Qubit stabilization via learning capable materials

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Quantum chemistry

Applications of category theory in physics

FAST Foundation

Research

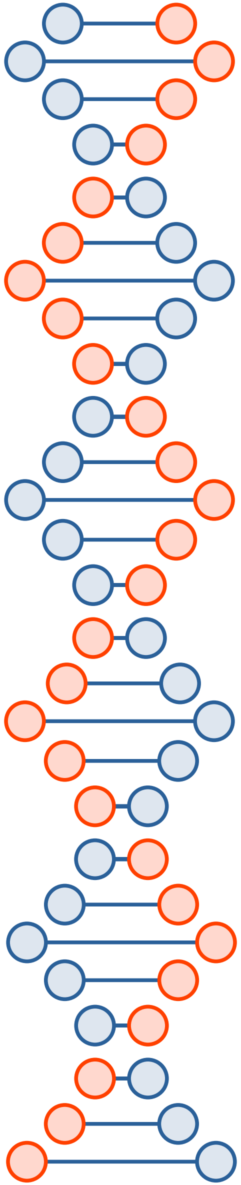
Quantum information

Gauge theory, QFT, string theory

Machine learning

Cvasi-classical / Cvasi-quantum neural networks

Neural networks



Cvasi-classical / Cvasi-quantum neural networks

Quantum only at small scales

Quantum only at low temperatures

Quantum only without external interactions

Right?

Wrong!

Going to classical limit is more than just taking h go to zero

Different problems require different ways to go to the classical limit

Ultimately nothing is purely classical. There is always relative quantum phase somewhere

Practically to go to h=0 it is useful to work with the Wigner-Weyl representation

We want a quantum distribution over the phase space

Technically, that's impossible

We want expectation values

They involve distributions and operators

Quantum phenomena will be fully described by the Wigner distribution in conjunction with Weyl transformed operators

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

$$\widetilde{A}(x,p) = \int e^{-ipy/\hbar}\langle x + y/2|\hat{A}|x - y/2\rangle dy$$

Weyl transf.

$$\langle x|\hat{\rho}|x'\rangle = \psi(x)\psi^*(x')$$

$$W(x,p) = \widetilde{\rho}/h = \frac{1}{h}\int e^{-ipy/\hbar}\psi(x+y/2)\psi^*(x-y/2)dy$$

Wigner distrib.

$$\mathrm{Tr}[\hat{A}\hat{B}] = \frac{1}{h}\int\int \widetilde{A}(x,p)\widetilde{B}(x,p)dx\,dp$$

$$\langle A\rangle = \int\int W(x,p)\widetilde{A}(x,p)dx\,dp$$

We also need dynamics, hence time dependence

$$\frac{\partial W}{\partial t} = \frac{1}{h}\int e^{-ipy/\hbar}\left[\frac{\partial \psi^*(x-y/2)}{\partial t}\psi(x+y/2) + \frac{\partial \psi(x+y/2)}{\partial t}\psi^*(x-y/2)\right]dy$$

By Schrodinger Eq. $\frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar}{2im}\frac{\partial^2 \psi(x,t)}{\partial x^2} + \frac{1}{i\hbar}U(x)\psi(x,t)$

$$\frac{\partial W}{\partial t} = \frac{\partial W_T}{\partial t} + \frac{\partial W_U}{\partial t}$$

$$\frac{\partial W_T}{\partial t} = \frac{1}{4\pi i m}\int e^{-ipy/\hbar}\left[\frac{\partial^2 \psi^*(x-y/2)}{\partial x^2}\psi(x+y/2) - \psi^*(x-y/2)\frac{\partial^2 \psi(x+y/2)}{\partial x^2}\right]dy,$$

$$\frac{\partial W_T}{\partial t} = -\frac{p}{m}\frac{\partial W(x,p)}{\partial x}$$

$$\frac{\partial W_U}{\partial t} = \frac{2\pi}{ih^2}\int e^{-ipy/\hbar}[U(x+y/2) - U(x-y/2)]\psi^*(x-y/2) \times \psi(x+y/2)dy.$$

$$\frac{\partial W_U}{\partial t} = \sum_{s=0}(-\hbar^2)^s\frac{1}{(2s+1)!}\left(\frac{1}{2}\right)^{2s}\frac{\partial^{2s+1}U(x)}{\partial x^{2s+1}} \times \left(\frac{\partial}{\partial p}\right)^{2s+1} W(x,p).$$

So, just taking h to 0? What about the 2s+1 derivative on W there?

Classical distributions are linear in phase space ⟷ Wigner functions are not

$$\psi = \psi_\alpha + \psi_\beta \not\Rightarrow W_\psi = W_\alpha + W_\beta$$

We introduce mixed states

$$\hat{\rho} = \sum_j P_j |\psi_j\rangle\langle\psi_j|$$

→ $W(x,p) = \tilde{\rho}/h = \sum_j P_j W_j(x,p)$ → Recover linearity like in classical phase space distributions when using mixed states

Basic example of harmonic oscillator ground state

Wigner function of harmonic oscillator ground state

Pure state

$$\psi = A[\psi_0(x-b) + \psi_0(x+b)]$$

$$\psi_0(x) = \frac{1}{\sqrt[4]{\pi}\sqrt{a}} e^{-x^2/(2a^2)}$$

$$W(x,p) = \frac{1}{h(1+e^{-b^2/a^2})} e^{-(ap)^2/\hbar^2}\left[e^{-(x-b)^2/a^2} + e^{-(x+b)^2/a^2}\right.$$

$$\left. + 2e^{-x^2/a^2}\cos(2bp/\hbar)\right].$$

Mixed state

$$W(x,p) = \frac{1}{2}[W_0(x-b,p) + W_0(x+b,p)]$$

$$W_0(x,p) = \frac{2}{h}\exp(-a^2p^2/\hbar^2 - x^2/a^2)$$

$$W(x,p) = \frac{1}{2}[W_0(x-b,p) + W_0(x+b,p)]$$

$$= \frac{1}{h} e^{-a^2p^2/\hbar^2}\left[e^{-(x-b)^2/a^2} + e^{-(x+b)^2/a^2}\right]$$

$(\partial / \partial p)^{2s+1}$ on the Wigner functions → $(-2a^2 p/\hbar^2)^{2s+1} \exp[-a^2 p^2/\hbar^2]$

↓

$\hbar \to 0$ → $p \sim \hbar / a$ → $\left(\frac{\partial W_0}{\partial p}\right)^{(2s+1)} \to \frac{1}{\hbar^{2s+1}}$

But

$$\frac{\partial W_U}{\partial t} = \sum_{s=0} (-\hbar^2)^s \frac{1}{(2s+1)!} \left(\frac{1}{2}\right)^{2s} \frac{\partial^{2s+1} U(x)}{\partial x^{2s+1}} \times \left(\frac{\partial}{\partial p}\right)^{2s+1} W(x,p).$$

→ $1/\hbar$

Probability density:

$$P(p_0) = \frac{1}{c\sqrt{\pi}} e^{-p_0^2/c^2}$$

Wigner function of mixed state

$$W(x,p) = \int W_0(x,p-p_0) P(p_0) dp_0$$

$$= \frac{1}{\pi\sqrt{a^2 c^2 + \hbar^2}} e^{-x^2/a^2} e^{p^2/(c^2+\hbar^2/a^2)}$$

We also cannot fix p or x, because the uncertainty relation links them through h. Dynamics can reverse the places of x and p (we can get squeezed states, etc.) so fixing one is not realistic.

If we fix the width on x, then width on p cannot be arbitrarily small.

But in a mixed state, the widths of other p-states can be arbitrary (large or small)

$$W(x,p) = \frac{1}{2\pi ac(1 + e^{-(b/a)^2})} e^{-p^2/c^2}[e^{-(x-b)^2/a^2} + e^{-(x+b)^2/a^2} + 2e^{-x^2/a^2} e^{-b^2/a^2}].$$

Now we can take the h → 0 limit, for example in the double coherent pure state above:

Level of suppression of quantum properties

But non-linear classical dynamics may result in very large higher order derivatives. Combined with our h → 0 limit this may amplify quantum effects by means of classical dynamics.

One example: Chaos ?

Another example: a neural network

Yet another example: chaotic neural networks...

Neural network as system of differential equations

$$\frac{d}{dt}y(t) = F(\mathrm{in}(t), y(t), W(t))$$

Activation dynamics

Neuron output

Neuron input

Weight dynamics

$$\frac{dW}{dt}(t) = G(\mathrm{in}(t), y(t), W(t))$$

$$\frac{dW_{ij}}{ds}(t, s) = \sum_m \frac{\partial E}{\partial y_m}(t, s) \cdot \frac{\partial y_m}{\partial W_{ij}}(t, s) + \frac{\partial E}{\partial W_{ij}}(t, s)$$

Learning dynamics

Lyapunov or cost

Two time scales: Neuron dynamics (t), and learning dynamics (s)

Define an observable of the neural network say J (e.g. mean error is observable of supervised learning)

Dynamics of observable

$$D(t, y, W, J, \partial J/\partial t, \partial J/\partial y, \partial J/\partial W) = 0.$$

Consider NN in phase space as a dynamical system

$$\frac{\partial J}{\partial t} + H(t, y, \Delta, W, M) = 0$$

Hamilton Jacobi eq.

$$\Delta_i := \frac{\partial J}{\partial y_i} \qquad M_{ij} := \frac{\partial J}{\partial W_{ij}}$$

Associate characteristic equations (Hamilton eq)

$$\frac{dy_i}{dt} = \frac{\partial H}{\partial \Delta_i} \qquad \frac{d\Delta_i}{dt} = -\frac{\partial H}{\partial y_i} \qquad \frac{dW_{ij}}{dt} = \frac{\partial H}{\partial M_{ij}}$$

$$\frac{dJ}{dt} = \frac{\partial J}{\partial t} + \sum_j \Delta_j(t) \cdot \frac{\partial H}{\partial \Delta_j}(t) + \sum_{i,j} M_{ij}(t) \cdot \frac{\partial H}{\partial M_{ij}}(t)$$

A solution of HJ equation is generator of a transformation to a set of variables that are constants of motion. Such a solution that depends on all n variables is called a complete integral. Once we know a complete integral all we need to do is to substitute into the previous coordinates and obtain the solution:

$$\Delta_i := \frac{\partial J}{\partial y_i} \qquad M_{ij} := \frac{\partial J}{\partial W_{ij}}$$

$$y^i = y^i(\alpha^j, \beta_k, t)$$
$$W^{ij} = W^{ij}(\alpha^p, \beta_k, t)$$

However: a HJ solution may depend on fewer integration constants.

Non-invertible

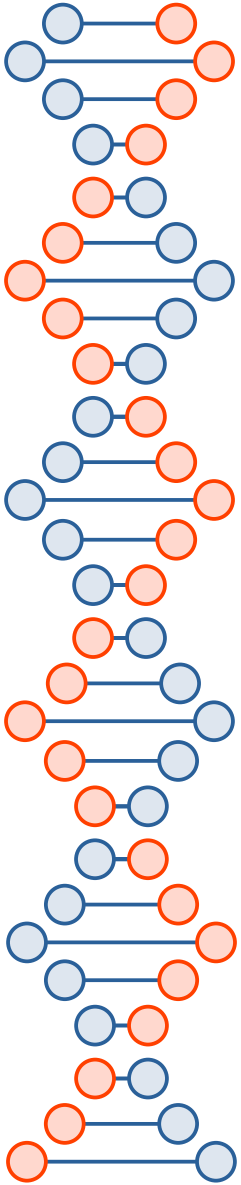
$$det\left(\frac{\partial^2 J}{\partial \alpha^j \partial y^i}\right) = 0$$

New variables, constants of motion

Old variables

$$J(y^i, \alpha^A, t) = J(y^i, \alpha^A, \alpha_a = 0, t)$$

We fix some integration constants



The dependence on those constants set to zero is lost → Their conjugate momenta become fully undetermined

But this happens in quantum mechanics

Take Schrodinger eq.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$

Insert

$$\psi = R\, e^{iS/\hbar}$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} = 0$$

$$\frac{\partial R^2}{\partial t} + \nabla\left(R^2 \frac{\nabla S}{m}\right) = 0$$

$$J(y^i, \alpha^A, \alpha_a, t) \rightarrow J(y^i, t)$$

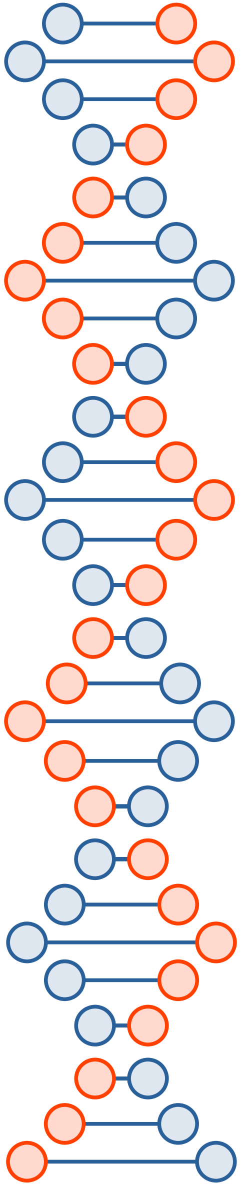
If we complete "loose" all dependence on the constants of motion we obtain quantum mechanics

$$\psi = exp(\tfrac{i}{\hbar} \cdot J(y^i, \beta, t))$$

"classical" wavefunction

Here let be dragons… or "interpretations"

How do we "loose" Beta? (thermodynamics, statistics, probabilistic, random potential, all wrong)



The reason we “loose” beta in QM is the same as in gauge theory: non-invertibility which leads to quantization constraints like in a classical HJ theory with constraints

$$rank\left(\frac{\partial^2 S}{\partial \alpha^A \partial y^i}\right) = N - m$$

Rank diminishes, the “incomplete” integral cannot determine the unique solutions of the e.o.m anymore

How is that represented in neural networks?

As showed in first part: neural networks are only “cvasi-classical”. Dynamics can make quantum effects manifest

Some non-reversible learning dynamics

Not truly a problem as most neural learning dynamics is non-invertible all by itself

$$W_{ij}(nT+\tau) = W_{ij}((n-1)T+\tau) + \Delta W(nT+\tau)$$

What has been learned

What will be learned (change of weights during one epoch)

$$t = n \cdot T + \tau,\ 0 \leqq \tau < T,\ n = 0,\ 1,\ 2,\ \ldots .$$

Kinetic

$$H = \sum_{k} \Delta_k \cdot F_k(t, y, S_T W) + \frac{1}{2\omega} \sum_{k,l} \sum_{v=0}^{n+1} (S_{vT} M)^2_{kl} + E(t, y, W)$$

$$F_k := \frac{1}{\lambda} \cdot \left[-y_k + f_k\left(\sum_{j=-1}^{N} S_T W_{kj} \cdot y_j \right) \right]$$

$$(S_{vT} X)_{kl}(t) := X_{kl}(t - vT), \quad X = W, M$$

$$\frac{dy_i}{dt}(t) = F_i(t)$$

$$\frac{d\Delta_i}{dt}(t) = \frac{1}{\lambda} \cdot \left[\Delta_i(t) - \sum_j \Delta_j(t) \cdot f_j'(t) \cdot W_{ji}(t - T) \right] - \frac{\partial E}{\partial y_i}(t).$$

Potential

And basically irreversible dynamics

Dynamics as source of quantum behavior

- Let an input be a quantum state (this means the synaptic particles have non-trivial relative phases and present quantum correlations, they are in a state in which some of their properties are fundamentally not known, say, for example their momenta)
- Let us consider the neuron as a de-coherent system
- But let us consider the dynamics of a neural network described by the rules of an open quantum system

Interaction

Coherence → Decoherence

Not quite...

pointer states

H_{int}

Interaction

Coherent state H_S → Coherent state H_S

With a special environment

H_E

Special due to constituents

Special due to inner dynamics

However:

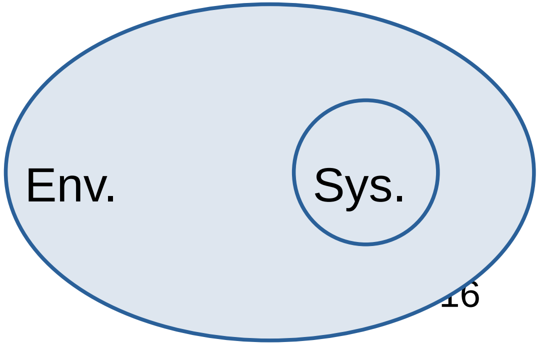
H_{int} → Induces an environment super-selection rule → Dynamical filter on the state space selecting states that can be robustly prepared and observed

$$e^{-iH_{int}t/\hbar}\,|s_i\rangle\,|E_0\rangle = \lambda\,|s_i\rangle\,e^{-iH_{int}t/\hbar}\,|E_0\rangle = |s_i\rangle\,|E_i(t)\rangle$$

States that remain unaffected by the interaction with environment

$$\{|s_i\rangle\}$$

Never mind interactions



Consider the "quantum measurement limit"

$$H \sim H_{int}$$

Diagonalize the interaction Hamiltonian in the subspace of the system

Obtain the "pointer states"

The system-environment product state initially unentangled stays unentangled as time advances

Define a pointer observable

$$\mathcal{O}_S = \sum_i o_i \left| s_i \right\rangle \left\langle s_i \right|$$

Pointer state is eigenstate of interaction Hamiltonian

$$[\mathcal{O}_S, H_{int}] = 0$$

Consider the interaction and the environment being those of a neural network

Describe decoherence through it by

$$\mathcal{L}(\rho) = \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}_+$$

(Lindblad operator)

The Lindblad equation

$$\frac{d\,|\rho\rangle}{dt} = \mathcal{L}\,|\rho\rangle$$

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}\right)$$

Summation over all possible channels of the environment interaction

In a quantum simulator we represent time as an auxiliary register

$$\{|t\rangle\}_{t=0}^{T}$$

Channels: dynamics of neural network

Lindblad operator

$$L_i = |0\rangle_i \langle 1| \times |0\rangle_t \langle 0|$$

$$\mathcal{L} = -i\mathbb{I}\otimes H + iH\otimes\mathbb{I} + \sum_k L_k^*\otimes L_k - \frac{1}{2}\mathbb{I}\otimes(L_k^\dagger L_k) - \frac{1}{2}L_k^T L_k^*\otimes\mathbb{I}$$

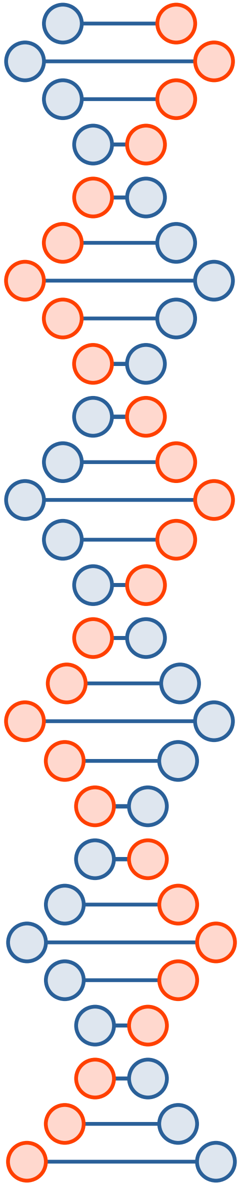
In terms of time development:

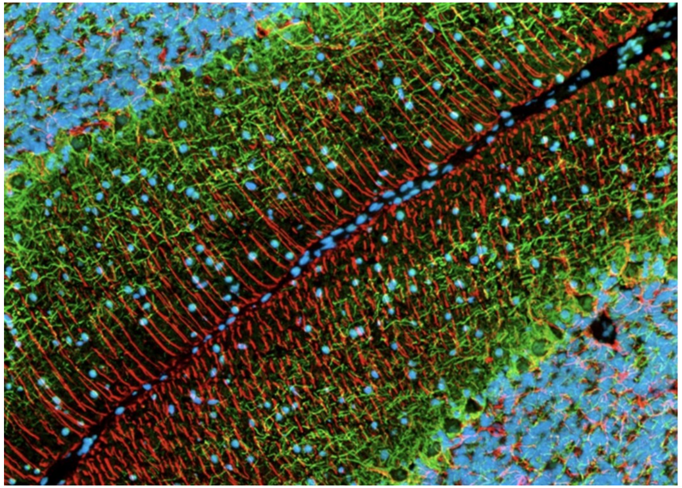
$$L_t = U_t \times |t+1\rangle \langle t| + U_t^\dagger \times |t\rangle \langle t+1|$$

Time propagation

$$|\rho(t)\rangle = e^{\mathcal{L}t}\,|\rho(0)\rangle$$

However: we have here non-Markovian open systems → Cannot be described by a closed master equation with a time-independent generator in Lindblad form!





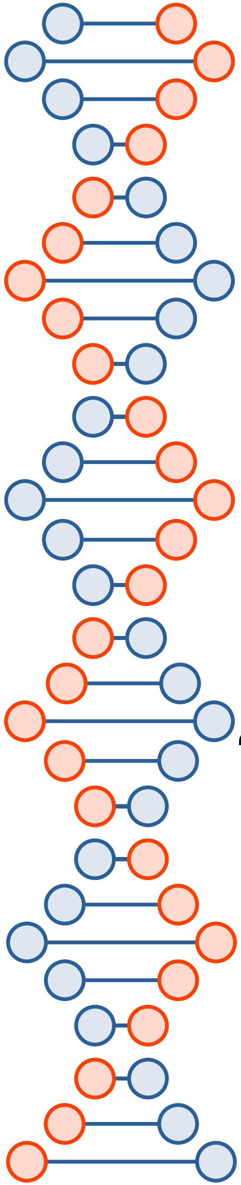
The interaction Hamiltonian must encode connection between different layers

$$\frac{d\rho}{dt}$$

Strong couplings, correlation, memory effects (certainly non-Markovian)

Lindbladian must implement neural dynamics

Further simplification: Projection operator technique (must encode non-Markovianity)



Projection operator technique: Non-Markovian dynamics (memory effects,cvasi-classical)

Projection super-operator acting on states of the total system (including environment)

$$\rho \longrightarrow \mathcal{P}\rho$$

(Elimination of d.o.f. from total sys.)

$$\rho = \mathcal{P}\rho_{rel} + \mathcal{Q}\rho_{irel}$$

"Relevant" dynamics

Also relevant but called "irrelevant"

$$\mathcal{P} + \mathcal{Q} = I$$

Because we need memory effects and correlation between environment and system we look for the Projected density matrix as

$$\mathcal{P}\rho = Tr_E(A_i\rho) \otimes B_i$$

Acting on $\mathcal{H} = \mathcal{H}_S \otimes \bar{\mathcal{H}}_E$

The representation of the projection operators is not unique

$$A_i' = \sum_j u_{ij} A_j$$
$$B_i' = \sum_j v_{ij} B_j$$

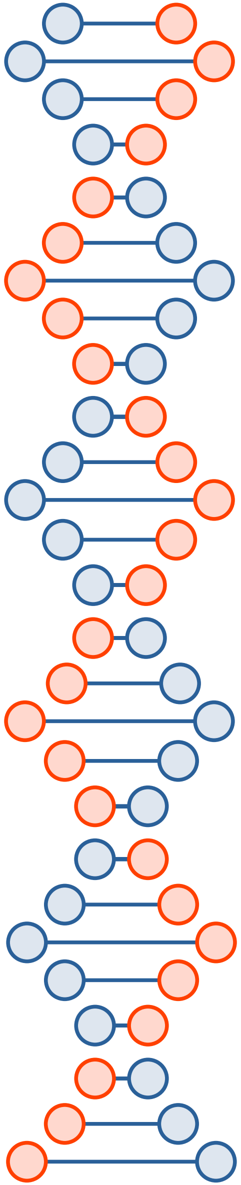
Given a projection super-operator we define the relevant states as those for which $\mathcal{P}\rho_{rel} = \rho_{rel}$

$$\rho_{rel} = \sum_i \rho \otimes B_i$$

And for observables

$$tr\{\mathcal{O}_{rel}(\mathcal{P}\rho)\} = tr\{\mathcal{O}_{rel}\rho\}$$

The relevant observables will be $\mathcal{O}_{rel} = \sum_i \mathcal{O}_S^i \otimes A_i$



$$\rho(t) = U(t)\rho(0)U^\dagger(t)$$ → Unitary time evolution

Density matrix is an operator: characterised by the A-operators → Dynamical variables are

$$\rho_i(t) tr_E\{A_i\rho(t)\}$$

↓

Reduced density matrix of the system

Start with a state belonging to the manifold of relevant states

$$\rho(0) = \mathcal{P}\rho(0) = \sum_i \rho_i(0) \otimes B_i$$

Initial state projected into states with classical correlation between environment and system

Via dynamics:

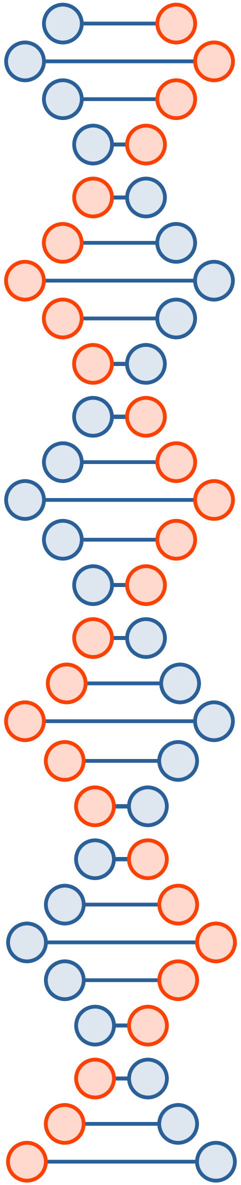
$$\rho_i(t) = \sum_j tr_E\{A_i U(t)\rho_j(0) \otimes B_j U^\dagger(t)\}$$

Introduce projection to mixed states (classical correlation)

$$A_i \geq 0 \qquad \rho_i \geq 0$$

$$\rho_S(t) = \sum_i \rho_i(t)$$

Non-normalised density matrix



The dynamics is described by

$$\frac{d}{dt}\mathcal{P}\rho(t) = \mathcal{K}^t(\mathcal{P}\rho(t))$$

Given

$$\mathcal{P}\rho = \sum_i tr_E\{A_i\rho\} \otimes B_i$$

For general A_i, B_i

$$\frac{d}{dt}\rho_i = \mathcal{K}_i^t(\rho_1, ..., \rho_n)$$

The “relevant” part corresponds to memory and “slow fluctuations”

The “irrelevant” part corresponds to rapid fluctuations

Putting together the two effects :

$$\frac{\partial}{\partial t}\mathcal{P}\rho(t) = \alpha\mathcal{P}L(t)\cdot\mathcal{I}\cdot\rho(t) = \alpha\mathcal{P}L(t)(\mathcal{P}+\mathcal{Q})\rho(t) = \alpha\mathcal{P}L(t)\mathcal{P}\rho(t) + \alpha\mathcal{P}L(t)\mathcal{Q}\rho(t)$$

$$\frac{\partial}{\partial t}\mathcal{Q}\rho(t) = \alpha\mathcal{Q}L(t)\cdot\mathcal{I}\cdot\rho(t) = \alpha\mathcal{Q}L(t)(\mathcal{P}+\mathcal{Q})\rho(t) = \alpha\mathcal{Q}L(t)\mathcal{P}\rho(t) + \alpha\mathcal{Q}L(t)\mathcal{Q}\rho(t)$$

With a formal solution

$$\mathcal{Q}\rho(t) = K(t,t_0)\mathcal{Q}\rho(t_0) + \alpha\int_{t_0}^{t} ds K(t,s)\mathcal{Q}L(s)\mathcal{P}\rho(s)$$

With propagator

$$K(t,s) = \mathcal{T}exp[\alpha\int_s^t ds'\mathcal{Q}L(s')]$$

satisfying

$$\frac{\partial}{\partial t}K(t,s) = \alpha\mathcal{Q}L(t)K(t,s)$$

We obtain $\frac{\partial}{\partial t}\mathcal{P}\rho(t) = \alpha\mathcal{P}L(t)\mathcal{P}\rho(t) + \alpha\mathcal{P}L(t)[K(t,t_0)\mathcal{Q}\rho(t_0) + \alpha\int_{t_0}^{t} ds K(t,s)\mathcal{Q}L(s)\mathcal{P}\rho(s)] =$

$$\alpha\mathcal{P}L(t)\mathcal{P}\rho(t) + \alpha\mathcal{P}L(t)K(t,t_0)\mathcal{Q}\rho(t_0) + \alpha^2\int_{t_0}^{t} ds K(t,s)\mathcal{Q}L(s)\mathcal{P}\rho(s)$$

This projection does not represent a simple product state : it projects onto correlated states (even entangled states)

Introduce auxiliary Hilbert space with a basis

$$\{|i\rangle\}$$

Allows us to analyze additional degrees of freedom describing the correlations induced by the super-operator

$$\mathcal{L}(\sum_i \rho_i \otimes |i\rangle\langle i|) = \sum_i \mathcal{K}_i(\rho_1, ..., \rho_n) \otimes |i\rangle\langle i|$$

$$\sum_i \rho_i(t) \otimes |i\rangle\langle i| = e^{\mathcal{L}t}(\sum_i \rho_i(0) \otimes |i\rangle\langle i|)$$

$$\mathcal{K}_i(\rho_1, \rho_2, ..., \rho_n) = -i[H^i, \rho_i] + \sum_{j\lambda}(S_\lambda^{ij}\rho_j S_\lambda^{ij\dagger} - \frac{1}{2}\{S_\lambda^{ij\dagger}S_\lambda^{ij}, \eta\})$$

$$H = \sum_i H^i \otimes |i\rangle\langle i|$$

$$S_\lambda^{ij} = R_\lambda^{ij} \otimes |i\rangle\langle j|$$

We replaced quantum superposition with mixed states (as in cvasi-classical)

What if we start from a system already entangled to the environment?

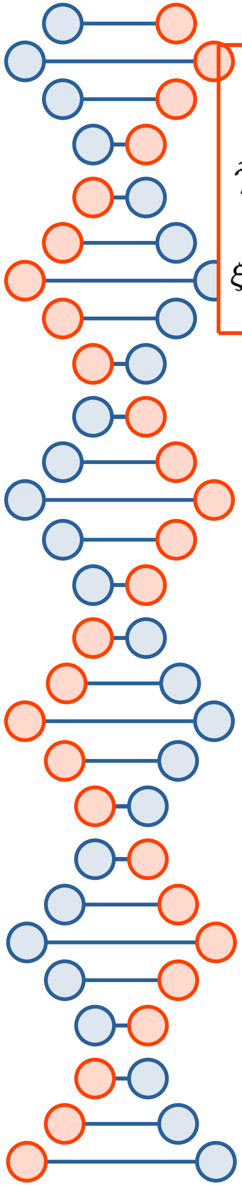
$$\rho_{AB} = \frac{1}{NM}\left(\mathbb{1}_{AB} + \alpha_i \sigma_i \otimes \mathbb{1}_B + \beta_j \mathbb{1}_A \otimes \tau_j + \gamma_{ij}\sigma_i \otimes \tau_j\right)$$

$$\rho'_A = \mathrm{Tr}_B \left(U_{AB}\, \rho_{AB}\, U^\dagger_{AB}\right)$$

$$\rho'_A = \sum_{\mu,\nu} M_{\mu\nu} \rho_A M^\dagger_{\mu\nu} + \sum_{\mu} \langle \mu | U_{AB}\ \gamma'_{ij} \sigma_i \otimes \sigma_j\ U^\dagger_{AB} | \mu \rangle$$

Standard Krauss

Initial quantum correlations
Make equation non-homogeneous



$$\rho_A(t) = \widehat{\mathcal{T}}(t)\ \rho_A(0) + \xi(t)$$

$$\widehat{\mathcal{T}}(t)\ \rho_A(0) = \sum_{\mu,\nu} M_{\mu\nu}\rho_A(0)M^{\dagger}_{\mu\nu}$$

$$\xi(t) = \sum_{\mu i j}\langle\mu|U_{AB}\ \gamma'_{ij}\sigma_i \otimes \sigma_j\ U^{\dagger}_{AB}|\mu\rangle$$

$$\left(\frac{\partial}{\partial t} - \widehat{\mathcal{X}}\right)\ [\rho_A(t) - \xi(t)] = 0$$

$$\widehat{\mathcal{X}} = \left(\frac{\partial}{\partial t}\widehat{\mathcal{T}}(t)\right)\ \frac{1}{\widehat{\mathcal{T}}(t)}$$ Liouvillian superoperator

Cvasi-classical results:

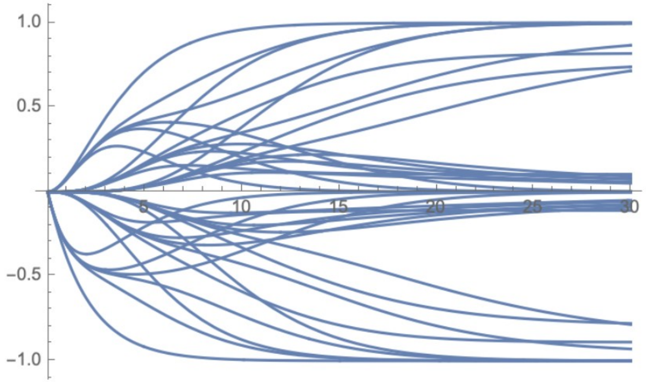


FIG. 1: evolution of the diagonal and off diagonal density matrix elements in a non-engineered environment which is not a learning material

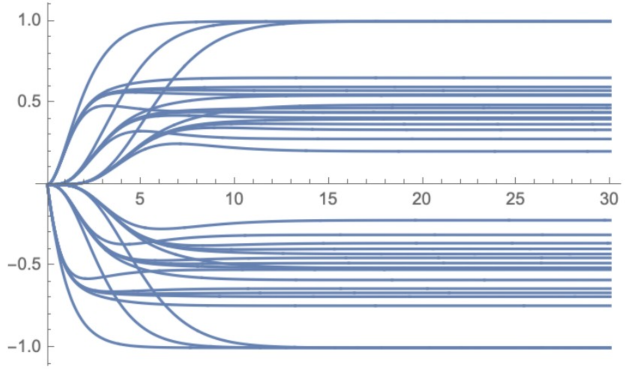


FIG. 2: evolution of the diagonal and off diagonal density matrix elements in a neural learning material, a material capable of implementing a learning dynamics according to a cost function represented in a Hamiltonian form