations as simple as possible.

Example 2 Identify Missing Coordinates

Name the missing coordinates of isosceles triangle *XYZ*.



$\left(\frac{a}{2}, b\right)$

b

X(0 0)

Z(a,0)

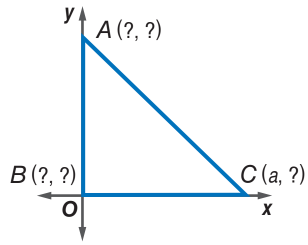
Vertex *X* is positioned at the origin; its coordinates are (0, 0).

Vertex *Z* is on the *x*-axis, so its *y*-coordinate is 0. The coordinates of vertex *Z* are (*a*, 0).

$\triangle XYZ$ is isosceles, so using a vertical segment from *Y* to to the *x*-axis and the Hypotenuse-Leg Theorem shows that the *x*-coordinate of *Y* is halfway between 0 and *a* or $\frac{a}{2}$. We cannot write the *y*-coordinate in terms of *a*, so call it *b*. The coordinates of point *Y* are $\left(\frac{a}{2}, b\right)$.

GuidedPractice

2. Name the missing coordinates of isosceles right triangle *ABC*.



B(0,0)
A(0,b)
C(a,0)

StudyTip

Right Angle The intersection of the *x*- and *y*-axis forms a right angle, so it is a convenient place to locate the right angle of a figure such as a right triangle.

2 Write Coordinate Proofs

After a triangle is placed on the coordinate plane and labeled, we can use coordinate proofs to verify properties and to prove theorems.

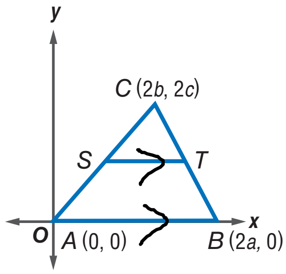
StudyTip

Coordinate Proof The guidelines and methods used in this lesson apply to all polygons, not just triangles.

Example 3 Write a Coordinate Proof

Write a coordinate proof to show that a line segment joining the midpoints of two sides of a triangle is parallel to the third side.

Place a vertex at the origin and label it *A*. Use coordinates that are multiples of 2 because the Midpoint Formula involves dividing the sum of the coordinates by 2.



Given: $\triangle ABC$
S is the midpoint of $\overline{AC}$.
T is the midpoint of $\overline{BC}$.

Prove: $\overline{ST} \parallel \overline{AB}$ • CAP Converse

or

Proof: • AIA converse
• slopes equal — $\frac{y_2 - y_1}{x_2 - x_1}$

By the Midpoint Formula, the coordinates of *S* are $\left(\frac{2b+0}{2}, \frac{2c+0}{2}\right)$ or (*b*, *c*) and the coordinates of *T* are $\left(\frac{2a+2b}{2}, \frac{0+2c}{2}\right)$ or (*a* + *b*, *c*).

By the Slope Formula, the slope of $\overline{ST}$ is $\frac{c-c}{a+b-b}$ or 0 and the slope of $\overline{AB}$ is $\frac{0-0}{2a-0}$ or 0.

Since $\overline{ST}$ and $\overline{AB}$ have the same slope, $\overline{ST} \parallel \overline{AB}$.

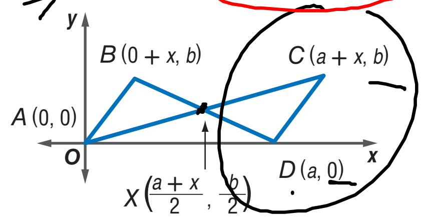
$CD = \sqrt{(a+x-a)^2 + (b-0)^2}$ Show $\overline{AB} \cong \overline{CD}$ $A(0,0)$ $B(x, b)$ $AB = \sqrt{(x-0)^2 + (b-0)^2}$

$\sqrt{x^2+b^2}$ $\sqrt{x^2+b^2}$

$\overline{AB} \cong \overline{CD}$

Guided Practice

3. Write a coordinate proof to show that $\triangle ABX \cong \triangle CDX$.



Use dist. form & show corr. sides are $\cong$

The techniques used for coordinate proofs can be used to solve real-world problems.

Real-World Link

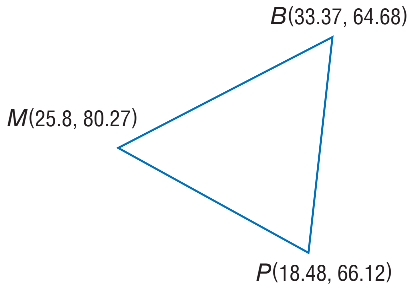
More than 50 ships and 20 airplanes have mysteriously disappeared from a section of the North Atlantic Ocean off of North America commonly referred to as the Bermuda Triangle.

Source: *Encyclopaedia Britannica*

Real-World Example 4 Classify Triangles

GEOGRAPHY The Bermuda Triangle is a region formed by Miami, Florida, San Jose, Puerto Rico, and Bermuda. The approximate coordinates of each location, respectively, are 25.8°N 80.27°W, 18.48°N 66.12°W, and 33.37°N 64.68°W. Write a coordinate proof to prove that the Bermuda Triangle is scalene.

The first step is to label the coordinates of each location. Let M represent Miami, B represent Bermuda, and P represent Puerto Rico.



If no two sides of $\triangle MPB$ are congruent, then the Bermuda Triangle is scalene. Use the Distance Formula and a calculator to find the distance between each location.

$$MB = \sqrt{(33.37 - 25.8)^2 + (64.68 - 80.27)^2}$$
$$\approx 17.33$$

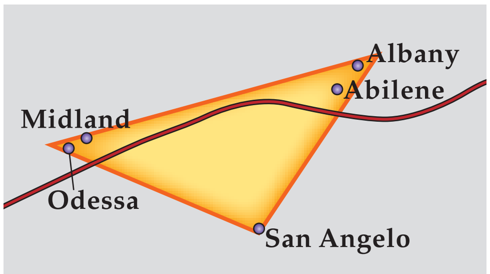
$$MP = \sqrt{(25.8 - 18.48)^2 + (80.27 - 66.12)^2}$$
$$\approx 15.93$$

$$PB = \sqrt{(33.37 - 18.48)^2 + (64.68 - 66.12)^2}$$
$$\approx 14.96$$

Since each side is a different length, $\triangle MPB$ is scalene. Therefore, the Bermuda Triangle is scalene.

Guided Practice

4. GEOGRAPHY In 2006, a group of art museums collaborated to form the West Texas Triangle to promote their collections. This region is formed by the cities of Odessa, Albany, and San Angelo. The approximate coordinates of each location, respectively, are 31.9°N 102.3°W, 32.7°N 99.3°W, and 31.4°N 100.5°W. Write a coordinate proof to prove that the West Texas Triangle is approximately isosceles.



Check Your Understanding

= Step-by-Step Solutions begin on page R14.

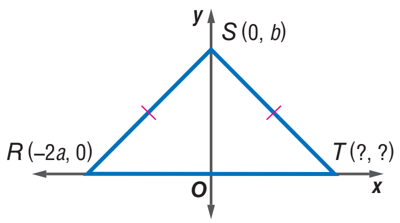
Example 1 **Position and label each triangle on the coordinate plane.**

1. right $\triangle ABC$ with legs $\overline{AC}$ and $\overline{AB}$ so that $\overline{AC}$ is $2a$ units long and leg $\overline{AB}$ is $2b$ units long

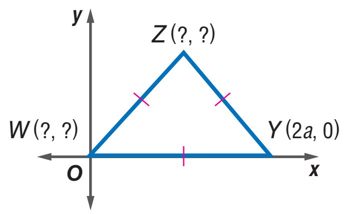
2. isosceles $\triangle FGH$ with base $\overline{FG}$ that is $2a$ units long

Example 2 **Name the missing coordinate(s) of each triangle.**

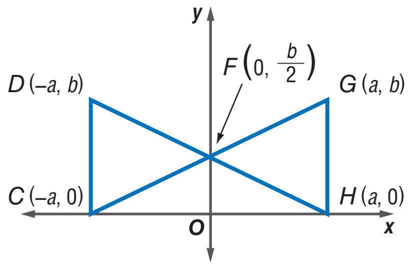
3.



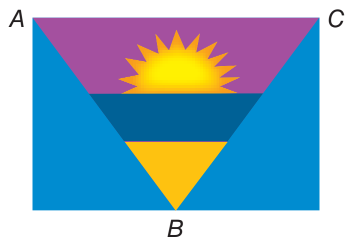
4.



Example 3 5. **CCSS ARGUMENTS** Write a coordinate proof to show that $\triangle FGH \cong \triangle FDC$.



Example 4 6. **FLAGS** Write a coordinate proof to prove that the large triangle in the center of the flag is isosceles. The dimensions of the flag are 4 feet by 6 feet and point B of the triangle bisects the bottom of the flag.



Practice and Problem Solving

Extra Practice is on page R4.

Example 1 **Position and label each triangle on the coordinate plane.**

7. isosceles $\triangle ABC$ with base $\overline{AB}$ that is a units long

8. right $\triangle XYZ$ with hypotenuse $\overline{YZ}$, the length of $\overline{XY}$ is b units long, and the length of $\overline{XZ}$ is three times the length of $\overline{XY}$

9. isosceles right $\triangle RST$ with hypotenuse $\overline{RS}$ and legs $3a$ units long

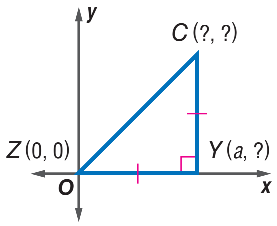
10. right $\triangle JKL$ with legs $\overline{JK}$ and $\overline{KL}$ so that $\overline{JK}$ is a units long and leg $\overline{KL}$ is $4b$ units long

11. equilateral $\triangle GHJ$ with sides $\frac{1}{2}a$ units long

12. equilateral $\triangle DEF$ with sides $4b$ units long

Example 2 **Name the missing coordinate(s) of each triangle.**

13.



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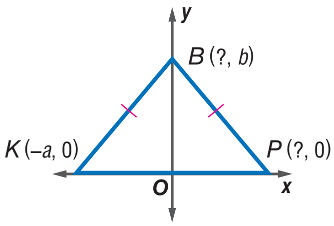

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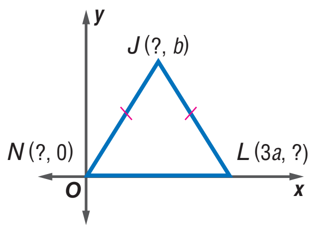

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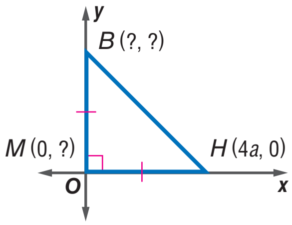

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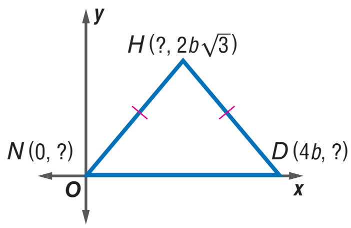

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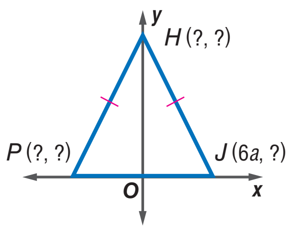


Example 3 CCSS **ARGUMENTS** **Write a coordinate proof for each statement.**

19. The segments joining the base vertices to the midpoints of the legs of an isosceles triangle are congruent.

20. The three segments joining the midpoints of the sides of an isosceles triangle form another isosceles triangle.

Example 4 **PROOF** **Write a coordinate proof for each statement.**

21. The measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

22. If a line segment joins the midpoints of two sides of a triangle, then its length is equal to one half the length of the third side.

23. **RESEARCH TRIANGLE** The cities of Raleigh, Durham, and Chapel Hill, North Carolina, form what is known as the Research Triangle. The approximate latitude and longitude of Raleigh are 35.82°N 78.64°W, of Durham are 35.99°N 78.91°W, and of Chapel Hill are 35.92°N 79.04°W. Show that the triangle formed by these three cities is scalene.

24. **PARTY PLANNING** Three friends live in houses with backyards adjacent to a neighborhood bike path. They decide to have a round-robin party using their three homes, inviting their friends to start at one house and then move to each of the other two. If one friend's house is centered at the origin, then the location of the other homes are (5, 12) and (13, 0). Write a coordinate proof to prove that the triangle formed by these three homes is isosceles.

Draw $\triangle XYZ$ and find the slope of each side of the triangle. Determine whether the triangle is a right triangle. Explain.

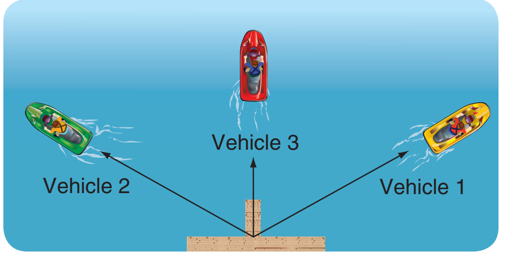
25. $X(0, 0)$, $Y(2h, 2h)$, $Z(4h, 0)$

26. $X(0, 0)$, $Y(1, h)$, $Z(2h, 0)$

27. **CAMPING** Two families set up tents at a state park. If the ranger's station is located at (0, 0), and the locations of the tents are (0, 25) and (12, 9), write a coordinate proof to prove that the figure formed by the locations of the ranger's station and the two tents is a right triangle.

28. **PROOF** Write a coordinate proof to prove that $\triangle ABC$ is an isosceles triangle if the vertices are $A(0, 0)$, $B(a, b)$, and $C(2a, 0)$.

29 **WATER SPORTS** Three personal watercraft vehicles launch from the same dock. The first vehicle leaves the dock traveling due northeast, while the second vehicle travels due northwest. Meanwhile, the third vehicle leaves the dock traveling due north.



The first and second vehicles stop about 300 yards from the dock, while the third stops about 212 yards from the dock.

a. If the dock is located at (0, 0), sketch a graph to represent this situation. What is the equation of the line along which the first vehicle lies? What is the equation of the line along which the second vehicle lies? Explain your reasoning.

b. Write a coordinate proof to prove that the dock, the first vehicle, and the second vehicle form an isosceles right triangle.

c. Find the coordinates of the locations of all three watercrafts. Explain your reasoning.

d. Write a coordinate proof to prove that the positions of all three watercrafts are approximately collinear and that the third watercraft is at the midpoint between the other two.

H.O.T. Problems Use Higher-Order Thinking Skills

30. **REASONING** The midpoints of the sides of a triangle are located at $(a, 0)$, $(2a, b)$ and (a, b). If one vertex is located at the origin, what are the coordinates of the other vertices? Explain your reasoning.

CHALLENGE **Find the coordinates of point L so $\triangle JKL$ is the indicated type of triangle. Point J has coordinates (0, 0) and point K has coordinates $(2a, 2b)$.**

31. scalene triangle **32.** right triangle **33.** isosceles triangle

34. **OPEN ENDED** Draw an isosceles right triangle on the coordinate plane so that the midpoint of its hypotenuse is the origin. Label the coordinates of each vertex.

35. **CHALLENGE** Use a coordinate proof to show that if you add n units to each x-coordinate of the vertices of a triangle and m to each y-coordinate, the resulting figure is congruent to the original triangle.

36. CCSS **REASONING** A triangle has vertex coordinates (0, 0) and $(a, 0)$. If the coordinates of the third vertex are in terms of a, and the triangle is isosceles, identify the coordinates and position the triangle on the coordinate plane.

37. **WRITING IN MATH** Explain why following each guideline below for placing a triangle on the coordinate plane is helpful in proving coordinate proofs.

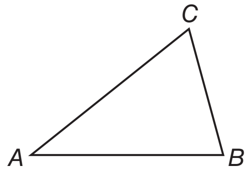
a. Use the origin as a vertex of the triangle.

b. Place at least one side of the triangle on the x- or y-axis.

c. Keep the triangle within the first quadrant if possible.

Standardized Test Practice

38. GRIDDED RESPONSE In the figure below, $m\angle B = 76$. The measure of $\angle A$ is half the measure of $\angle B$. What is $m\angle C$?



39. ALGEBRA What is the x-coordinate of the solution to the system of equations shown below?

$$\begin{cases} 2x - 3y = 3 \\ -4x + 2y = -18 \end{cases}$$

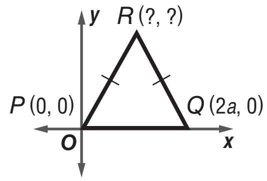
A -6 **C** 3

B -3 **D** 6

40. What are the coordinates of point R in the triangle?

F $\left(\frac{a}{2}, a\right)$ **H** $\left(\frac{b}{2}, a\right)$

G (a, b) **J** $\left(\frac{b}{2}, \frac{a}{2}\right)$



41. SAT/ACT For all x,

$17x^5 + 3x^2 + 2 - (-4x^5 + 3x^3 - 2) =$

A $13x^5 + 3x^3 + 3x^2$

B $13x^5 + 6x^2 + 4$

C $21x^5 - 3x^3 + 3x^2 + 4$

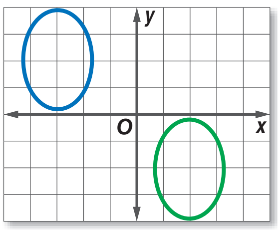
D $21x^5 + 3x^2 + 3x^3$

E $21x^5 + 3x^3 + 3x^2 + 4$

Spiral Review

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*. (Lesson 4-7)

42.



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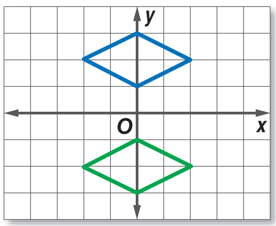
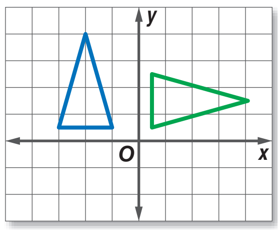


44.

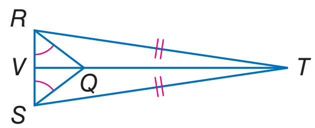


Refer to the figure at the right. (Lesson 4-6)

45. Name two congruent angles.

46. Name two congruent segments.

47. Name a pair of congruent triangles.



48. RAMPS The Americans with Disabilities Act requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch. (Lesson 3-3)

a. Determine the slope represented by this requirement.

b. The maximum length that the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp?

Skills Review

Find the distance between each pair of points. Round to the nearest tenth.

49. $X(5, 4)$ and $Y(2, 1)$

50. $A(1, 5)$ and $B(-2, -3)$

51. $J(-2, 6)$ and $K(1, 4)$

CHAPTER 4

Study Guide and Review

Study Guide

KeyConcepts

Classifying Triangles (Lesson 4-1)

- Triangles can be classified by their angles as acute, obtuse, or right, and by their sides as scalene, isosceles, or equilateral.

Angles of Triangles (Lesson 4-2)

- The measure of an exterior angle is equal to the sum of its two remote interior angles.

Congruent Triangles (Lesson 4-3 through 4-5)

- SSS: If all of the corresponding sides of two triangles are congruent, then the triangles are congruent.
- SAS: If two pairs of corresponding sides of two triangles and the included angles are congruent, then the triangles are congruent.
- ASA: If two pairs of corresponding angles of two triangles and the included sides are congruent, then the triangles are congruent.
- AAS: If two pairs of corresponding angles of two triangles are congruent, and a corresponding pair of nonincluded sides is congruent, then the triangles are congruent.

Isosceles and Equilateral Triangles (Lesson 4-6)

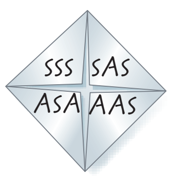
- The base angles of an isosceles triangle are congruent and a triangle is equilateral if it is equiangular.

Transformations and Coordinate Proofs (Lessons 4-7 and 4-8)

- In a congruence transformation, the position of the image may differ from the preimage, but the two figures remain congruent.
- Coordinate proofs use algebra to prove geometric concepts.

FOLDABLES StudyOrganizer

Be sure the Key Concepts are noted in your Foldable.



KeyVocabulary

- **acute triangle** (p. 237)
- **auxiliary line** (p. 246)
- **base angles** (p. 285)
- **congruence transformation** (p. 296)
- **congruent polygons** (p. 255)
- **coordinate proof** (p. 303)
- **corollary** (p. 249)
- **corresponding parts** (p. 255)
- **equiangular triangle** (p. 237)
- **equilateral triangle** (p. 238)
- **exterior angle** (p. 248)
- **flow proof** (p. 248)
- **included angle** (p. 266)
- **included side** (p. 275)
- **isosceles triangle** (p. 238)
- **obtuse triangle** (p. 237)
- **reflection** (p. 296)
- **remote interior angles** (p. 248)
- **right triangle** (p. 237)
- **rotation** (p. 296)
- **scalene triangle** (p. 238)
- **translation** (p. 296)
- **vertex angle** (p. 285)

VocabularyCheck

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. An equiangular triangle is also an example of an acute triangle.
2. A triangle with an angle that measures greater than 90° is a right triangle.
3. An equilateral triangle is always equiangular.
4. A scalene triangle has at least two congruent sides.
5. The vertex angles of an isosceles triangle are congruent.
6. An included side is the side located between two consecutive angles of a polygon.
7. The three types of congruence transformations are rotation, reflection, and translation.
8. A rotation moves all points of a figure the same distance and in the same direction.
9. A flow proof uses figures in the coordinate plane and algebra to prove geometric concepts.
10. The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Lesson-by-Lesson Review

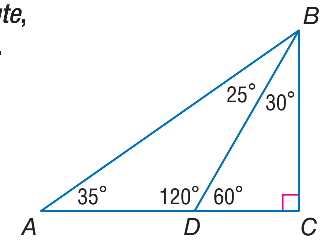
4-1 Classifying Triangles

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

11. $\triangle ADB$

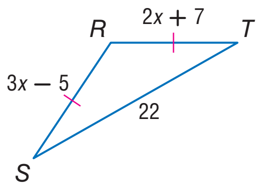
12. $\triangle BCD$

13. $\triangle ABC$



ALGEBRA Find x and the measures of the unknown sides of each triangle.

14.



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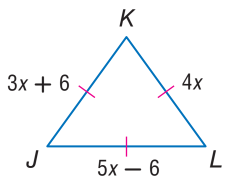
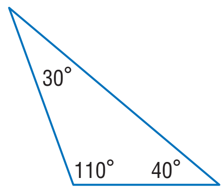


16. **MAPS** The distance from Chicago to Cleveland to Cincinnati and back to Chicago is 900 miles. The distance from Chicago to Cleveland is 50 miles more than the distance from Cincinnati to Chicago, and the distance from Cleveland to Cincinnati is 50 miles less than the distance from Cincinnati to Chicago. Find each distance and classify the triangle formed by the three cities.

Example 1

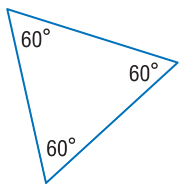
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

a.



Since the triangle has one obtuse angle, it is an obtuse triangle.

b.



The triangle has three acute angles that are all equal. It is an equiangular triangle.

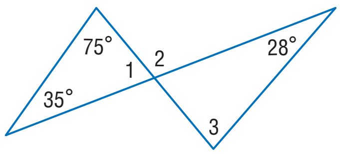
4-2 Angles of Triangles

Find the measure of each numbered angle.

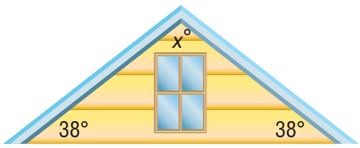
17. $\angle 1$

18. $\angle 2$

19. $\angle 3$

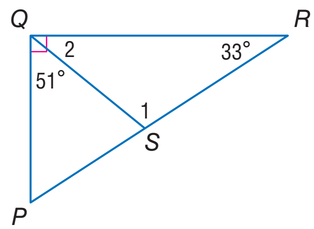


20. **HOUSES** The roof support on Lamar's house is in the shape of an isosceles triangle with base angles of 38°. Find x.



Example 2

Find the measure of each numbered angle.

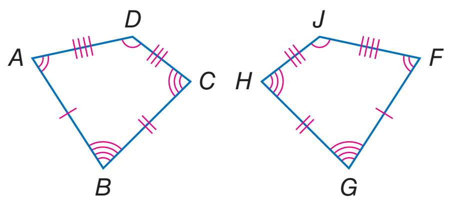


$m\angle 2 + m\angle PQS = 90$	
$m\angle 2 + 51 = 90$	Substitution
$m\angle 2 = 39$	Subtract 51 from each side.
$m\angle 1 + m\angle 2 + 33 = 180$	Triangle Sum Theorem
$m\angle 1 + 39 + 33 = 180$	Substitution
$m\angle 1 + 72 = 180$	Simplify.
$m\angle 1 = 108$	Subtract.

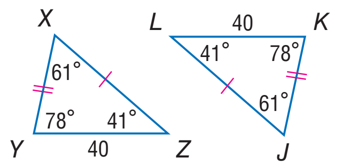
4-3 Congruent Triangles

Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

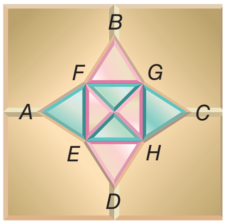
21.



22.

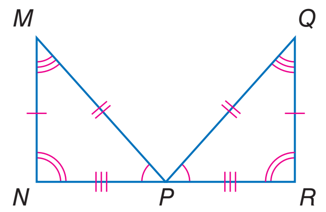


23. **MOSAIC TILING** A section of a mosaic tiling is shown. Name the triangles that appear to be congruent.



Example 3

Show that the polygons are congruent by identifying all the congruent corresponding parts. Then write a congruence statement.



Angles: $\angle N \cong \angle R$, $\angle M \cong \angle Q$, $\angle MPN \cong \angle QPR$

Sides: $\overline{MN} \cong \overline{QR}$, $\overline{MP} \cong \overline{QP}$, $\overline{NP} \cong \overline{RP}$

All corresponding parts of the two triangles are congruent. Therefore, $\triangle MNP \cong \triangle QRP$.

4-4 Proving Triangles Congruent—SSS, SAS

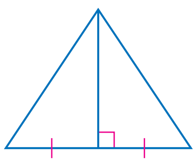
Determine whether $\triangle ABC \cong \triangle XYZ$. Explain.

24. $A(5, 2)$, $B(1, 5)$, $C(0, 0)$, $X(-3, 3)$, $Y(-7, 6)$, $Z(-8, 1)$

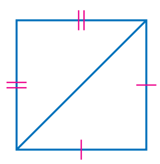
25. $A(3, -1)$, $B(3, 7)$, $C(7, 7)$, $X(-7, 0)$, $Y(-7, 4)$, $Z(1, 4)$

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

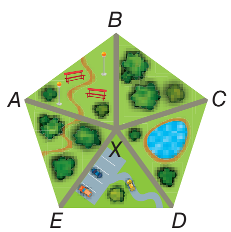
26.



27.



28. **PARKS** The diagram shows a park in the shape of a pentagon with five sidewalks of equal length leading to a central point. If all the angles at the central point have the same measure, how could you prove that $\triangle ABX \cong \triangle DCX$?

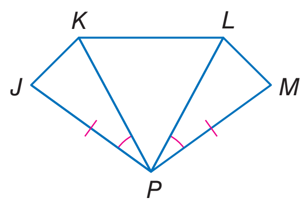


Example 4

Write a two-column proof.

Given: $\triangle KPL$ is equilateral.
$\overline{JP} \cong \overline{MP}$,
$\angle JPK \cong \angle MPL$

Prove: $\triangle JPK \cong \triangle MPL$



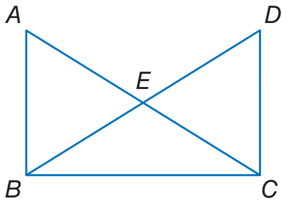
Statements	Reasons
1. $\triangle KPL$ is equilateral.	**1.** Given
2. $\overline{PK} \cong \overline{PL}$	**2.** Def. of Equilateral $\triangle$
3. $\overline{JP} \cong \overline{MP}$	**3.** Given
4. $\angle JPK \cong \angle MPL$	**4.** Given
5. $\triangle JPK \cong \triangle MPL$	**5.** SAS

4-5 Proving Triangles Congruent—ASA, AAS

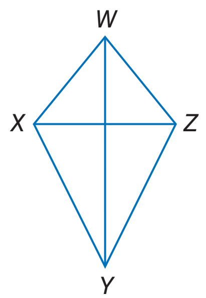
Write a two-column proof.

29. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABE \cong \triangle CDE$



30. **KITES** Denise's kite is shown in the figure at the right. Given that $\overline{WY}$ bisects both $\angle XWZ$ and $\angle XYZ$, prove that $\triangle WXY \cong \triangle WZY$.

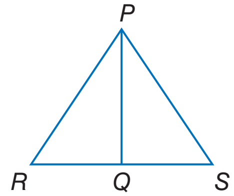


Example 5

Write a flow proof.

Given: $\overline{PQ}$ bisects $\angle RPS$.
$\angle R \cong \angle S$

Prove: $\triangle RPQ \cong \triangle SPQ$



Flow Proof:

$\overline{PQ}$ bisects $\angle RPS$.
Given

$\angle R \cong \angle S$
Given

$\overline{PQ} \cong \overline{PQ}$
Reflexive

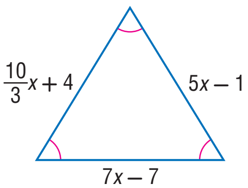
$\angle RPQ \cong \angle SPQ$
Def. of $\angle$ Bisector

$\triangle RPQ \cong \triangle SPQ$
AAS

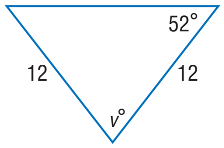
4-6 Isosceles and Equilateral Triangles

Find the value of each variable.

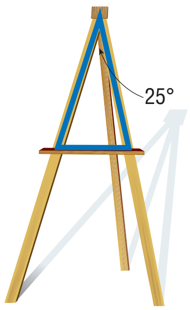
31.



32.

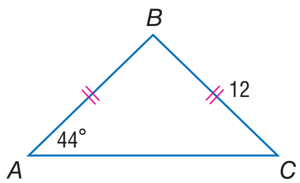


33. **PAINTING** Pam is painting using a wooden easel. The support bar on the easel forms an isosceles triangle with the two front supports. According to the figure below, what are the measures of the base angles of the triangle?



Example 6

Find each measure.



a. $m\angle B$

Since $AB = BC$, $\overline{AB} \cong \overline{BC}$. By the Isosceles Triangle Theorem, base angles A and C are congruent, so $m\angle A = m\angle C$. Use the Triangle Sum Theorem to write and solve an equation to find $m\angle B$.

$m\angle A + m\angle B + m\angle C = 180$	$\triangle$ Sum Theorem
$44 + m\angle B + 44 = 180$	$m\angle A = m\angle C = 44$
$88 + m\angle B = 180$	Simplify.
$m\angle B = 92$	Subtract.

b. AB

$AB = BC$, so $\triangle ABC$ is isosceles. Since $BC = 12$, $AB = 12$ by substitution.

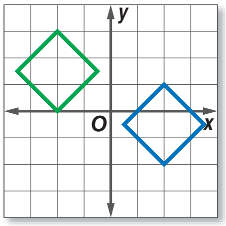
CHAPTER 4

Study Guide and Review *Continued*

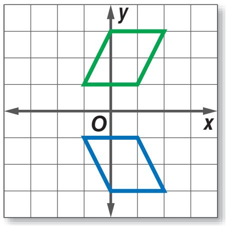
4-7 Congruence Transformations

Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

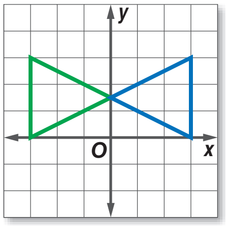
34.



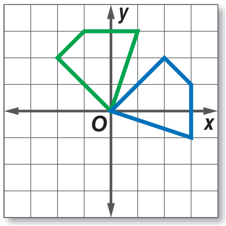
35.



36.



37.



38. Triangle ABC with vertices $A(1, 1)$, $B(2, 3)$, and $C(3, -1)$ is a transformation of $\triangle$MNO with vertices $M(-1, 1)$, $N(-2, 3)$, and $O(-3, -1)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.

Example 7

Triangle *RST* with vertices $R(4, 1)$, $S(2, 5)$, and $T(-1, 0)$ is a transformation of $\triangle CDF$ with vertices $C(1, -3)$, $D(-1, 1)$, and $F(-4, -4)$. Identify the transformation and verify that it is a congruence transformation.

Graph each figure. The transformation appears to be a translation. Find the lengths of the sides of each triangle.

$$RS = \sqrt{(4-2)^2 + (1-5)^2} \text{ or } \sqrt{20}$$
$$TS = \sqrt{(-1-2)^2 + (0-5)^2} \text{ or } \sqrt{34}$$
$$RT = \sqrt{(-1-4)^2 + (0-1)^2} \text{ or } \sqrt{26}$$
$$CD = \sqrt{(-1-1)^2 + [1-(-3)]^2} \text{ or } \sqrt{20}$$
$$DF = \sqrt{[-4-(-1)]^2 + (-4-1)^2} \text{ or } \sqrt{34}$$
$$CF = \sqrt{(-4-1)^2 + [-4-(-3)]^2} \text{ or } \sqrt{26}$$

Since each vertex of $\triangle CDF$ has undergone a transformation 3 units to the right and 4 units up, this is a translation.

Since $RS = CD$, $TS = DF$, and $RT = CF$, $\overline{RS} \cong \overline{CD}$, $\overline{TS} \cong \overline{DF}$, and $\overline{RT} \cong \overline{CF}$. By SSS, $\triangle RST \cong \triangle CDF$.

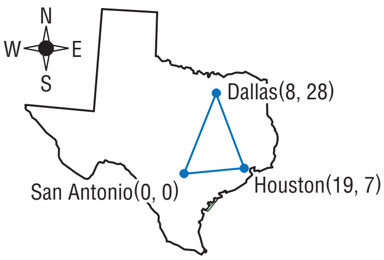
4-8 Triangles and Coordinate Proof

Position and label each triangle on the coordinate plane.

39. right $\triangle MNO$ with right angle at point M and legs of lengths a and $2a$.

40. isosceles $\triangle WXY$ with height h and base $\overline{WY}$ with length $2a$.

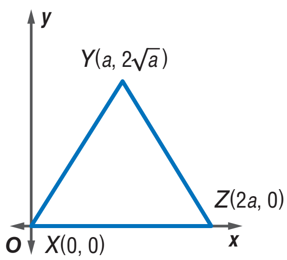
41. **GEOGRAPHY** Jorge plotted the cities of Dallas, San Antonio, and Houston as shown. Write a coordinate proof to show that the triangle formed by these cities is scalene.



Example 8

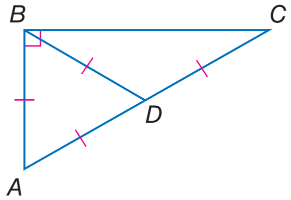
Position and label an equilateral triangle $\triangle XYZ$ with side lengths of $2a$.

- Use the origin for one of the three vertices of the triangle.
- Place one side of the triangle along the positive side of the *x*-axis.
- The third point should be located above the midpoint of the base of the triangle.



Practice Test

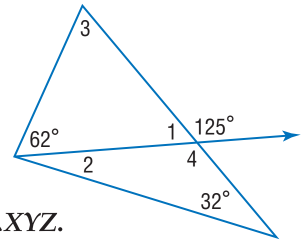
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.



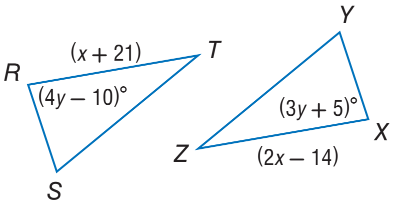
1. $\triangle ABD$
2. $\triangle ABC$
3. $\triangle BDC$

Find the measure of each numbered angle.

4. $\angle 1$
5. $\angle 2$
6. $\angle 3$
7. $\angle 4$



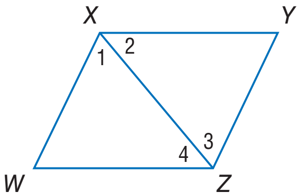
In the diagram, $\triangle RST \cong \triangle XYZ$.



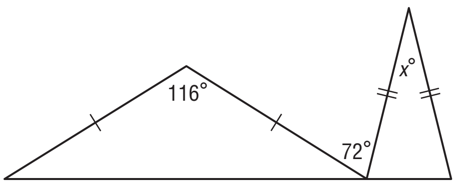
8. Find x.
9. Find y.

10. **PROOF** Write a flow proof.

Given: $\overline{XY} \parallel \overline{WZ}$ and $\overline{XW} \parallel \overline{YZ}$
Prove: $\triangle XWZ \cong \triangle ZYX$



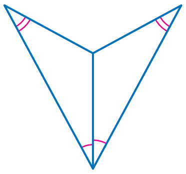
11. **MULTIPLE CHOICE** Find x.

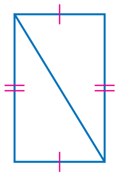


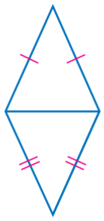
A 36
B 32
C 28
D 22

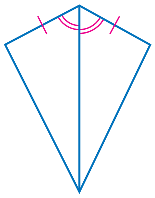
12. Determine whether $\triangle TJD \cong \triangle SEK$ given $T(-4, -2)$, $J(0, 5)$, $D(1, -1)$, $S(-1, 3)$, $E(3, 10)$, and $K(4, 4)$. Explain.

Determine which postulate or theorem can be used to prove each pair of triangles congruent. If it is not possible to prove them congruent, write *not possible*.

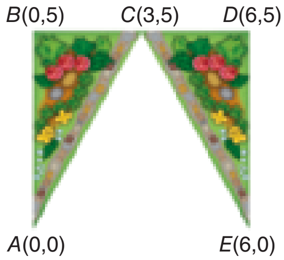
13. 

14. 

15. 

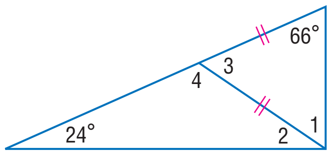
16. 

17. **LANDSCAPING** Angie has laid out a design for a garden consisting of two triangular areas as shown below. The points are $A(0, 0)$, $B(0, 5)$, $C(3, 5)$, $D(6, 5)$, and $E(6, 0)$. Name the type of congruence transformation for the preimage $\triangle ABC$ to $\triangle EDC$.



Find the measure of each numbered angle.

18. $\angle 1$
19. $\angle 2$



20. **PROOF** $\triangle ABC$ is a right isosceles triangle with hypotenuse $\overline{AB}$. M is the midpoint of $\overline{AB}$. Write a coordinate proof to show that $\overline{CM}$ is perpendicular to $\overline{AB}$.

CHAPTER 4 Preparing for Standardized Tests

Short-Answer Questions

Short-answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.

Short-answer questions are typically graded using a **rubric**, or a scoring guide.

The following is an example of a short-answer question scoring rubric.

Scoring Rubric		
Criteria		Score
Full Credit	The answer is correct and a full explanation is provided that shows each step.	2
Partial Credit	• The answer is correct, but the explanation is incomplete. • The answer is incorrect, but the explanation is correct.	1 1
No Credit	Either an answer is not provided or the answer does not make sense.	0

Strategies for Solving Short-Answer Questions

Step 1

Read the problem to gain an understanding of what you are trying to solve.

- Identify relevant facts.
- Look for key words and mathematical terms.

Step 2

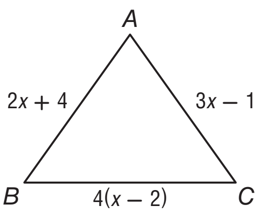
Make a plan and solve the problem.

- Explain your reasoning or state your approach to solving the problem.
- Show all of your work or steps.
- Check your answer if time permits.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

Triangle ABC is an isosceles triangle with base $\overline{BC}$. What is the perimeter of the triangle?



Read the problem carefully. You are told that $\triangle ABC$ is isosceles with base $\overline{BC}$. You are asked to find the perimeter of the triangle.

Make a plan and solve the problem.

The legs of an isosceles triangle are congruent. So, $\overline{AB} \cong \overline{AC}$ or $AB = AC$. Solve for x.

$$\begin{aligned} AB &= AC \\ 2x + 4 &= 3x - 1 \\ 2x - 3x &= -1 - 4 \\ -x &= -5 \\ x &= 5 \end{aligned}$$

Next, find the length of each side.

$AB = 2(5) + 4 = 14$ units

$AC = 3(5) - 1 = 14$ units

$BC = 4(5 - 2) = 12$ units

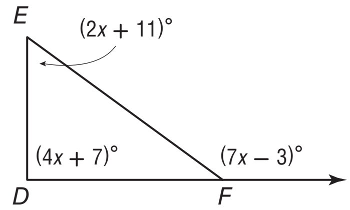
The perimeter of $\triangle ABC$ is $14 + 14 + 12 = 40$ units.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

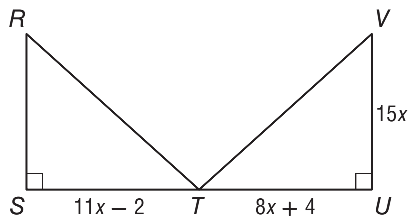
Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. Classify $\triangle DEF$ according to its angle measures.

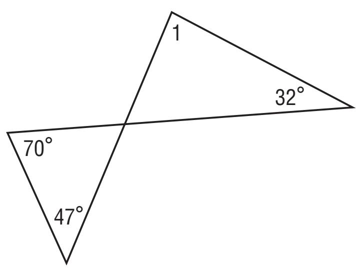


2. In the figure below, $\triangle RST \cong \triangle VUT$. What is the area of $\triangle RST$?



3. A farmer needs to make a 48-square-foot rectangular enclosure for chickens. He wants to save money by purchasing the least amount of fencing possible to enclose the area. What whole-number dimensions will require the least amount of fencing?

4. What is $m\angle 1$ in degrees?

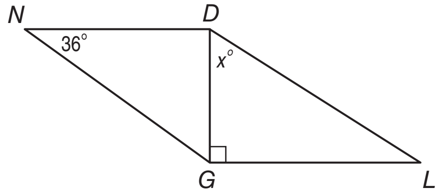


5. Write an equation of the line containing the points (2, 4) and (0, –2).

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. **GRIDDED RESPONSE** In the figure below, $\triangle NDG \cong \triangle LGD$. What is the value of x?

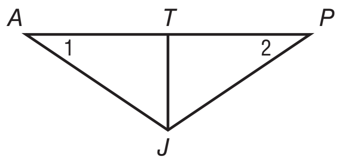


9. **GRIDDED RESPONSE** Suppose line ℓ contains points A, B, and C. If $AB = 7$ inches, $AC = 32$ inches, and point B is between points A and C, what is the length of $\overline{BC}$? Express your answer in inches.

10. Write the converse of the statement.

    If you are the winner, then I am the loser.

11. Use the figure and the given information below.

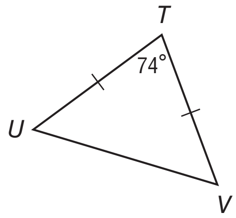


Given: $\overline{JT} \perp \overline{AP}$
$\angle 1 \cong \angle 2$

Which congruence theorem could you use to prove $\triangle PTJ \cong \triangle ATJ$ with only the information given? Explain.

12. Write an equation in slope intercept form for the line which goes through the points (0, 3) and (4, −5).

13. **GRIDDED RESPONSE** Find $m\angle TUV$ in the figure.



14. Suppose two sides of triangle ABC are congruent to two sides of triangle MNO. Also, suppose one of the nonincluded angles of $\triangle ABC$ is congruent to one of the nonincluded angles of $\triangle MNO$. Are the triangles congruent? If so, write a paragraph proof showing the congruence. If not, sketch a counterexample.

Extended Response

Record your answers on a sheet of paper. Show your work.

15. Use a coordinate grid to write a coordinate proof of the following statement.

    If the vertices of a triangle are A(0, 0), B(2a, b), and C(4a, 0), then the triangle is isosceles.

    a. Plot the vertices on a coordinate grid to model the problem.

    b. Use the Distance Formula to write an expression for AB.

    c. Use the Distance Formula to write an expression for BC.

    d. Use your results from parts **b** and **c** to draw a conclusion about $\triangle ABC$.

Need ExtraHelp?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Go to Lesson...	3-2	4-7	4-1	4-3	1-7	4-2	4-6	4-3	1-2	2-3	4-5	3-4	4-6	4-4	4-8