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Interval estimation of an extended capability index with application in educational systems

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Abstract

Most evaluations on process capability indices focus on point estimates, which may result in unreliable assessments of process potential. The new index $C_{\tilde{p}}$ introduced by Parchami et al. (2010) can be used in various manufacturing industries to provide a quantitative measurement of the potential of a fuzzy process. In this paper, based on the fuzzy set theory, we propose to construct a $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$ index, where instead of precise specification limits we have two membership functions for upper and lower specification limits. A simulation study is given to explore the coverage probabilities of the introduced confidence interval. Some educational applications are given to show the performance of the proposed method.

Keywords: Fuzzy quality, confidence interval, process capability index, simulation, fuzzy specification limits

1. Introduction

When a process is in a perfect state of statistical control, the process capability indices (PCIs) are widely used to measure the capability of the process to manufacture items within the required tolerance limits. Several PCIs are introduced in the literatures which are used to estimate the capability of a manufacturing process. Let the corresponding random variate be denoted by X and the expected value and the standard deviation of X be denoted by μ and σ, respectively. When univariate measurements are concerned and $\mu = M$, with $M = (USL + LSL)/2$, the first generation of PCIs (Kotz and Johnson, 1993, 2002; Montgomery, 2005) is introduced by

$$C_{\tilde{p}} = \frac{USL - LSL}{2}, \tag{1}$$

where the upper and lower specification limits denoted by *USL* and *LSL*, respectively. In quality control, we may confront imprecise concepts. One case is a situation in which upper and lower specification limits (SLs) are imprecise. If we introduce vagueness into SLs and express them by fuzzy terms, we face quite new, reasonable and interesting processes, and the ordinary capability indices are not appropriate for measuring the capability of these processes. Recently, several PCIs developed for such situation by authors (Lee, 2001; Moeti et al., 2006; Parchami and Mashinchi, 2005, 2007, 2010; Ramezani et al., 2011; Tsai and Chen, 2006; Yongting, 1996) which are applied in different real situations (Kahraman and Kaya, 2009a, 2009b; Kaya and Kahraman, 2007, 2008, 2009a, 2009b, 2010a, 2010b; Mashinchi et al., 2005). In this paper, based on fuzzy set theory, we are going to discuss on the point and interval estimators of the new capability index $C_{\tilde{p}}$ which has been introduced in Parchami and Mashinchi (2010). Then an application of this index in an educational system will be given.

The organization of this paper is as follows. In Section 2, we reintroduce and recall some preliminary definitions such as fuzzy SLs, arithmetic operation on fuzzy SLs, the extended PCI and a point estimator for the extended PCI. In Section 3, a $100(1-\gamma)\%$ confidence interval for the extended capability index $C_{\tilde{p}}$ is constructed. A simulation study is done in Section 4 to explore the coverage probabilities of the introduced confidence interval. In Section 5, the presented approach is applied to different teaching processes. Conclusions and future research works are drawn in Section 6.

2. Preliminaries

Let R be the set of all real numbers and $F(R) = \{\tilde{A} \mid \tilde{A} : R \to [0,1], \tilde{A}$ is a continuous function$\}$ be the set of all fuzzy sets on R. The α-cut of $\tilde{A} \in F(R)$ is the crisp set given by $\tilde{A}_\alpha = \{x \mid \tilde{A}(x) \geq \alpha\}$, for any $\alpha \in [0,1]$.

In the following some revised definitions are given from Parchami and Mashinchi (2010).

Definition 1 Let $\widetilde{USL} \in F(R)$ be a non-increasing function and there exists $u_1 \in R$ such that $\widetilde{USL}(x) = 1$ for $x \leq u_1$. Then $\widetilde{USL}$ is called an upper fuzzy specification limit.

Definition 2 Let $\widetilde{LSL} \in F(R)$ be a non-decreasing function and there exists $l_1 \in R$ such that $\widetilde{LSL}(x) = 1$ for $x \geq l_1$. Then $\widetilde{LSL}$ is called a lower fuzzy specification limit.

$F_U(R)$ and $F_L(R)$ denote the set of all upper fuzzy specification limits and the set of all lower fuzzy specification limits, respectively.

In the following definition the subtraction of two upper and lower fuzzy specification limits is quoted from Parchami and Mashinchi (2010).

Definition 3 Let $\widetilde{USL} \in F_U(R)$ and $\widetilde{LSL} \in F_L(R)$ are the upper and the lower fuzzy specification limits, such that $l_1 \leq u_1$. Then

$$\widetilde{USL} \text{ Ä } \widetilde{LSL} = \int_0^1 g(\alpha)(u_\alpha - l_\alpha)d\alpha, \tag{2}$$

is called subtraction of $\widetilde{USL}$ and $\widetilde{LSL}$, in which

$$\widetilde{USL}_\alpha = (-\infty, u_\alpha] \text{ , } \widetilde{LSL}_\alpha = [l_\alpha + \infty) \text{, for any } \alpha \in (0,1], \tag{3}$$

u_0, l_0 are finite real numbers and g is a non-decreasing function on [0,1] with $g(0) = 0$ and $\int_0^1 g(\alpha)\,d\alpha = 1$.

As an example of the function g in Definition 3, one can consider the class of functions $g(\alpha) = (m+1)\alpha^m$, $m = 1, 2, 3, \ldots$. For more details on the function g, see (Parchami and Mashinchi, 2010).

For observing some properties about the arithmetic operations on fuzzy limits, and also numerical examples about fuzzy SLs see (Parchami and Mashinchi, 2010). Now, by extending the subtraction operation for the fuzzy limits, one can use A operation to generalize the classical PCIs, when the SLs are fuzzy rather than crisp (Parchami and Mashinchi, 2010). For instance, let in a fuzzy process $\widetilde{USL} \in F_U(R)$ and $\widetilde{LSL} \in F_L(R)$ be the engineering upper and lower fuzzy specification limits, respectively, where $l_1 \leq u_1$. Then the extended process capability indices can be defined as follows

$$C_{\tilde{p}} = \frac{\widetilde{USL} \text{ Ä } \widetilde{LSL}}{6\sigma}. \tag{4}$$

Note that $C_{\tilde{p}}$ is useful when $\mu = \frac{l_1 + u_1}{2}$. Similar to the traditional PCIs, substituting the sample mean and the sample standard deviation in each of the extended capability indices, will provide a point estimate and a large value of estimate implies a better distribution of the quality characteristic.

3. Interval estimation of $C_{\tilde{p}}$

Sometimes, a range of plausible values for an unknown parameter is preferred to a single estimate. We shall discuss how to turn data into what is called confidence interval and show that this can be done in such a manner that definite statements can be made about how confident we are that the true parameter value is in the reported interval. This level of confidence is something that a user or an experimenter can choose. In this section, we propose a new method to obtain a confidence interval for the unknown parameter $C_{\tilde{p}}$, based on the knowledge of the sampling distributions of corresponding estimators.

Theorem 1 Suppose $X_1, \ldots, X_n$ are independent, identically distributed random variables with $N(\mu, \sigma^2)$ and $\widetilde{USL} \in F_U(R)$ and $\widetilde{LSL} \in F_L(R)$ are the engineering fuzzy specification limits, where $l_1 \leq u_1$. Then random interval

$$\left[\hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}, \hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right], \tag{5}$$

is a $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$, in which $\hat{C}_{\tilde{p}} = \dfrac{\widetilde{USL}\,\text{Ä}\,\widetilde{LSL}}{6S}$ is the point estimator of $C_{\tilde{p}}$.

Proof. The only parameter in (4) which should be estimated is *σ*, the standard deviation of *X*. A natural estimator of *σ* is

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2}\,. \tag{6}$$

By assumption, S^2 is distributed as $\sigma^2 \times \chi^2_{n-1}/(n-1)$, see Section 1.5 of Kotz and Johnson (1993). Since $P\{\chi^2_{n-1} \le \chi^2_{n-1,\varepsilon}\} = \varepsilon$, we have

$$P\left\{\chi^2_{n-1,\alpha/2} \le \frac{(n-1)S^2}{\sigma^2} \le \chi^2_{n-1,1-\alpha/2}\right\} = 1-\alpha.$$

Therefore

$$P\left\{\frac{1}{6S}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} \le \frac{1}{6\sigma} \le \frac{1}{6S}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right\} = 1-\alpha,$$

$$P\left\{\frac{\widetilde{USL}\,\text{Ä}\,\widetilde{LSL}}{6S}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} \le \frac{\widetilde{USL}\,\text{Ä}\,\widetilde{LSL}}{6\sigma} \le \frac{\widetilde{USL}\,\text{Ä}\,\widetilde{LSL}}{6S}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right\} = 1-\alpha, \tag{7}$$

$$P\left\{\hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} \le C_{\tilde{p}} \le \hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right\} = 1-\alpha. \quad \blacksquare \tag{8}$$

To show the behavior of the given confidence interval in Theorem 1, a table is prepared. Table 1 contains the 95% confidence intervals of $C_{\tilde{p}}$ for the various $\hat{C}_{\tilde{p}}$ and sample size, which are computed by (5). These intervals imply the perfect reaction of the introduced $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$ to the sample size *n*. In fact as *n* increases, the length of this interval decreases.

Table 1. The 95% interval estimates for several given values of $\hat{C}_{\tilde{p}}$ and *n* in a fuzzy process

$\hat{C}_{\tilde{p}}$	*n* = 25	*n* = 50	*n* = 10	*n* = 150	*n* = 200
0.75	[0.54, 0.96]	[0.60, 0.90]	[0.65, 0.85]	[0.66, 0.83]	[0.68, 0.82]
1.00	[0.72, 1.28]	[0.80, 1.20]	[0.86, 1.14]	[0.89, 1.11]	[0.90, 1.10]
1.33	[0.96, 1.70]	[1.07, 1.59]	[1.14, 1.51]	[1.18, 1.48]	[1.20, 1.46]
1.50	[1.08, 1.92]	[1.20, 1.80]	[1.29, 1.71]	[1.33, 1.67]	[1.35, 1.65]
1.67	[1.20, 2.14]	[1.34, 2.00]	[1.44, 1.90]	[1.48, 1.86]	[1.51, 1.83]
2.00	[1.44, 2.56]	[1.60, 2.39]	[1.72, 2.28]	[1.77, 2.23]	[1.80, 2.20]

Definition 4 $\widetilde{USL}_{\mathrm{L}} \in F_U(R)$ and $\widetilde{LSL}_{\mathrm{L}} \in F_L(R)$ are said to be linear upper and lower fuzzy SLs, respectively, if their membership functions can be expressed as

$$\widetilde{USL}_{\mathrm{L}}(x)=\begin{cases} 1 & \text{if} \quad x \le u_1, \\ \dfrac{x-u_0}{u_1-u_0} & \text{if} \quad u_1 < x < u_0, \\ 0 & \text{if} \quad u_0 \le x, \end{cases} \tag{9}$$

and

$$\widetilde{LSL}_{\mathrm{L}}(x)=\begin{cases} 0 & \text{if} \quad x \le l_0, \\ \dfrac{x-l_0}{l_1-l_0} & \text{if} \quad l_0 < x < l_1, \\ 1 & \text{if} \quad l_1 \le x, \end{cases} \tag{10}$$

where $u_1 \le u_0$, $l_0 \le l_1$ and $u_0, u_1, l_0, l_1 \in R$. For convenient, we denote them by $\widetilde{USL}_{\mathrm{L}} = (u_1, u_0)_{UL}$ and $\widetilde{LSL}_{\mathrm{L}} = (l_0, l_1)_{LL}$, respectively (Parchami and Mashinchi, 2010).

Definition 5 $\widetilde{USL}_E \in F_U(R)$ and $\widetilde{LSL}_E \in F_L(R)$ are said to be exponential upper and lower fuzzy SLs, respectively, if their membership functions can be expressed as

$$\widetilde{USL}_E(x)=\begin{cases} 1 & \text{if} \quad x \le u_1, \\ e^{-(\frac{x-u_1}{s_u})^2} & \text{if} \quad x > u_1, \end{cases} \tag{11}$$

and

$$\widetilde{LSL}_E(x)=\begin{cases} e^{-(\frac{x-l_1}{s_l})^2} & \text{if} \quad x < l_1, \\ 1 & \text{if} \quad x \ge l_1, \end{cases} \tag{12}$$

where u_1, s_u, $l_1, s_l \in R$ and $s_u, s_l > 0$. For convenient, we denote them by $\widetilde{USL}_{\mathrm{E}} = (u_1, s_u)_{UE}$ and $\widetilde{LSL}_{\mathrm{E}} = (l_1, s_l)_{LE}$, respectively (Parchami et al. 2010).

Using the introduced operation in Definition 3, we are going to construct two $100(1-\gamma)\%$ confidence intervals for the extended capability index $C_{\tilde{p}}$ in a fuzzy process. By using linear fuzzy SLs and exponential fuzzy SLs we have the following two corollaries.

Corollary 1 Suppose $X_1,...,X_n$ are independent, identically distributed random variables with $N(\mu,\sigma^2)$ and, $\widetilde{USL}_{\mathrm{L}} = (u_1, u_0)_{UL} \in F_U(R)$ and $\widetilde{LSL}_{\mathrm{L}} = (l_0, l_1)_{LL} \in F_L(R)$ are the fuzzy linear specification limits where $l_1 \le u_1$. If $g(\alpha) = (m+1)\alpha^m$, $m = 1, 2, 3, \ldots$, then $\frac{1}{6S(m+2)}[(m+1)(u_1 - l_1) + (u_0 - l_0)]$ is a point estimator of $C_{\tilde{p}}$, and random interval

$$\left[\frac{(m+1)(u_1-l_1)+(u_0-l_0)}{6S(m+2)} \sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}} \,,\, \frac{(m+1)(u_1-l_1)+(u_0-l_0)}{6S(m+2)} \sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}} \right] \tag{13}$$

is a $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$.

Proof. The cut sets of the linear fuzzy SLs $\widetilde{USL}_{\mathrm{L}}$ and $\widetilde{LSL}_{\mathrm{L}}$ can be expressed as $\widetilde{USL}_{\mathrm{L}\alpha} = (-\infty, u_\alpha]$ and $\widetilde{LSL}_{\mathrm{L}\alpha} = [l_\alpha, +\infty)$, respectively, where $u_\alpha = u_0 - \alpha(u_0 - u_1)$ and

$l_\alpha = l_0 + \alpha(l_1 - l_0)$. Hence, for $m = 1, 2, 3, \ldots$, the point estimator of $C_{\tilde{p}}$ can compute as follow

$$\hat{C}_{\tilde{p}} = \frac{\widetilde{USL}_{\mathrm{L}} \text{ Ä } \widetilde{LSL}_{\mathrm{L}}}{6S}$$

$$= \frac{\int_0^1 (m+1)\alpha^m (u_\alpha - l_\alpha) d\alpha}{6S}$$

$$= \frac{1}{6S}\int_0^1 (m+1)\alpha^m \left[u_0 - \alpha(u_0 - u_1) - l_0 - \alpha(l_1 - l_0)\right] d\alpha$$

$$= \frac{m+1}{6S}\int_0^1 \alpha^{m+1}\left[(u_1 - l_1) + (l_0 - u_0)\right] d\alpha + \frac{m+1}{6S}\int_0^1 \alpha^m \left[u_0 - l_0\right] d\alpha$$

$$= \frac{m+1}{6S\,(m+2)}\left[(u_1 - l_1) + (l_0 - u_0)\right] + \frac{1}{6S}\left[u_0 - l_0\right]$$

$$= \frac{1}{6S\,(m+2)}\left[(m+1)(u_1 - l_1) + (u_0 - l_0)\right].$$

Therefore, by Theorem 1, one can assert that

$$\left[\hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\,,\ \hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right] =$$

$$\left[\frac{(m+1)(u_1 - l_1) + (u_0 - l_0)}{6S\,(m+2)}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\,,\ \frac{(m+1)(u_1 - l_1) + (u_0 - l_0)}{6S\,(m+2)}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right]$$

be a $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$. ■

Corollary 2 Under the same assumption as in Corollary 1, but by using $\widetilde{USL}_{\mathrm{E}} = (u_1, s_u)_{UE} \in F_U(R)$ and $\widetilde{LSL}_{\mathrm{E}} = (l_1, s_l)_{LE} \in F_L(R)$ as exponential fuzzy specific limits, $\frac{u_1 - l_1}{6S} + \frac{s_u + s_l}{12S}\sqrt{\frac{\pi}{m+1}}$ is a point estimator of $C_{\tilde{p}}$, and random interval

$$\left[\left(\frac{u_1 - l_1}{6S} + \frac{s_u + s_l}{12S}\sqrt{\frac{\pi}{m+1}}\right)\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\,,\ \left(\frac{u_1 - l_1}{6S} + \frac{s_u + s_l}{12S}\sqrt{\frac{\pi}{m+1}}\right)\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right] \quad (14)$$

is a $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$.

Proof. The cut sets of the exponential fuzzy SLs $\widetilde{USL}_{\mathrm{E}}$ and $\widetilde{LSL}_{\mathrm{E}}$ can be expressed as $\widetilde{USL}_{\mathrm{E}\alpha} = (-\infty, u_\alpha]$ and $\widetilde{LSL}_{\mathrm{E}\alpha} = [l_\alpha, +\infty)$, respectively, where $u_\alpha = u_1 + s_u\sqrt{-Ln\,\alpha}$ and $l_\alpha = l_1 - s_l\sqrt{-Ln\,\alpha}$. Hence, for $m = 1, 2, 3, \ldots$, the point estimator of $C_{\tilde{p}}$ can compute as follow

$$\hat{C}_{\tilde{p}} = \frac{\widetilde{USL}_{\mathrm{E}} \text{ Ä } \widetilde{LSL}_{\mathrm{E}}}{6S}$$

$$= \frac{\int_0^1 (m+1)\alpha^m (u_\alpha - l_\alpha) d\alpha}{6S}$$

$$= \frac{1}{6S}\int_0^1 (m+1)\alpha^m \left[u_1 + s_u\sqrt{-Ln\,\alpha} - l_1 + s_l\sqrt{-Ln\,\alpha}\right] d\alpha$$
$$= \frac{m+1}{6S}\int_0^1 \alpha^m \left[u_1 - l_1\right] d\alpha + \frac{m+1}{6S}\int_0^1 \alpha^m \sqrt{-Ln\,\alpha}\,(s_u + s_l)\, d\alpha$$
$$= \frac{u_1 - l_1}{6S} + \frac{(m+1)(s_u + s_l)}{6S}\int_0^1 \alpha^m \sqrt{-Ln\,\alpha}\, d\alpha$$
$$= \frac{u_1 - l_1}{6S} + \frac{s_u + s_l}{12S}\sqrt{\frac{\pi}{m+1}}.$$

Therefore, by Theorem 1, one can assert that

$$\left[\hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\, , \hat{C}_{\tilde{p}}\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right] =$$
$$\left[\left(\frac{u_1 - l_1}{6S} + \frac{s_u + s_l}{12S}\sqrt{\frac{\pi}{m+1}}\right)\sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}\, , \left(\frac{u_1 - l_1}{6S} + \frac{s_u + s_l}{12S}\sqrt{\frac{\pi}{m+1}}\right)\sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right]$$

be a $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$. ■

Remark 1 By Corollary 1 and Corollary 2, one can see that the introduced $100(1-\gamma)\%$ confidence interval in Theorem 1 could be computationally simplified, by using the linear and the exponential fuzzy SLs. Moreover, these two kinds of the fuzzy SLs are convenient and they can be used in many industrial applications (Parchami and Mashinchi, 2010).

4. Simulation study

We did a simulation study for Corollary 1 to investigate the proportion of time that the interval estimators of $C_{\tilde{p}}$ contained $C_{\tilde{p}}$. For $\widetilde{LSL}_{\mathrm{L}} = (-3,-1)_{LL} \in F_L(R)$ and $\widetilde{USL}_{\mathrm{L}} = (4,9)_{UL} \in F_U(R)$, where the mean, standard deviation and sample size are changed over the following ranges

μ = −2, −1, 0, 1 and 2,
σ = 0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75 and 2.0,
n = 25, 50, 75, 100, 125, 150, 175 and 200.

For each combination of μ, σ and n (320 cases) 6000 simulated independent random samples from a normal distribution were generated by R software. For each sample we obtained the sample average $(\bar{x})$, sample standard deviation (s) and the capability index $C_{\tilde{p}}$ by (4). Then, (13) is used to obtain two confidence regions of $C_{\tilde{p}}$ in 95% and 99% levels.

The simulations captured the proportion of time that the true $C_{\tilde{p}}$ was contained in the intervals, as well as proportion of time that $C_{\tilde{p}}$ was below or above the intervals (Zhang et al., 1990). The results of simulation show that the true $C_{\tilde{p}}$ lies within 95% confidence intervals roughly 94.99% of the time and lies within 99% confidence intervals roughly 98.99% of the time.

Tables 2 and 3 can be used to place some measure of confidence on the interval estimate. They show, for 320 cases in the study, the average, minimum and maximum proportion of times that the true $C_{\tilde{p}}$ was below, within and above 95% and 99% confidence intervals. Our calculations indicate that the sampling distribution of $C_{\tilde{p}}$ is slightly skewed but that the skewness is not great and the suggested interval is a symmetric interval estimator.

Table 2. Statistics for 95% interval estimation coverage of $C_{\tilde{p}}$ based on the 320 cases in simulation study with linear SLs

	Below	Within	Above
minimum proportion	0.01916667	0.9415000	0.01950000
average proportion	0.02485521	0.9499724	0.02517240
maximum proportion	0.03116667	0.9576667	0.03116667

Table 3. Statistics for 99% interval estimation coverage of $C_{\tilde{p}}$ based on the 320 cases in simulation study with linear SLs

	Below	Within	Above
minimum proportion	0.002500000	0.9860000	0.002666667
average proportion	0.005120833	0.9899240	0.004955208
maximum proportion	0.007833333	0.9931667	0.007666667

The result of a similar simulation study on the proportion of time that the interval estimator of $C_{\tilde{p}}$ proposed in Corollary 2 contained $C_{\tilde{p}}$ has been shown in Tables 4 and 5, where $\widetilde{USL}_{\mathrm{E}} = (4, 2.5)_{UE} \in F_U(R)$ and $\widetilde{LSL}_{\mathrm{E}} = (-1,1)_{LE} \in F_L(R)$ considered as the exponential fuzzy specification limits. Our simulation results confirm that the proportion of time that the true $C_{\tilde{p}}$ was above the interval was not too different from the proportion of time that it was below the interval.

Table 4. Statistics for 95% interval estimation coverage of $C_{\tilde{p}}$ based on the 320 cases in simulation study with exponential SLs

	Below	Within	Above
minimum proportion	0.02066667	0.9420000	0.01916667
average proportion	0.02496563	0.9498932	0.02514115
maximum proportion	0.03133333	0.9578333	0.03083333

Table 5. Statistics for 99% interval estimation coverage of $C_{\tilde{p}}$ based on the 320 cases in simulation study with exponential SLs

	Below	Within	Above
minimum proportion	0.002333333	0.9856667	0.002833333
average proportion	0.005046354	0.9899484	0.005005208
maximum proportion	0.007500000	0.9935000	0.008000000

5. A real educational application

To understand why and when the specification limits are considered non-precise sets, one can see several real application examples in Kahraman and Kaya (2009a, 2009b), Kaya and Kahraman (2007, 2008, 2009a, 2009b, 2010a, 2010b) and Mashinchi et al. (2005). In the following we are going to discuss this situation for the evaluation of educational systems.

Example 1 We are going to analyze several educational processes in the following real application, by the aid of introduced fuzzy limits. We wish to perform interval estimation from the capability of educational processes at Shahid Bahonar University of Kerman (SBUK), where their students are taken the quality control course with one specific teacher, see (Mashinchi et al., 2005). The grades in two classes are as follow:

Class *A*: 15.25, 12, 12, 13.75, 17, 14.75, 14.5, 19.5, 13.25, 13.5, 15.75, 15.5, 16.5, 15.75, 18.25, 16.5, 13.25.

Class *B*: 14.5, 14.25, 18.75, 16.5, 16.5, 12.25, 12, 13.25, 18, 12.5, 15.25, 14, 18, 13.75, 15.5, 16.25, 14.25, 12, 18.75, 13, 14.25, 12, 14, 15.75, 13.25, 9.5, 16, 14.75, 16.25.

From the data, the following sufficient statistics are computed:

$n_A = 17$, $\bar{x}_A = 15.12$, $s_A = 2.07$, $n_B = 29$, $\bar{x}_B = 14.66$ and $s_B = 2.23$.

We want to compare the students of two classes *A* and *B*. If the processes are evaluated by taking the fuzzy set theory into account, the result will be more sensitive and informative (Kaya and Kahraman, 2009a). Therefore in this situation, using fuzzy SLs is more suitable than using crisp SLs. According to situations and our goals we decide to use the extended capability index $C_{\tilde{p}}$ for this analysis. An expert person decides to employ $\widetilde{LSL}_{\mathrm{L}} = (9,11)_{LL} \in F_L(R)$ and $\widetilde{USL}_{\mathrm{L}} = (17,\ 19.5)_{UL} \in F_U(R)$, as fuzzy linear SLs.

By Anderson-Darling normality test our observations approximately satisfy in the normality condition. Substituting the sample size, mean and standard deviation of grades in the results of Corollary 1, provides the point and interval estimates for the extended capability index in each class, see Table 6. For example in Class *A*, $\hat{C}_{\tilde{p}}(A)$ = 0.6039 is a point estimate and [0.397, 0.810] is a 95% confidence interval for $C_{\tilde{p}}$ where m = 1. Similarly in Class *B*, one can compute $\hat{C}_{\tilde{p}}(B)$ = 0.5605 as a point estimate and [0.414, 0.706] as a 95% confidence interval. Comparison between point estimations of $C_{\tilde{p}}$ in two classes, lead to the comparison between their students, since two classes have one teacher with a similar teaching method and a similar experiment. Therefore, we can conclude that the students in class *A* are more active and capable than the students in class *B*. As previously mentioned one can provide point estimates for PCIs and then compare them for obtaining the best educational system or the most capable procedure. But, since the point estimates of any PCI, like other statistics, is subject to sampling variation; it is important to compute a confidence interval to provide a range which includes the true PCI with a certain and high probability. Also, it must be mentioned that the most evaluations on PCIs focus on only point estimates, which may result in unreliable assessments of process potential, but we obtain two confidence intervals at levels 95% and 99% for $C_{\tilde{p}}$ in Table 6.

Table 6. Point and interval estimates of $C_{\tilde{p}}$ for various *m* in two classes *A* and *B*

Class	*m*	$\hat{C}_{\tilde{p}}$	95% confidence interval	99% confidence interval
A	1	0.6039	[0.397, 0.810]	[0.342, 0.884]
A	2	0.5737	[0.377, 0.770]	[0.325, 0.840]
A	3	0.5556	[0.365, 0.746]	[0.315, 0.813]
A	4	0.5435	[0.357, 0.730]	[0.308, 0.795]
A	5	0.5349	[0.351, 0.718]	[0.303, 0.783]
A	6	0.5284	[0.347, 0.710]	[0.300, 0.773]
B	1	0.5605	[0.414, 0.706]	[0.374, 0.756]
B	2	0.5325	[0.394, 0.671]	[0.355, 0.719]
B	3	0.5157	[0.381, 0.650]	[0.344, 0.696]
B	4	0.5045	[0.373, 0.636]	[0.337, 0.681]
B	5	0.4965	[0.367, 0.626]	[0.331, 0.670]
B	6	0.4905	[0.363, 0.618]	[0.327, 0.662]

Example 2 We wish to perform a comparison between three teachers; Mathematics, Statistics and Quality control, at SBUK. Among the students we choose all those students who have taken 3 mentioned courses with 3 specified teachers. In this application, we just present the computed sufficient statistics rather than all observed grades. From the data the following statistics are computed:

Mathematics: $n_M = 63$, $\bar{x}_M = 13.3$, $s_M = 2.31$;

Statistics: $n_S = 47$, $\bar{x}_S = 14.27$, $s_S = 1.94$;

Quality Control: $n_Q = 19$, $\bar{x}_Q = 14.92$, $s_Q = 1.87$.

An expert person decides to employ the following fuzzy exponential SLs for each course:

Mathematics: $\widetilde{LSL}_{\mathrm{E}} = (9,\ 1.25)_{LE} \in F_L(R)$ and $\widetilde{USL}_{\mathrm{E}} = (17.5,\ 1.5)_{UE} \in F_U(R)$;

Statistics: $\widetilde{LSL}_{\mathrm{E}} = (10,\ 1.2)_{LE} \in F_L(R)$ and $\widetilde{USL}_{\mathrm{E}} = (18,\ 1.4)_{UE} \in F_U(R)$;

Quality control: $\widetilde{LSL}_{\mathrm{E}} = (9.5,\ 0.8)_{LE} \in F_L(R)$ and $\widetilde{USL}_{\mathrm{E}} = (18.5,\ 1.25)_{UE} \in F_U(R)$.

According to the Anderson-Darling normality test the observations approximately satisfy in the normality condition. Substituting the sample size, mean and standard deviation of grades in the results of Corollary 2, provides the point and interval estimates for the extended capability index for each course which are recorded in Table 7. Comparison between point estimations of $C_{\tilde{p}}$ for three courses, leads to the comparison between their three different teaching methods, since the students of three classes are exactly similar. Therefore, according to the point estimation results of Table 7, one can conclude that the teaching method for Quality Control is better than the teaching method for Statistics and Mathematics at SBUK. Also in this university, one can conclude that the teaching method for Statistics is capable and better than the teaching method for Mathematics at SBUK.

Table 7. Point and interval estimates of $C_{\tilde{p}}$ for Mathematics, Statistics and Quality control

	m	$\hat{C}_{\tilde{p}}$	95% confidence interval	99% confidence interval
Mathematics	1	0.738	[0.608, 0.867]	[0.570, 0.910]
Mathematics	2	0.715	[0.589, 0.840]	[0.553, 0.882]
Mathematics	3	0.701	[0.578, 0.824]	[0.542, 0.865]
Mathematics	4	0.692	[0.570, 0.813]	[0.535, 0.854]
Statistics	1	0.827	[0.659, 0.996]	[0.610, 1.052]
Statistics	2	0.802	[0.638, 0.965]	[0.591, 1.020]
Statistics	3	0.786	[0.626, 0.946]	[0.580, 1.000]
Statistics	4	0.776	[0.618, 0.934]	[0.572, 0.987]
Quality Control	1	0.917	[0.620, 1.213]	[0.541, 1.317]
Quality Control	2	0.896	[0.606, 1.185]	[0.528, 1.287]
Quality Control	3	0.883	[0.597, 1.169]	[0.521, 1.269]
Quality Control	4	0.875	[0.591, 1.157]	[0.516, 1.257]

6. Conclusions and future research works

We note that, it is more appropriate some industrial products be evaluated and qualified by an imprecise (fuzzy) quality. In this idea, the products evaluated using two membership functions of specification limits, instead of the traditional precise specification limits. The process capability index $C_{\tilde{p}}$ is used as an easy index to evaluate quality characteristics and process yields in a fuzzy process when the engineering specification limits are imprecise. In this paper, we construct a $100(1-\gamma)\%$ confidence interval for $C_{\tilde{p}}$ index, where instead of real numbers we have two membership functions for upper and lower specification limits. A simulation study is given to explore the coverage probabilities of the introduced confidence interval. Also, the results of this paper are applied in analysis and comparison between the students of two different classes and three different teaching processes. This is done by using capability index $C_{\tilde{p}}$ when specification limits are considered fuzzy. This application can bring up the extended capability index $C_{\tilde{p}}$ to the comparison of other educational systems, such as comparing capability indices of two or more teachers, schools and so on. As a future research, this approach can be applied to other process capability indices such as C_{pm}, C_{pk} and C_{pmk}. Note that finding confidence intervals for $C_{\widetilde{pm}}$, $C_{\widetilde{pk}}$ and $C_{\widetilde{pmk}}$ needs more insight and research and it is not an easy task to be included in this paper which we leave it to a future work. Also as another future work, fuzzy mean and fuzzy standard deviation can be incorporated into this analysis under fuzzy environment.

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On discriminating fuzzy numbers

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Abstract

Ranking fuzzy numbers plays a very important role in linguistic decision making and some other fuzzy application systems such as data analysis, artificial intelligence and socio economic systems. Various approaches have been proposed in the literature for the ranking of fuzzy numbers and most of the methods seem to suffer from drawbacks. In this paper a new method is proposed to rank fuzzy numbers. This method is based on the centroid of centroids of generalized trapezoidal fuzzy numbers and allows the participation of decision maker by using an index of optimism to reflect the decision maker's optimistic attitude and also an index of modality that represents the importance of considering the areas of spreads by the decision maker. This method is relatively simple and easier in computation and ranks various types of fuzzy numbers along with crisp fuzzy numbers as special case of fuzzy numbers.

Keywords: Fuzzy numbers, centroid points, ranking index, index of optimism, index of modality.

1. Introduction

Non-random impreciseness or vagueness of numeric quantities occur frequently in fields like decision making, social sciences and control theory. Fuzzy numbers provide a useful tool for modeling these kinds of situations. In fuzzy decision analysis, fuzzy quantities are used to describe the performance of alternatives in modeling a real-world problem. As fuzzy numbers are represented by possibility distributions, they may overlap with each other and hence it is not possible to order them. Ranking fuzzy numbers is an important aspect of decision making in a fuzzy environment. Since the inception of fuzzy sets by (Zadeh, 1965), many authors have proposed different methods for ranking fuzzy numbers. However, due to the complexity of the problem, there is no method which gives a satisfactory result to all situations. It is true that fuzzy numbers are frequently partial order and cannot be compared like real numbers which can be linearly ordered.

Ranking fuzzy numbers was first proposed by (Jain, 1976) for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. (Jain, 1976, 1977) proposed a method using the concept of maximizing set to order the fuzzy numbers and the decision maker considers only the right side membership function. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. (Yager, 1980, 1981) proposed four indices to order fuzzy quantities in [*0*, *1*]. (Adamo, 1980) fuzzy decision trees was an important breakthrough in ranking procedures. (Bortolan and Degani, 1985) reviewed some of these ranking methods for ranking fuzzy subsets. (Chen, 1985) presented ranking fuzzy numbers with maximizing set and minimizing set and recently, this paper was revised by (Chou et al., 2011). (Kim and Park, 1990) presented a method of ranking fuzzy numbers with index of optimism. (Liou and Wang, 1992) presented ranking fuzzy numbers with integral value. (Choobineh and Li, 1993) presented an index for ordering fuzzy numbers. (Fortemps and Roubens, 1996) proposed ranking fuzzy numbers using area compensation. (Yao and Wu, 2000) proposed ranking fuzzy quantities by decomposition principle and signed distance. (Chen and Lu, 2001) proposed ranking fuzzy numbers by left and right dominance.

(Wang and Kerre, 2001) classified all the existing ranking procedures into three classes. The first class consists of ranking procedures where each index is associated with a mapping F from the set of fuzzy quantities to the real line R in order to transform the involved fuzzy quantities into real numbers and compared according to the corresponding real numbers. In the second class the fuzzy quantities to be ranked are compared with the reference set(s), whereas the third class consists of methods based on construction of a fuzzy relation to make pair wise comparisons between the fuzzy quantities and concluded that the ordering procedures associated with first class are relatively reasonable for the ordering of fuzzy numbers specially the ranking procedure presented by (Adamo, 1980) which satisfies all the reasonable properties for the ordering of fuzzy quantities. The methods presented in the second class are not doing well and the methods which belong to class three are reasonable.

Ranking fuzzy numbers based on centroids was proposed by (Cheng, 1998) where he used a ranking index which calculates, the distance between the centroid point of generalized trapezoidal fuzzy number and origin. Later on, (Chu and Tsao, 2002) proposed a ranking index which finds the area between the centroid point and original point to compare fuzzy quantities. But, these methods failed to rank crisp numbers and (Wang et al., 2006) pointed out that their centroid formulas are incorrect and proposed the correct centroid formula for ranking fuzzy quantities. Later (Garcia and Lamata, 2007) modified the index of (Liou and Wang, 1992) for ranking fuzzy numbers, by stating that the index of optimism is not alone sufficient to discriminate fuzzy numbers and proposed an index of modality to rank fuzzy numbers. (Wang et al., 2008) proposed the revised method of ranking fuzzy numbers with an area between the centroid and original points.

Recently, (Kumar et al., 2010) presented a procedure on ranking generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread (Kumar et al., 2011) also extended the work of (Chen et al., 2008) for ranking nonnormal p-norm trapezoidal fuzzy numbers with integral value with equal height to different heights. (Rao and Shankar, 2011) presented a method on ranking fuzzy numbers using circumcenter of centroids and index of modality. Most of the methods proposed so far are non-discriminating, counter-intuitive and some produce different rankings for the same situation whereas, some methods cannot rank crisp numbers which are a special case of fuzzy numbers.

In order to rank fuzzy quantities, each fuzzy quantity is converted into a real number and compared by defining a ranking index from the set of fuzzy numbers to a set of real numbers which assigns a real number to each fuzzy number where a natural order exists. Usually by reducing the whole of any analysis to a single number, much of the information is lost and most of the ranking methods consider only one point of view in comparing fuzzy quantities. Hence an attempt is to be made to minimize this loss.
In this paper a new method is proposed which is based on Centroid of Centroids to rank fuzzy quantities. In a trapezoidal fuzzy number, first the trapezoid is split into three parts where the first, second and third parts are a triangle, a rectangle and a triangle respectively. Then the centroids of these three parts are calculated followed by the calculation of the centroid of these centroids. Finally, a ranking procedure is defined which is the convex combination of the x -coordinate and y -coordinate of centroid of centroids using an index of optimism to reflect the decision maker's optimistic attitude and also uses an index of modality that represents the importance of considering the areas of spreads by the decision maker in those cases where the centroid of centroids is not sufficient to discriminate fuzzy quantities. The method proposed in this paper falls in first class of the methods as classified by (Wang and Kerre, 2001).

The rest of the paper is organized as follows. Section 2 briefly introduces the basic concepts and definitions of fuzzy numbers. Section 3 presents the proposed new method. In Section 4, validity of some natural properties of centroids are proposed and proved. In Section 5 the proposed method has been explained with examples which describe the advantages and the efficiency of the method which ranks generalized fuzzy numbers, images of fuzzy numbers and even crisp numbers. In Section 6 the method demonstrates its robustness by comparing with other methods like Liou and Wang, Yager and others where the methods cannot discriminate fuzzy quantities and do not agree with human intuition. The conclusions of the work are presented in Section 7.

2. Basic Definitions

In this section, some basic definitions are reviewed.

Definition 2.1 Let U be a universe set. A fuzzy set $\tilde{A}$ of U is defined by a membership function $f_{\tilde{A}} : U \to [0,1]$ where $f_{\tilde{A}}(x)$ is the degree of x in $\tilde{A}$, $\forall x \in U$.

Definition 2.2 A fuzzy set $\tilde{A}$ of universe set U is normal if and only if $Sup_{x \in U} f_{\tilde{A}}(x) = 1$

Definition 2.3 A fuzzy set $\tilde{A}$ of universe set U is convex if and only if

$$f_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min\left(f_{\tilde{A}}(x), f_{\tilde{A}}(y)\right), \forall x, y \in U \text{ and } \lambda \in [0,1]$$

Definition 2.4 A fuzzy set $\tilde{A}$ of universe set *U* is a fuzzy number if and only if $\tilde{A}$ is normal bounded (bounded support) and convex on *U*.

Definition 2.5 A real fuzzy number $\tilde{A}$ is described as any fuzzy subset of the real line *R* with membership function $f_{\tilde{A}}(x)$ possessing the following properties:

(i) $f_{\tilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to the closed interval $[0,w]; 0<w\leq 1$

(ii) $f_{\tilde{A}}(x)=0$, for all $x\in(-\infty,a]\cup[d,\infty)$

(iii) $f_{\tilde{A}}(x)$ is strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$

(iv) $f_{\tilde{A}}(x)=1$, for all $x\in[b,c]$ where *a, b, c, d* are real numbers.

Definition 2.6 The membership function of the real fuzzy number $\tilde{A}$ is given by

$$f_{\tilde{A}}(x)=\begin{cases} f^{L}_{\tilde{A}}(x), & a\leq x<b,\\ w, & b\leq x\leq c,\\ f^{R}_{\tilde{A}}(x), & c<x\leq d,\\ 0, & otherwise,\end{cases}$$

where $0<w\leq 1$ is a constant, *a, b, c, d* are real numbers and $f^{L}_{\tilde{A}}:[a,b]\rightarrow[0,w]$ $f^{R}_{\tilde{A}}:[c,d]\rightarrow[0,w]$ are two strictly monotonic and continuous functions from *R* to the closed interval $[0,w]$. It is customary to write a fuzzy number as $\tilde{A}=(a,b,c,d;w)$. If $w=1$, then $\tilde{A}=(a,b,c,d;1)$ is a normalized fuzzy number, otherwise $\tilde{A}$ is said to be a generalized or non-normal fuzzy number if $0<w<1$.

If the membership function is $f_{\tilde{A}}(x)$ piecewise linear, then $\tilde{A}$ is said to be a trapezoidal fuzzy number. The membership function of a trapezoidal fuzzy number is given by:

$$f_{\tilde{A}}(x)=\begin{cases} \dfrac{w(x-a)}{b-a}, & a\leq x<b,\\ w, & b\leq x\leq c,\\ \dfrac{w(x-d)}{c-d}, & c<x\leq d,\\ 0, & otherwise.\end{cases}$$

If *w*=1, then $\tilde{A}=(a,b,c,d;1)$ is a normalized trapezoidal fuzzy number, otherwise $\tilde{A}$ is a generalized or non normal trapezoidal fuzzy number if $0<w<1$.

The image of $\tilde{A}=(a,b,c,d;w)$ is given by $-\tilde{A}=(-d,-c,-b,-a;w)$.

As a particular case if $b = c$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A} = (a, b, d; w)$. The value of '*b*' corresponds with the mode or core and [*a*, *d*] with the support. If *w*=1, then $\tilde{A} = (a, b, d)$ is a normalized triangular fuzzy number, otherwise $\tilde{A}$ is a generalized or non normal triangular fuzzy number if $0 < w < 1$.

3. Proposed Method

The centroid of a trapezoid is considered to be the balancing point of the trapezoid (Fig.1). Divide the trapezoid into three plane figures. These three plane figures are a triangle (APB), a rectangle (BPQC) and again a triangle (CQD) respectively. Let the centroids of the three plane figures be $G_1, G_2 \& G_3$ respectively. The centroid of these centroids $G_1, G_2 \& G_3$ is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point (G_1 of triangle APB, G_2 of rectangle BPQC and G_3 of triangle CQD) are balancing points of each individual plane figure and the centroid of these centroid points is a better balancing point than the centroid of the trapezoid because it is a centroid.

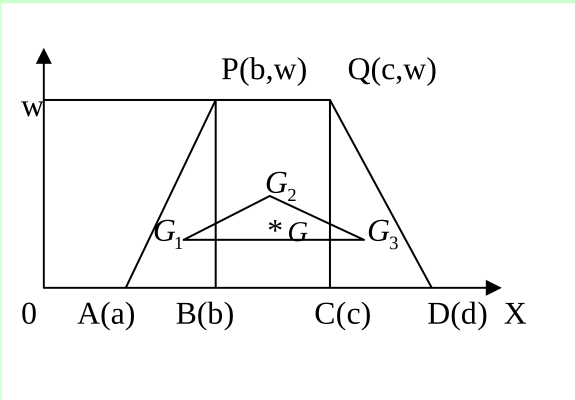

Figure 1. Centroid of Centroids

Consider a generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ (Fig. 1)

The Centroids of the three plane figures are:

$G_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right), G_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right)$ and $G_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right)$ respectively.

Equation of the line $\overleftrightarrow{G_1G_3}$ is $y = \frac{w}{3}$ and G_2 does not lie on the line $\overleftrightarrow{G_1G_3}$. Therefore, G_1, G_2 and G_3 are non-collinear and they form a triangle.

We define the Centroid $G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right)$ of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ as

$$G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right) = \left(\frac{2a + 7b + 7c + 2d}{18}, \frac{7w}{18}\right) \quad (1)$$

As a special case, for triangular fuzzy number $\tilde{A} = (a, b, d; w)$ i.e., $c = b$ the Centroid of Centroids is given by

$$G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right) = \left(\frac{a + 7b + d}{9}, \frac{7w}{18}\right) \quad (2)$$

We define the areas of spreads of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ with reference to *y*-axis as:

The mode spread area of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ with reference to *y*-axis is defined as:

Mode spread area = $\frac{1}{2}\int_0^w (b + c)\, dx = \frac{w}{2}(b + c)$

The total spread area of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ with reference to *y*-axis is defined as:

Total Spread area = $\int_0^w (d - a)\, dx = w(d - a)$

The left spread area of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ with reference to *y*-axis is defined as:

Left spread area = $\int_0^w (b - a)\, dx = w(b - a)$

The middle spread area of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ with reference to *y*-axis is defined as:

Middle spread area = $\int_0^w (c - b)\, dx = w(c - b)$

The right spread area of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ with reference to *y*-axis is defined as:

Right spread area = $\int_0^w (d - c)\, dx = w(d - c)$

For a generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, with Centroid of Centroids $G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right)$ defined by (1) we define the index associated with the ranking as

$$I_\alpha\left(\tilde{A}\right) = \alpha \overline{y_0} + (1 - \alpha)\overline{x_0} \quad (3)$$

which is a convex combination of the coordinates of the centroid of centroids. Here $\alpha \in [0,1]$ is the index of optimism which represents the degree of optimism of a decision maker. $\alpha = 0$ represent a pessimistic decision maker's view point of assigning a value to generalized fuzzy number $\tilde{A} = (a,b,c,d;w)$ for ranking, which is equal to the distance of the Centroid of Centroids from *y*-axis. $\alpha = 1$ represent an optimistic decision maker's view point of assigning a value to generalized fuzzy number $\tilde{A} = (a,b,c,d;w)$ for ranking, and is equal to the distance of the Centroid of Centroids from *x*-axis, and $\alpha = 0.5$ represent the moderate decision maker's view point of assigning a value to generalized fuzzy number $\tilde{A} = (a,b,c,d;w)$ for ranking, and is equal to the mean of the distances of Centroid of Centroids from *y* and *x* axes. The larger the value of α is, the higher the degree of optimism of the decision maker. The index of optimism is not alone sufficient to discriminate fuzzy numbers as this uses only the coordinates of the Centroids of Centroids. Due to the factor $\frac{7w}{18}$ in the centroid formula which depends on *w*, an optimistic decision maker always assign the value $\frac{7w}{18}$ for ranking to all the fuzzy numbers with same *w* value and tends to conclude that the fuzzy numbers are equal for which *w* is same irrespective of spreads. This is not true and hence, we upgrade this by using an index of modality which represents the importance of area of spreads along with index of optimism.

For a generalized trapezoidal fuzzy number $\tilde{A} = (a,b,c,d;w)$, with centroid of centroids $G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right)$ defined by (1) we define the following indexes associated with the ranking as:

$$I^1_{\alpha,\beta}\left(\tilde{A}\right) = \beta w\left(\frac{b+c}{2}\right) + (1-\beta)\, I_\alpha\left(\tilde{A}\right) \tag{4}$$

$$I^2_{\alpha,\beta}\left(\tilde{A}\right) = \beta w(d-a) + (1-\beta)\, I^1_{\alpha,\beta}\left(\tilde{A}\right) \tag{5}$$

$$I^3_{\alpha,\beta}\left(\tilde{A}\right) = \beta w(b-a) + (1-\beta)\, I^2_{\alpha,\beta}\left(\tilde{A}\right) \tag{6}$$

$$I^4_{\alpha,\beta}\left(\tilde{A}\right) = \beta w(d-c) + (1-\beta)\, I^3_{\alpha,\beta}\left(\tilde{A}\right) \tag{7}$$

where $\beta \in (0,1]$ is the index of modality which represents the importance of area of spreads against the coordinates of centroid of centroids $\overline{x_0}$ and $\overline{y_0}$ and $I_\alpha\left(\tilde{A}\right)$ is the one which is defined in (3). Here β represents the weight of some particular area spread and $(1-\beta)$ is the weight associated with coordinates of centroid of centroids $\overline{x_0}$ and $\overline{y_0}$.

Now, we present an algorithm for ranking fuzzy numbers as follows:

Let $\tilde{A}_i$ and $\tilde{A}_j$ be two fuzzy numbers, $\alpha \in [0,1]$ and $\beta \in (0,1]$

Step a) If $I^1_{\alpha,\beta}\left(\tilde{A}_i\right) > I^1_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \succ \tilde{A}_j$, end of algorithm;

Step b) If $I^1_{\alpha,\beta}\left(\tilde{A}_i\right) < I^1_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \prec \tilde{A}_j$, end of algorithm;

Step c) If $I^1_{\alpha,\beta}\left(\tilde{A}_i\right) = I^1_{\alpha,\beta}\left(\tilde{A}_j\right)$ for at least one decision maker then:

Step c1) If $I^2_{\alpha,\beta}\left(\tilde{A}_i\right) > I^2_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \prec \tilde{A}_j$, end of algorithm;

Step c2) If $I^2_{\alpha,\beta}\left(\tilde{A}_i\right) < I^2_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \succ \tilde{A}_j$, end of algorithm;

Step c3) If $I^2_{\alpha,\beta}\left(\tilde{A}_i\right) = I^2_{\alpha,\beta}\left(\tilde{A}_j\right)$ at least one decision maker then:

Step c3.1) If $I^3_{\alpha,\beta}\left(\tilde{A}_i\right) > I^3_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \succ \tilde{A}_j$, end of algorithm;

Step c3.2) If $I^3_{\alpha,\beta}\left(\tilde{A}_i\right) < I^3_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \prec \tilde{A}_j$, end of algorithm;

Step c3.3) If $I^3_{\alpha,\beta}\left(\tilde{A}_i\right) = I^3_{\alpha,\beta}\left(\tilde{A}_j\right)$ at least one decision maker then:

Step c3.3.1) If $I^4_{\alpha,\beta}\left(\tilde{A}_i\right) > I^4_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \succ \tilde{A}_j$, end of algorithm;

Step c3.3.2) If $I^4_{\alpha,\beta}\left(\tilde{A}_i\right) < I^4_{\alpha,\beta}\left(\tilde{A}_j\right)$ then $\tilde{A}_i \prec \tilde{A}_j$, end of algorithm;

Step c3.3.3) If $I^4_{\alpha,\beta}\left(\tilde{A}_i\right) = I^4_{\alpha,\beta}\left(\tilde{A}_j\right)$ at least one decision maker then:

Step c3.3.3.1) If $w_1 > w_2$ then $\tilde{A}_i \succ \tilde{A}_j$, end of algorithm;

Step c3.3.3.2) If $w_1 < w_2$ then $\tilde{A}_i \prec \tilde{A}_j$ **,** end of algorithm;

Step c3.3.3.3) If $w_1 = w_2$ then $\tilde{A}_i \approx \tilde{A}_j$ **,** end of algorithm;

4. Properties of centroid of centroids

In this section, we demonstrate the validity of natural properties that a correct centroid formula should possess (Wang et al. 2006).

Property 1: If $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers and $\tilde{B}$ is the right or left translation of $\tilde{A}$ along horizontal axis, then such a translation make the centroid point of $\tilde{B}$ do exactly same movement along the horizontal axis, and should not change the coordinate on vertical axis.

Proof: Let $\tilde{A} = (a,b,c,d;w)$ and $\tilde{B} = (a+\delta, b+\delta, c+\delta, d+\delta; w)$ be two fuzzy numbers and $\tilde{B}$ is the translation of $\tilde{A}$ about the horizontal axis where δ is a non-zero constant, then

$$G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right) = \left(\frac{2a+7b+7c+2d}{18}, \frac{7w}{18}\right)$$

$$G_{\tilde{B}}\left(\overline{x_0}, \overline{y_0}\right) = \left(\frac{2(a+\delta)+7(b+\delta)+7(c+\delta)+2(d+\delta)}{18}, \frac{7w}{18}\right) = \left(\frac{2a+7b+7c+2d}{18}+\delta, \frac{7w}{18}\right)$$

From the above we observe that $\overline{x_0}\left(\tilde{B}\right) = \overline{x_0}\left(\tilde{A}\right) + \delta$ and $\overline{y_0}\left(\tilde{A}\right) = \overline{y_0}\left(\tilde{B}\right)$.

Property 2: If $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers, then the changes of w should only affect $\overline{y_0}\left(\tilde{A}\right)$ and $\overline{y_0}\left(\tilde{B}\right)$, and should not result in any changes of $\overline{x_0}\left(\tilde{A}\right)$ and $\overline{x_0}\left(\tilde{B}\right)$.

Proof: Let $\tilde{A} = (a,b,c,d;1)$ and $\tilde{B} = (a,b,c,d;w)$ be two fuzzy numbers, then

$$G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right) = \left(\frac{2a+7b+7c+2d}{18}, \frac{7}{18}\right) \text{ and } G_{\tilde{B}}\left(\overline{x_0}, \overline{y_0}\right) = \left(\frac{2a+7b+7c+2d}{18}, \frac{7w}{18}\right)$$

From the above we observe that $\overline{x_0}\left(\tilde{A}\right) = \overline{x_0}\left(\tilde{B}\right)$ and $\overline{y_0}\left(\tilde{B}\right) = w\,\overline{y_0}\left(\tilde{A}\right)$.

5. Numerical Examples

In this section we demonstrate our ranking method by considering various types of fuzzy numbers like generalized and normal trapezoidal and triangular fuzzy numbers along with images of fuzzy numbers and crisp numbers.

Example 5.1

Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$ and $\tilde{B} = (0.1, 0.3, 0.5; 1)$

Then $G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right) = (0.3, 0.3888)$ and $G_{\tilde{B}}\left(\overline{x_0}, \overline{y_0}\right) = (0.3, 0.3888)$

$$I^1_{\alpha,\beta}\left(\tilde{A}\right) = 0.3\beta + (1-\beta)\left[0.3888\alpha + (1-\alpha)0.3\right]$$

$$I^1_{\alpha,\beta}\left(\tilde{B}\right) = 0.3\beta + (1-\beta)\left[0.3888\alpha + (1-\alpha)0.3\right]$$

An optimistic decision maker $(\alpha = 1)$, pessimistic decision maker $(\alpha = 0)$ and neutral decision maker $(\alpha = 0.5)$ conclude that $\tilde{A} \approx \tilde{B}$ since, $I^1_{\alpha,\beta}\left(\tilde{A}\right) = I^1_{\alpha,\beta}\left(\tilde{B}\right)$ **(Step c)**

$$I^2_{\alpha,\beta}\left(\tilde{A}\right) = 0.4\beta + (1-\beta)\left[0.3888\alpha + (1-\alpha)0.3\right]$$

$$I^2_{\alpha,\beta}\left(\tilde{B}\right)=0.4\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)0.3\right]$$

An optimistic decision maker $(\alpha=1)$, pessimistic decision maker $(\alpha=0)$ and neutral decision maker $(\alpha=0.5)$ conclude that $\tilde{A}\approx\tilde{B}$ since, $I^2_{\alpha,\beta}\left(\tilde{A}\right)=I^2_{\alpha,\beta}\left(\tilde{B}\right)$ **(Step c3)**

$$I^3_{\alpha,\beta}\left(\tilde{A}\right)=0.1\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)0.3\right]$$

$$I^3_{\alpha,\beta}\left(\tilde{B}\right)=0.2\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)0.3\right]$$

For any decision maker whether pessimistic $(\alpha=0)$, neutral $(\alpha=0.5)$ or optimistic $(\alpha=1)$ we have $I^3_{\alpha,\beta}\left(\tilde{A}\right)<I^3_{\alpha,\beta}\left(\tilde{B}\right)\Rightarrow\tilde{A}\prec\tilde{B}$ **(Step c3.2)**

Example 5.2

Let $\tilde{A}=(3,5,7;1)$ and $\tilde{B}=\left(4,5,\frac{51}{8};1\right)$

Then $G_{\tilde{A}}\left(\overline{x_0},\overline{y_0}\right)=(5,0.3888)$ and $G_{\tilde{B}}\left(\overline{x_0},\overline{y_0}\right)=(5.0416,0.3888)$

$$I^1_{\alpha,\beta}\left(\tilde{A}\right)=5\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)5\right]$$

$$I^1_{\alpha,\beta}\left(\tilde{B}\right)=5\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)5.0416\right]$$

For an optimistic decision maker $(\alpha=1)$ we have $I^1_{\alpha,\beta}\left(\tilde{A}\right)=I^1_{\alpha,\beta}\left(\tilde{B}\right)$ **(Step c)**

$$I^2_{\alpha,\beta}\left(\tilde{A}\right)=4\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)5\right]$$

$$I^2_{\alpha,\beta}\left(\tilde{B}\right)=2.375\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)5.0416\right]$$

For any decision maker whether pessimistic $(\alpha=0)$, neutral $(\alpha=0.5)$ or optimistic $(\alpha=1)$ we have $I^2_{\alpha,\beta}\left(\tilde{A}\right)<I^2_{\alpha,\beta}\left(\tilde{B}\right)\Rightarrow\tilde{A}\prec\tilde{B}$ **(Step c2)**

Example 5.3

Let $\tilde{A}=(0,1,2;1)$, $\tilde{B}=\left(\frac{1}{5},1,\frac{7}{4};1\right)$

Then $G_{\tilde{A}}\left(\overline{x_0},\overline{y_0}\right)=(1,0.3888)$ and $G_{\tilde{B}}\left(\overline{x_0},\overline{y_0}\right)=(0.9944,0.3888)$

$$I^1_{\alpha,\beta}\left(\tilde{A}\right)=1\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)1\right]$$

$$I^1_{\alpha,\beta}\left(\tilde{B}\right)=1\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)0.9944\right]$$

For an optimistic decision maker $(\alpha=1)$ we have $I^1_{\alpha,\beta}\left(\tilde{A}\right)=I^1_{\alpha,\beta}\left(\tilde{B}\right)$ **(Step c)**

$$I^2_{\alpha,\beta}\left(\tilde{A}\right)=2\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)1\right]$$

$$I^2_{\alpha,\beta}\left(\tilde{B}\right)=1.55\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)0.9944\right]$$

For any decision maker whether pessimistic$(\alpha=0)$, neutral $(\alpha=0.5)$ or optimistic $(\alpha=1)$ we have $I^2_{\alpha,\beta}\left(\tilde{A}\right)>I^2_{\alpha,\beta}\left(\tilde{B}\right)\Rightarrow\tilde{A}\succ\tilde{B}$ **(Step c1)**

Example 5.4

Let $\tilde{A}=(0,1,2;1)\Rightarrow-\tilde{A}=(-2,-1,0;1)$ and $\tilde{B}=\left(\frac{1}{5},1,\frac{7}{4};1\right)\Rightarrow-\tilde{B}=\left(\frac{-7}{4},-1,\frac{-1}{5};1\right)$

Then $G_{-\tilde{A}}\left(\overline{x_0},\overline{y_0}\right)=(-1,0.3888)$ and $G_{-\tilde{B}}\left(\overline{x_0},\overline{y_0}\right)=(-0.9944,0.3888)$

$$I^1_{\alpha,\beta}\left(-\tilde{A}\right)=(-1)\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)(-1)\right]$$

$$I^1_{\alpha,\beta}\left(-\tilde{B}\right)=(-1)\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)(-0.9944)\right]$$

For an optimistic decision maker $(\alpha=1)$ we have $I^1_{\alpha,\beta}\left(\tilde{A}\right)=I^1_{\alpha,\beta}\left(\tilde{B}\right)$ **(Step c)**

$$I^2_{\alpha,\beta}\left(\tilde{A}\right)=2\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)(-1)\right]$$

$$I^2_{\alpha,\beta}\left(\tilde{B}\right)=1.55\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)(-0.9944)\right]$$

For any decision maker whether pessimistic$(\alpha=0)$, neutral $(\alpha=0.5)$ or optimistic $(\alpha=1)$ we have $I^2_{\alpha,\beta}\left(-\tilde{A}\right)<I^2_{\alpha,\beta}\left(-\tilde{B}\right)\Rightarrow-\tilde{A}\prec-\tilde{B}$ **(Step c2)**

From examples 5.3 and 5.4 we see that $\tilde{A}\succ\tilde{B}\Rightarrow-\tilde{A}\prec-\tilde{B}$.

Example 5.5

Let $\tilde{A}=(0.1,0.2,0.4,0.5;1)$, $\tilde{B}=(1,1,1,1;1)$

Then $G_{\tilde{A}}\left(\overline{x_0},\overline{y_0}\right)=(0.3,0.3888)$ and $G_{\tilde{B}}\left(\overline{x_0},\overline{y_0}\right)=(1,0.3888)$

$$I^1_{\alpha,\beta}\left(\tilde{A}\right)=0.3\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)0.3\right]$$

$$I^1_{\alpha,\beta}\left(\tilde{B}\right)=1\beta+(1-\beta)\left[0.3888\alpha+(1-\alpha)1\right]$$

For any decision maker whether pessimistic $(\alpha = 0)$, neutral $(\alpha = 0.5)$ or optimistic $(\alpha = 1)$ we have $I^1_{\alpha,\beta}\left(\tilde{A}\right) < I^1_{\alpha,\beta}\left(\tilde{B}\right) \Rightarrow \tilde{A} \prec \tilde{B}$ **(Step b)**

From example 5.5 it is clear that this new method can rank crisp numbers.

Example 5.6

Let $\tilde{A} = (0.1, 0.3, 0.5; 0.8)$, $\tilde{B} = (0.1, 0.3, 0.5; 1)$

Then $G_{\tilde{A}}\left(\overline{x_0}, \overline{y_0}\right) = (0.3, 0.30004)$ and $G_{\tilde{B}}\left(\overline{x_0}, \overline{y_0}\right) = (0.3, 0.3888)$

$$I^1_{\alpha,\beta}\left(\tilde{A}\right) = 0.24\beta + (1-\beta)\left[0.30004\alpha + (1-\alpha)0.3\right]$$

$$I^1_{\alpha,\beta}\left(\tilde{B}\right) = 0.3\beta + (1-\beta)\left[0.3888\alpha + (1-\alpha)0.3\right]$$

For any decision maker whether pessimistic $(\alpha = 0)$, neutral $(\alpha = 0.5)$ or optimistic $(\alpha = 1)$ we have $I^1_{\alpha,\beta}\left(\tilde{A}\right) < I^1_{\alpha,\beta}\left(\tilde{B}\right) \Rightarrow \tilde{A} \prec \tilde{B}$ **(Step b)**

From example 5.6 it is clear that this new method can rank generalized fuzzy numbers.

6. Comparative study

In this section, we compare the proposed method by taking different sets of fuzzy numbers with various ranking procedures available in literature and the results are presented in Table1 and Table 2.

Case 1: Consider two fuzzy numbers *A*=(1, 4, 5;1) and *B*=(2, 3, 6;1)

By (Liou and Wang method, 1992) it is clear that the two fuzzy numbers are equal for all the decision maker's as $I^{\alpha}_T(A) = 4.5\alpha + (1-\alpha)2.5$ and $I^{\alpha}_T(B) = 4.5\alpha + (1-\alpha)2.5$ which is not even true by intuition. By using our method we have:

$G_A\left(\overline{x_0}, \overline{y_0}\right) = (3.7777, 0.3888)$ and $G_B\left(\overline{x_0}, \overline{y_0}\right) = (3.2222, 0.3888)$

$$I^1_{\alpha,\beta}\left(\tilde{A}\right) = 4\beta + (1-\beta)\left[0.3888\alpha + (1-\alpha)3.7777\right]$$

$$I^1_{\alpha,\beta}\left(\tilde{B}\right) = 3\beta + (1-\beta)\left[0.3888\alpha + (1-\alpha)3.2222\right]$$

For any decision maker whether pessimistic $(\alpha = 0)$, neutral $(\alpha = 0.5)$ or optimistic $(\alpha = 1)$ we have $I^1_{\alpha,\beta}\left(\tilde{A}\right) > I^1_{\alpha,\beta}\left(\tilde{B}\right) \Rightarrow \tilde{A} \succ \tilde{B}$ **(Step a)**

Case 2: Let $\tilde{A} = (0.1, 0.3, 0.5; 1)$ and $\tilde{B} = (1,1,1,1;1)$

(Cheng, 1998) proposed a ranking function which is the distance from centroid point of a generalized trapezoidal fuzzy number and the origin. (Chu, 2002) proposed a ranking function which is the area between the centroid point and original point. Their centroid formulae are given by:

$$\left(\frac{w\left(d^2-2c^2+2b^2-a^2+dc-ab\right)+3\left(c^2-b^2\right)}{3w\left(d-c+b-a\right)+6\left(c-b\right)},\frac{w}{3}\left(1+\frac{\left(b+c\right)-\left(a+d\right)\left(1-w\right)}{\left(b+c-a-d\right)+2\left(a+d\right)w}\right)\right)$$

And $\left(\frac{w\left(d^2-2c^2+2b^2-a^2+dc-ab\right)+3\left(c^2-b^2\right)}{3w\left(d-c+b-a\right)+6\left(c-b\right)},\frac{w}{3}\left(1+\frac{b+c}{a+b+c+d}\right)\right)$

Both these Centroid formulae cannot rank crisp numbers which are a special case of fuzzy numbers as, it can be seen from the above formulae that the denominator in the first coordinate of their Centroid formulae is zero and hence, Centroid of crisp numbers are undefined for their formulae. By using our method we have:

$G_{\tilde{A}}\left(\overline{x_0},\overline{y_0}\right)=\left(0.3,0.3888\right)$ and $G_{\tilde{B}}\left(\overline{x_0},\overline{y_0}\right)=\left(1,0.3888\right)$

$$I^1_{\alpha,\beta}\left(\tilde{A}\right)=0.3\beta+\left(1-\beta\right)\left[0.3888\alpha+\left(1-\alpha\right)0.3\right]$$

$$I^1_{\alpha,\beta}\left(\tilde{B}\right)=1\beta+\left(1-\beta\right)\left[0.3888\alpha+\left(1-\alpha\right)1\right]$$

For any decision maker whether pessimistic $\left(\alpha=0\right)$, neutral $\left(\alpha=0.5\right)$ or optimistic $\left(\alpha=1\right)$ we have $I^1_{\alpha,\beta}\left(\tilde{A}\right)<I^1_{\alpha,\beta}\left(\tilde{B}\right)\Rightarrow\tilde{A}\prec\tilde{B}$ **(Step b)**

Case 3: Consider the fuzzy numbers

$A_1=\left(0.1,0.2,0.3;1\right)$; $A_2=\left(0.2,0.5,0.8;1\right)$; $A_3=\left(0.3,0.4,0.9;1\right)$; $A_4=\left(0.6,0.7,0.8;1\right)$

ranked by different researchers and the results are presented in Table 1.

Table 1. A Comparison of the ranking results for different approaches

Method |Fuzzy | numbers	A_1	A_2	A_3	A_4
Yager (1981)	0.20	0.50	0.50	0.70
Fortemps & Roubens (1996)	0.20	0.50	0.50	0.70
Liou & Wang (1992)				
$\alpha=1$	0.25	0.65	0.65	0.75
$\alpha=0.5$	0.20	0.50	0.50	0.70
$\alpha=0$	0.15	0.35	0.35	0.65
Chen and Lu (2001)				
$\alpha=1$	-0.20	0.00	0.00	-0.20
$\alpha=0.5$	-0.20	0.00	0.00	-0.20
$\alpha=0$	-0.20	0.00	0.00	-0.20
Proposed method				
$\alpha=0,\beta\in\left(0,1\right]$	$0.2\beta+\left(1-\beta\right)0.2$	$0.5\beta+\left(1-\beta\right)0.5$	$0.4\beta+\left(1-\beta\right)0.4444$	$0.7\beta+\left(1-\beta\right)0.7$
$\alpha=1,\beta\in\left(0,1\right]$	$0.2\beta+\left(1-\beta\right)0.3888$	$0.5\beta+\left(1-\beta\right)0.3888$	$0.4\beta+\left(1-\beta\right)0.3888$	$0.7\beta+\left(1-\beta\right)0.3888$
$\alpha=0.5,\beta\in\left(0,1\right]$	$0.2\beta+\left(1-\beta\right)0.2944$	$0.5\beta+\left(1-\beta\right)0.4444$	$0.4\beta+\left(1-\beta\right)0.4166$	$0.7\beta+\left(1-\beta\right)0.5444$

Note: "▭" and "▭"denotes that the respective method cannot discriminate fuzzy numbers.

It can be seen from Table 1 that none of the methods discriminates fuzzy numbers. Yager's (1981) and Fortemps and Roubens methods (1996) failed to discriminate the fuzzy numbers A_2 and A_3 whereas, the methods of Liou and Wang (1992) and Chen and Lu (2001) failed to discriminate the fuzzy numbers A_2, A_3 and A_1, A_4.
By using our method we have:

$$G_{A_1}\left(\overline{x_0},\overline{y_0}\right)=\left(0.2,0.3888\right), G_{A_2}\left(\overline{x_0},\overline{y_0}\right)=\left(0.5,0.3888\right), G_{A_3}\left(\overline{x_0},\overline{y_0}\right)=\left(0.4444,0.3888\right)$$

$$G_{A_4}\left(\overline{x_0},\overline{y_0}\right)=\left(0.7,0.3888\right)$$

$$I^1_{\alpha,\beta}\left(A_1\right)=0.2\beta+\left(1-\beta\right)\left[0.3888\alpha+\left(1-\alpha\right)0.2\right]$$

$$I^1_{\alpha,\beta}\left(A_2\right)=0.5\beta+\left(1-\beta\right)\left[0.3888\alpha+\left(1-\alpha\right)0.5\right]$$

$$I^1_{\alpha,\beta}\left(A_3\right)=0.4\beta+\left(1-\beta\right)\left[0.3888\alpha+\left(1-\alpha\right)0.4444\right]$$

$$I^1_{\alpha,\beta}\left(A_4\right)=0.7\beta+\left(1-\beta\right)\left[0.3888\alpha+\left(1-\alpha\right)0.7\right]$$

For any decision maker whether pessimistic $\left(\alpha=0\right)$,neutral $\left(\alpha=0.5\right)$ or optimistic $(\alpha=1)$ we have $I^1_{\alpha,\beta}\left(A_1\right)<I^1_{\alpha,\beta}\left(A_3\right)<I^1_{\alpha,\beta}\left(A_2\right)<I^1_{\alpha,\beta}\left(A_4\right) \Rightarrow A_1 \prec A_3 \prec A_2 \prec A_4$

(Step b)

Case 4: In this case we consider seven sets of fuzzy numbers available in literature and the comparative study is presented in Table 2.

Set 1: $\tilde{A}=\left(0.2,0.4,0.6,0.8;0.35\right)$ and $\tilde{B}=\left(0.1,0.2,0.3,0.4;0.7\right)$

Set 2: $\tilde{A}=\left(0.1,0.2,0.4,0.5;1\right)$ and $\tilde{B}=\left(0.1,0.3,0.3,0.5;1\right)$

Set 3: $\tilde{A}=\left(0.1,0.2,0.4,0.5;1\right)$ and $\tilde{B}=\left(1,1,1,1;1\right)$

Set 4: $\tilde{A}=\left(-0.5,-0.3,-0.3,-0.1;1\right)$ and $\tilde{B}=\left(0.1,0.3,0.3,0.5;1\right)$

Set 5: $\tilde{A}=\left(0.3,0.5,0.5,1;1\right)$ and $\tilde{B}=\left(0.1,0.6,0.6,0.8;1\right)$

Set 6: $\tilde{A}=\left(0,0.4,0.6,0.8;1\right)$, $\tilde{B}=\left(0.2,0.5,0.5,0.9;1\right)$ and $\tilde{C}=\left(0.1,0.6,0.7,0.8;1\right)$

Set 7: $\tilde{A}=\left(0.1,0.2,0.4,0.5;1\right)$ and $\tilde{B}=\left(-2,0,0,2;1\right)$

Table 2. A Comparisons of the ranking results for different approaches

Methods	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
Cheng (1998)	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\approx\tilde{B}$	Not Comparable	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\prec\tilde{B}\prec\tilde{C}$	Not Comparable
Chu and Tsao (2002)	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\approx\tilde{B}$	Not Comparable	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\prec\tilde{B}\prec\tilde{C}$	Not Comparable
Chen and Chen (2007)	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\prec\tilde{C}\prec\tilde{B}$	$\tilde{A}\succ\tilde{B}$
Abbasbandy and Hajjari (2009)	Not Comparable	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\approx\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\prec\tilde{B}\prec\tilde{C}$	$\tilde{A}\succ\tilde{B}$
Chen and Chen (2009)	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\prec\tilde{B}$	$\tilde{A}\succ\tilde{B}$	$\tilde{A}\prec\tilde{B}\prec\tilde{C}$	$\tilde{A}\succ\tilde{B}$

Kumar et al. (2010)	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \approx \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \succ \tilde{B}$
Proposed method							
$\alpha = 0, \beta \in (0,1]$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \succ \tilde{B}$
$\alpha = 1, \beta \in (0,1]$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \succ \tilde{B}$
$\alpha = 0.5, \beta \in (0,1]$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \succ \tilde{B}$

7. Conclusions

This paper proposes a method that ranks fuzzy numbers which is simple and concrete. This method ranks trapezoidal as well as triangular fuzzy numbers with their images. This method also ranks crisp numbers which are a special case of fuzzy numbers, whereas methods proposed by Cheng and Chu cannot rank crisp numbers. This method uses an index of modality which represents the importance of areas of spreads against the coordinates of centroid of centroids of generalized fuzzy numbers, besides the decision maker's degree of optimism. This method which is simple and easier in calculation not only gives satisfactory results to well defined problems, but also discriminates fuzzy numbers. The methods proposed by Yager, Fortemps et al., Liou et al. and Chen et al. indexes failed to discriminate fuzzy numbers which is evidenced from Table 1 and 2 and this method also agrees with human intuition.

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Fuzzy philosophy of science and education

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Abstract

Scientific consequences are dependent on premisses that are logical propositions of the phenomena concerned. Propositions are verbal and linguistic statements, and at the initial philosophical thinking stage they all include vaguneness and imprecision. As more scientific evidence becomes available either rationally or empirically proposition validity degree increases at the cost of vagueness. In some societies scientific propositions are assumed directly as absolutely correct, but in philospohical thinking they are valid with some uncertainty. Recent trend of falsifaibility of sceintific propositions brings into view fuzzy ingradients, which have not yet have wide spread recognition. Objective probability attachment to scientific statements is a difficult task, and therefore, subjective (Bayesian) proportions are attached in practice. After a detailed account of what are the advacoters and opponents to scientific absolute correctness and probability, a fuzzy thinking and consequently, membership degree attachments rather than probability are presented by in this paper. Classical education systems are based on very systematic, crisp and organized framework on the basis of more than 25-century old Aristotalian logic with two opposite outcomes. Almost in every corner of information source gray fore and back grounds prevail. It is a big dilemma how to deal with gray information sources in order to arrive at scientific conclusions with crisp and deterministic logical principles. Fuzzy logic principles with linguistically valid propositions and rather vague categorization provide a sound ground for this purpose. The preliminary step is a genuine logical and uncertain conceptualization of the phenomenon with causal and result variables that are combined through the fuzzy logical propositions in a set of rules. Such an approach helps not only to visualize the relationships between different variables logically, but furnishes a philosophical background about the mechanism of the phenomenon that can be presented to anybody linguistically without any mathematical treatment. It is emphasized in this paper that in an innovative education system, the basic philosophy and fuzzy logic justifications in any problem solving should be given linguistically prior to any crisp bases such as mathematics or systematic, mechanical and crisp algorithms. In this way the student may be able to develop creative and analytical thinking capabilities with the support of teachers who can provide analytical thinking principles. Since, the modern philosophy of science insists on the falsification of current scientific results, there are always room for ambiguity, vagueness, imprecision and fuzziness in scientific research activities. Innovative education systems are adiced to lean more towards basic scientific philosophy with fuzzy logical principles.

Keywords: Crisp, education, fuzzy, innovation, language, logic, philosophy, proposition, reasoning, vagueness.

1. Introduction

In attempting to define what is meant by a "philosophy" of science, the first problem one encounters is the notorious vagueness of the term "philosophy" (McMullin, 1987). A direct consequent of this statement is to rise a question as to how the science itself is objective but its very foundation as philosophy is vague, imprecise, blurred and rather uncertain. How can scientific development become possible if the science and its philosophy are uncertain? The term philosophy has a wide meaning including from a cloudy speculative fancy to a piece of formal logic. Until recently, the formal logic in philosophy has been taken as the Aristotelian logic, which has alternatives of two completely defined classes as true or false; positive or negative; black or white; beatiful or ugly. All the scientific propositions, hypothesis, theories and ideas are measured first on the basis of this logic and consequently, classical scienticism followers believe in them dogmatically with insignificant scientific information, because they immediately defuzzify uncertainties through the crisp logic principles automatically. They become academicians who cannot have attributes of due to the crisp nature of the classical logic. In this logical domain even the cloudy, vague, uncertain, imprecise qualities are crisply classified to distinctive and mutually exclusive parts with the acceptance only one part as scientific. None of the scientific knowledge can be accounted as completely crisp without suspicions, otherwise scientific development cannot exist. The scientific development is possible due to its vague, imprecise and uncertain character. In the text so far the terms as vague, imprecise, uncertain, blurred, cloudy are altogether referred to as the fuzzy information (Zadeh, 1968). Most often common man expects or thinks that the science moves toward a unified account of the world, but the pictures of reality become ever more disparate. Especially, many scientific theories which were believed to be true turned out to be false or semi false or there are a lot of debates about their verifications or falsifications. Hence, in the domain of scientific philosophy, the scientists become rather uneasy in testing and providing demarcation for the distinction of scientific from, the so called non-scientific knowledge. It is not possible to have scientific thought without knowing or at least even unconcisely going through the process of philosophy, which provides complete freedom in scientific thinking. Although, today many academicians may think that they are producing scientific papers without thinking about the philosophical ingradients in their approach, in fact, their procedure has embedded scientific philosophical thinking. Complete freedom of philospohical thinking provides many scanerios about any phenomenon concerned, but logic eliminates many of them as opposed to common sense. Of course, common sense is not dependable on all the time. Philosophers of science seek for exploration of general scientific characteristics that mostly relate to its function as a knowledge-producing activity such as the nature of its validation procedures, its patterns of development, the truth-state of theories and the like.

In order to clearify the distinction between the formal classical logic and the fuzzy logic, one should remember that according to Aristotelian logic, if something is true or thought to be thrue, then it is given the number 1 and its completely opposite alternative number 0. Likewise, simply true statements are attached degree of belief as 1 and false ones as 0. The fuzzy logic attributes degrees to even a scientific belief (degree of verification or falsification) that assumes values between 0 and 1 inclusive. Verifiability of scientific knowledge or theories by logical positivists means on the classical grounds that the demarcation of sceince concerning a phenomenon is equal to 1 without giving room for falsification (Popper, 1952). The conflict between verifiability and falsifiability of scientific theories includes philosophical grounds that are fuzzy but many scientific philosophers concluded the case with Aristotelian logic of crispness, which is against the nature of scientific development. Although many science philosophers tried to resolve this problem by bringing into the argument the probability (Carnap, 1987) and at times the possibility of the scientific knowledge demarcation and scientific development, unfortunately so far the "fuzzy philosophy of science" has not been introduced into the literature. The scientists cannot be completely objective in their justification for scientific demarcation or progress, but ingredients of fuzziness are driving engine for the generation of new theories. All the scientific rule bases must be tested by fuzzy inference engine, which leads to fuzzy scientific domain but for classical understanding and dissemination of the knowledge, many render them into a defuzzified manner. In fact, the scientific phenomena are all fuzzy in nature and especially the foundations of scientific philosophy include embedded fuzzy components. Dogmatic nature of scientific knowledge, or belief in the science as if it is not suspectible, is the fruits of formal classical Aristotelian logic, whereas fuzzy logic holds the scientific arena vivid and fruitful for future scientific plantations and knowledge generation.

This paper proposes the use of fuzzy logic in the demarcation of scientific knowledge and education system. It is declared that whatever is scientific, it includes fuzzy information to a certain extent and Aristotelian logic cannot be valid in natural environment and human thoughts except after idealization (defuzzification) of the facts.

2. Historical perspective

Newton used a less restrictive conception of scientific knowledge than philosopher John Locke in natural philosophy. In his conception, science requires moral or practical certainty rather than metaphysics or absolute certainty, which implies that the scientific statements are naturally fuzzy in character. They had different understandings of what kind of uncertainty is required in scientific knowledge. Locke’s concept of scientific knowledge involved absolute certainty, which cannot be a matter of degree. He was able to preserve a sharp distinction between, on the one hand, scientific knowledge, and on the other “judgement”, this being his term for what had been called “probable opinion”, which is valid only in the case of vague information, in other words, when scientific statements include fuzziness. Newton’s practical certainty is a matter of degree, and to acknowledge the degrees of certainty is to acknowledge degreees of probability. In Newton’s philosophy, understanding of certainty could not be maintained as a sharp distinction between scientific knowledge and probable opinion. It is, therefore, necessary to benefit from fuzzy statements as intermediators. Newton agreed that his knowledge does not have absolute certainty until the integrity of experimental natural philosophy provides real knowledge. He had to supplement logic with rhetoric in order to sustain his distinction. Rheoteric statements are sets of vague or imprecise information in the forms of fuzzy statements. Here again, rheoteric words may be treated as fuzzy sets.

On the other hand, in 17th century especially Christian Huygens promoted Newton's rejection and he conceded no sharp distinction between the knowledge of nature that one counts as scientific and the judgements about nature as probable. Most recognized reasoning is probable reasoning in the sense that it generates conclusions, which have one or another degree of certainty, and therefore, one or another degree of probability and still, as a suggestion in this paper, one or another degree of fuzziness. Later, in the century with the idea of using especially experimental evidence for arguing, the truth or at least the credibility, the concept of probability become identical with the idea of something being credible or believable in the light of evidence. It is also alleged in this paper that all these last statements can be reconsidered under the light of fuzzy sets (Zadeh, 1965).

Not every probable reasoning is acceptable, but only to some degrees of uncertainty. For any probable resoning, it is necessary to have a measure of when and why such reasoning is acceptable. One should to find ways of measuring the degree of uncertainty, which will show how a conclusion should be quantified. Towards this purpose, Huygens himself published an account in 1675. He tried first time to show the treatment of quantitative probabilistic reasoning in games of chance. In his treatment, words are missing for customarily use of signifying the probability concepts.

On the other hand, Leibniz stated that for promotion of scientific achievements, investigators must recognize that absolute certainty is an ideal case that they can achieve. During scientific studies degrees of probability should be accepted attached with knowledge. Objective probability definition requires measurements that are impossible at the scientific thinking experiment stage. The best that can be done is the attachment of subjective probability values to each statement. However, it is preferrable and better to express the uncertainties in any scientific statement by words of vagueness, which can be expressed in terms of fuzzy subsets and degreees of memberships within each subset. Hence, reasonings with probable conclusions should be embraced rather than rejection. Leibniz probability is a relation between the evidence disclosed by investigators and the conclusions they draw. This suggests that Leibniz accepts a jurisprudential model for the logic of probable reasoning. As Leibniz says we need "a new logic in order to know degrees of probability, since this is necessary in judging the proofs of fact and of moral." Hence, such a new logic is suggested as the fuzzy logic which has been already suggested by Zadeh (1965).

Although Leibniz was exploring for the first time the philosophy of probability, the Swiss mathematician Jacob Bernoulli was achieving some success by pursuing this practical line of thought. As the example, Gracchus might be considered with the accusation of a crime. He turns pale when accused, and on the basis of this fact, it is possible that we conclude in an argument for his guilt. However, this can only be a probable argument, because Gracchus's pale may have a cause other than the presumed guilt. Hume had claimed that "all reasonings concerning matter of fact seem to be founded on the reflection of cause and effect and the probable arguments with which Bayes was concerned can indeed be understood as reasoning from effect to cause".

Keynes (1921) viewed that probabilities are not relative frequencies based on observation, as Russell (1948) believed and likewise as so many of his hard-headed empiricist colleagues at Cambridge supposed, but degrees of rational belief determined by reason. Furthermore, for Keynes probabilities were degrees of belief, and it was necessary not only to attribute probabilities primarily to propositions, but also to recognize that propositions are always probable in relation to other propositions. He, in fact, endosed the “conceptual” or “classical” idea of probability associated with Leibniz and Laplace. A degree of rational belief has nothing to do with how frequently conclusions “of this kind” follow when one uses reasoning “of this type”, rather it is a function of the logical relations between conclusions and the reason one gives for its truth. Hence, in a fuzzy proposition, the premise includes the reasons for the evidence and the consequent part represents the partial conclusion in harmony with the premise. In this manner, each position carves a portion from the overall representative logic and yields its consequent part, which is also partial. That is to say, logicians should recognise not only that propositions can entail, or contradict other propositions, but also that propositions can partially entail or contradict other propositions. Hence, one may say that the conclusions of some measuring must be true when it is entailed by premisses one accepts, so one can say that a conclusion is probably or possibly true when it is “partially” entailed by acceptable premisesses. In other words, where entailed conclusions are necessary in relation to premisses, partially entailed conclusions are probable in relation to premisses. According to Keynes probability comprises “that part of logic, which deals with arguments that are rational but not conclusive. The same sentence can be read by replacing probability with possibility, which means to say that premisses include fuzzy subsets. Hence, it is a part of logic, but not mathematics.

When evidence changes, vaguenes, and therefore, degrees of belief also change in thought rather than in experience, because they are logical relations of partial entailment between propositions expressing conclusions in which one has degrees of belief and propositions expressing the evidence for the conclusions. Probabilities as degree of belief are subjective rather than objective; they represent psychological states (Ramsey, 1978). One should not understand the rationality of the probability judgements expressing partial beliefs arising from scientific investigations as a matter, not of their correspondence to something external to them, or of their derivability from a supposedly objective indifference principle, but of the relation of the beliefs to each other.

Russell (1948) states that the aim of the inductive arguments is, given the truth of their premisesses, to make their conclusions probably true. However, in deductive arguments, one requires that conclusions are necessarily true, given the truth of their premisses. Instead of asserting that a regularity occurs in all cases, sometimes, assert that it occurs in only a certain percentage of cases. If the percentage is specified or if in some other way a quantitative statement is made about the relation of one event to another, then the statement is called a “statistical law” (Carnap, 1995).

The concepts of science, as well as those of everyday life, may be conveniently divided into three main groups, classificatory, comperative, and quantitative. Classificatory concept means simply a concept that places an object within a certain class. They vary widely in the amount of information that they give about an object. For example, if one says an object is blue, or warm, or cubical, he/she makes relatively weak statements about the object. In other words, all these statements include vagueness and consequently they can be conveniently expressed by fuzzy subsets. By placing the object in a narrower class, the information increases, even though it still remains relatively modest. Such a narrowness corresponds to the narrowness of the fuzzy subsets, which is also equivalent to information content increase in a fuzzy subset. For instance, a statement that "an object is a living organism" is very vague, but "it is an animal" says a bit more. As the classes continue to narrow one has increasing amounts of fuzzy sets but still vague information.

3. Uncertainty in science

Early humans had thinking in an entirely uncertain environment for their daily activities. It is possible to say that early knowledge and information were concepts derived from frequent observations and experience. Throughout the centuries, human thinking had support from scripts, drawings, calculations, logic and finally mathematical calculations. In the mean time science is separated from philosophy with its own axioms, hypotheses, laws and final formulations especially after the renaissance in the 17th century. It is possible to state that with Newtonian classical physics, science entered almost entirely a deterministic world where uncertainty was not even accounted among the scientific knowledge. However, today almost in all the branches of science, there are uncertainty ingredients and many scientific deterministic foundations became uncertain with fuzzy ingredients. Among such conceptions are quantum physics, fractal geometry, chaos theory and fuzzy inference systems. However, some others such as the natural and social sciences have never gone through the stages of determinism. With the advancement of numerical uncertainty techniques such as probability, statistics and stochastic principles scientific progresses in quantitative aspects had rapid developments, but still leaving aside the qualitative sources of knowledge and information, which can be tackled by the fuzzy logic principles only.

Recently, famous philosophers and scientists alike, started to spell out the uncertainty and fuzzy ingredients that are essential basis of scientific progress. For instance Russell (1923) stated that "All traditional logic habitually assumes that precise symbols are being employed. It is, therefore, not applicable to this terrestrial life but only to an imagined celestial existence". On the other hand, as for the fuzzy conception Zadeh (1965) said that "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics".

As an esteem of the eastern thinking, philosophical objects may be raised by logical premises and implications along three basic mental activities, namely, imagination, conceptualization and idealization jointly leading to idea generations. Since the existence of terrestrial life, human beings have interaction with nature, which has provided the basic material in the form of objects and events evolving with time and space for the human mental activity chain. At the early stages of human history or during the childhood of any individual these stages play roles in different proportions and with experience, they take final forms. Each thinking process includes uncertainty, because imagination, conceptualization and idealization stages are rather subjective depending on individual's grasp. At any stage of human thinking evolution, the premises include to a certain extent uncertainty elements such as vagueness, ambiguosness, possibilities, probabilities and fuzziness. Implication of mathematical structure from the mental thinking process might seem exact, but even today it is understood as a result of scientific development that at every stage of modeling, physical or mechanical, there are uncertainty pieces, if not in the macro scale, at least at the micro scale. It is clear today that mathematical conceptualization and idealization leading to satisfactory mathematical structure of any physical actuality is often an unrealistic requirement. As Einstein (1952) stated "So far as the law of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality".

At the very elementary stages of mental thinking, activity objects are thought as members or non-members of a given or physically plausible domain of variability. This brings into consideration sets, which include possible outcomes or basis of the investigated phenomenon. In formal sciences such as physics, geology, meteorology, etc. almost invariably and automatically, these elements are considered as either completely members of the set or completely outside the same set. Hence, the Aristotalian logic of pairs in the form of one or zero; positive or negative; yes or no, black or white, etc., are employed at the foundation of any scientific phenomenon for mathematical modeling. However, Zadeh (1965) suggested instead of crisp membership consideration, continuity of membership degrees between 0 and 1, inclusive. Hence, fuzzy sets play intuitively plausible philosophical basis at every stage of the aforementioned mental activity chain.

4. Fuzzy logic thinking and reasoning stages

Understanding, explanation and reasoning are the steps necessary in a complete thinking process. Each one of the steps cannot be explained in a crisp manner and each individual depending on his/her capabilities may benefit from this sequence. Although human wonder and mind are the sources of fuzziness, they also seek problem overcoming with human experience, i.e. expert views. The fuzzy concepts in understanding complex problems are dependent on observations, experiences and consciousness. In problem solution there is always fuzziness, which paves ways for further developments. Hence, the scientific solutions cannot be taken as absolute truths in a positivistic manner.

The precise knowledge is possible only when a phenomenon or process is isolated from its surrounding again with a set of restrictive assumptions, which render the problem into certainty world by ignoring all fuzzy uncertainty features. For instance, the factor of safety (FoS), also known as safety factor, is a multiplier applied to the deterministically (crisp logic) calculated maximum load (force, torque, bending moment or a combination) to which a component or assembly will be subjected in engineering design. Thus, a FoS accounts for imperfections in materials, flaws in assembly, material degradation, and uncertainty in load estimates. In fact, the FoS can be named as "ignorance factor" due to the exclusion of all fuzzy information about the engineering design. However, fuzzy logic and system help to solve the engineering design problem without considering safety factor, because the solution is based on fuzzy uncertain information domain. There are no isolated phenomena and processes in nature, and any knowledge about them is always fuzzy. The significance of fuzziness opens ways for changes, evolution, growth, and continuous scientific development. Figure 1 gives the steps in fuzzy thinking and problem solving. Fuzziology concentrates on the study of the human mind possibilities to know external objects by collecting information through observations or readings (Şen, 2010).

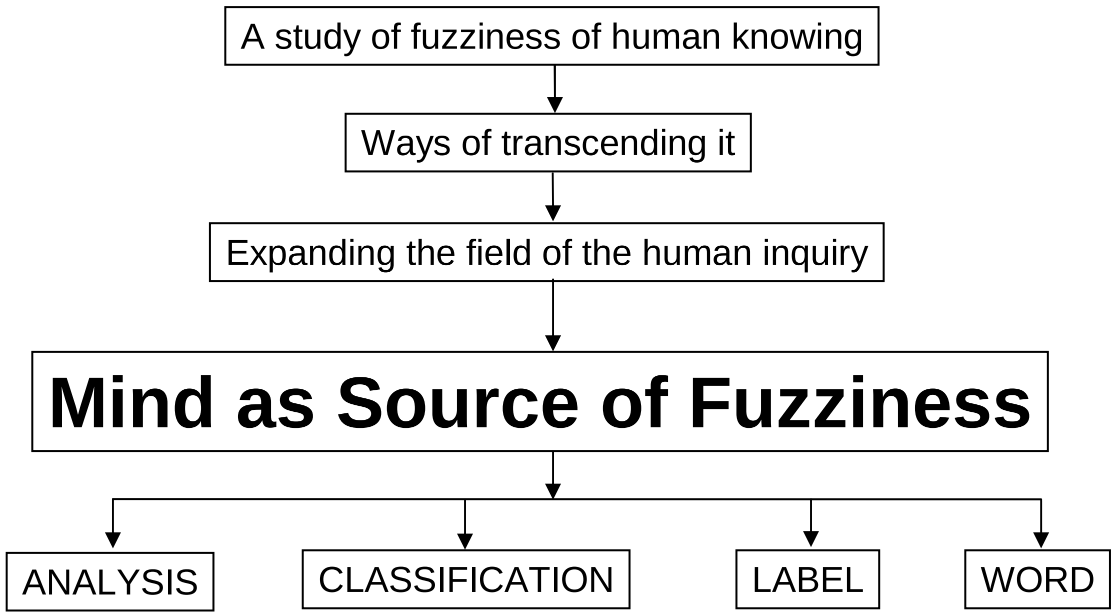

Figure 1. Fuzziology steps

Once the collection of such fuzzy linguistic information is completed then human inquiry expands the field of understanding along different directions. In the mind, the objects and their different visible properties are expressed first by words. This point indicates how significant the language is. Each item concerning the phenomenon under investigation is labeled by a word or a set of fuzzy words (statements, propositions). This is equivalent with the categorization of the objects into different classes again in a fuzzy manner and at this stage the crisp logic cannot be helpful. For instance, when some objects are labeled by a word, say "tree", one is certain that there is fuzzy uncertainty in this labeling. After all the stages in Figure 1, it is possible to carry on analytical thinking and solution procedures.

Fuzzy impressions and conceptions are generated by human mind, and it divides the seeable natural, environmental or engineering reality into fragments and categories, which are fundamental ingredients in classification, analysis and deduction of conclusions after labeling each fragment with a "word" such as a name, noun or adjective. The initial labeling by words is without interrelation between various categories. These words have very little to do with the wholeness of reality. Hence, common linguistic words help to imagine the same or very similar objects in our mind in a fuzzy manner. The fuzziness of knowing never ceases to exist. This is a paramount characteristic of the human knowing, which challenges humanity and constantly propels its search for truth and understanding the secrets of reality.

Every act of holistic understanding is inevitably fuzzy. The fuzziness and truth are not mutually exclusive as it is assumed in classical scientific research methodology, but they go hand in hand in every aspect of scientific research.

Conscious direction of attention towards an external object causes the object to be received by mind into the realm of our fuzziness, which causes in sequence to perception, experience, feeling, thinking, understanding, knowing, and finally, acting for meaningful description and analytical solution. Fuzzy statements have meaning and relevance only. "As the complexity of a system increases, human ability to make precise and relevant (meaningful) statements about its behavior diminishes until a threshold is reached beyond which the precision and the relevance become mutually exclusive characteristics" (Zadeh, 1968).

Fuzziness is an essential characteristic of our imaginations that raise and dissolve in our thoughts about the future plans. Human thoughts have blurred boundaries and consist of fuzzy immaterial “ubstance”. Having in mind how important is to think in images for the development of our intelligence and capacity to learn and know, to act and to create, to evolve and to transform, one should not underestimate the role of fuzziology. At this stage it is useful to mention about the three stages of human thinking in the middle eastern philosophy for reaching to a solution of any problem in general (Şen, 2010). These three words are "takhayyul" (imagination), "tasawwur" (geometric configuration) and "tafakkur" (idea generation). Any external object whether it exists materialistically or not, human beings try to imagine its different properties in a fuzzy world. This gives him/her the power of initializing individual and personal thinking domain with whole freedom in any direction. After the object comes into existence vaguely in the mind, then it is necessary to know its shape, which is related to geometry. It is essential that the geometric configuration of the phenomenon must be visualized in mind in some way even though it may be a simplification under a set of assumptions. Again the fuzzy shapes in the mind are put down as crisp geometrical shapes such as square, triangle, circle, ellipsoid, etc., or their mixtures for the mathematical treatments.

In 1932 Gödel proved that in any axiomatic mathematical system (theory), there are fuzzy propositions, that is, propositions which cannot be proved or disproved within the axioms of this system.

5. Approximate reasoning

Reasoning is the most important human brain operation that leads to creative ideal methodologies, algorithms and deductions giving way to sustainable research and development. Reasoning stage can be reached if there is stimulus for the initial driving of mental forces. Ignition of pondering on a phenomenon comes with the physical or mental effects that control an event of concern. These affects trust imaginations about the event and initial geometrical sketches of the imaginations by simple geometries or pieces and connections between them. The ideas begin to crystallize and they are expressed verbally by a native language to other individuals to get their criticisms, comments, suggestions and support for the betterment of the mental thinking and scientific achievement. Finally, all the conclusions must be expressed in a language, which can then be converted into universally used symbolic logic based on the principles of mathematics, statistics or probability statements. This explanation shows that linguistic (fuzzy) logic is followed by symbolic logic (mathematics). Unfortunately, in many education systems all over the world, this sequence of language and symbolism is overturned into the sequence of first symbolism (mathematics) and then linguistic understanding, which is against the natural perception abilities of human perception. This is especially true for countries or societies who are trained in some other language with symbols and those when they return to their community, the first difficulty is to convey the scientific messages in his/her native language, and therefore, in order to avoid such a dilemma the teacher bases the explanation on symbolic logic. This is one of the main reasons why scientific thinking and reasoning are missing in many countries including Turkey.

In fuzzy logic there are no mathematical symbols and it is intermingled with the linguistic statements, which can only be acceptable on the logical grounds. It is impossible to work with fuzzy logic if a sound as well as clear language and basic logical thinking are missing. It is possible to set down the necessary logical rules connecting the input and output variables of any phenomenon. After the completion of such logical deductions one can then enter the mathematical domain and express his/her ideas in the realm of symbolic logic, which is mathematics.

In the philosophical thinking for scientific and technological achievements, and another three essential steps are perceptions (feelings, chaos), sketches (geometry, design) and ideas (language, fuzzy). The perception part is very significant because it provides complete freedom of thinking without expressing it to others who can restrict the activity.

The subjectivity is the greatest at the perception stage and as one enters the sketch domain, the subjectivities decrease and at the final stage, since the ideas are exposed to other individuals, the objectivity becomes at least logical, but still there remains some uncertainty (vagueness, incompleteness, missing information, etc), and hence, the final conclusion is not crisp but fuzzy.

6. Scientific sense and thinking

Various phenomena in engineering, medicine and sciences take place in a complex world, where complexity arises from uncertainty in the forms of ambiguity. Scientists address problems of complexity and ambiguity at times sub-consciously, since they could think, these ubiquitous features pervade most natural, technical, and economical problems faced by the human race. The only way for computers to deal with complex and ambiguous issues is through fuzzy logic thinking, systemizing, controlling and decision procedures.

Common sense dictates that some form of empiricism is essential to make sense of the world. In traditional quantitative educational training, the classical dualism as the tension between subjectivity and objectivity is often addressed by adopting an objectivist, empiricist or positivistic approach, and then by applying a scientific research design. Even based on classical logic, scientific thinking starts in an entirely subjective medium. Subjective thinking penetrates objectivity domain by time through imagination and visualization, and hence there is not a crisp line between subjectivity and objectivity. Empirical works, which are based on either observations or measurements as experimental information help to decrease the degree of subjectivity on behalf of objectivity. None of the scientific formulations obtained up to now is completely crisp, but they are regarded as crisp information provided that the fundamental assumptions such as mutually exclusiveness and exhaustiveness are taken into consideration. The crispness of any scientific information can be shacked by modifying one of the basic assumptions. This implies that all the scientific principles are not crisp completely, but include vagueness, incompleteness and uncertainty even to a small extent, and hence they can be considered as fuzzy by nature or by human understanding.

As mentioned in the previous section, any scientific thinking has three major steps imagination, visualization and idea generations. Imagination part includes the setting up of suitable hypothesis or a set of logical rules for the problem at hand and the visualization stage is to defend the representative hypothesis and logical propositions. Scientists typically use variety of representations, including different kinds of figures (geometry) to represent and defend the hypotheses. On the basis of hypotheses, the scientists behave as a philosopher by generating relevant ideas and their subsequent dissemination, which should include new and even controversial ideas, so that other scientists can overtake and elaborate more on the basic hypotheses. Whatever are the means of thinking, the scientific arguments are expressed by verbal expressions prior to any symbolic and mathematical abstractions. Especially, in engineering and physical sciences visualization stage is represented by algorithms, graphs, diagrams, charts and figures, which include tremendous amount of condensed verbal information.

The scientific visualizations are conducted with geometry since the very early beginning of scientific thoughts. This is the reason why the geometry was developed and recognized by early philosophers and scientists than any other scientific tools such as algebra, trigonometry, and mathematical symbolism Al-Khawarizmi (died 840 A.D.) who is known in the west as his Latinized name "algorithm" solved second order equations by considering geometric shapes. For instance, he visualized x^2 as a square with sides equal to x, and terms such as ax are considered as rectangles with base length x and height equal to a. This geometrical thinking and visualization made him the father of "algebra". All his discussions were explained linguistically.

All the conceptual models deal with parts of something that is perceived by human mind. Of course, among the meaningful fragments of the phenomenon, there exist clear and hidden interrelationships, which are there for the exploration of human intellectual mind. Such possible relationships can be explained by a set of fuzzy rule statements (base). Among the fragments of thinking are perception, sensations, thoughts, which serve collectively to provide partial and distorted conceptual models of reality. The success in understanding of any scientific theory or publication is not only through the text, but additionally verbal expressions of the mathematical formulations and figures. Hence, the whole basic philosophy and working mechanism of any scientific work can be understood through the linguistic expressions, where there are not only crisp logic propositions, but most of the time vague, incomplete, uncertain statements that are more valuable for scientific developments. Such uncertain linguistic statements have fuzzy contents that can be assessed by fuzzy logic principles. Scientists treat figures as integral parts of their arguments, whose strength and soundness depend on visual representations as much as they do on linguistic representations. Arguments are expressed in terms of statements and this is one of the main reason why the scientific philosophy has paid little attention to figures.

In everyday life human beings make many predictions and estimations especially on the basis of qualitative data and past experience. Additionally, expert opinions help to shape and refine such predictions besides the mutual discussion and confidence. In predictions there are similarities, which are the input information about the phenomenon concerned, output clues and the logical connectivity between these two sources of information. On the basis of certain clues, it is possible to make judgments about output information. The default of these judgments is the commonly available scientific thinking and its sublime version of logic (crisp or fuzzy) leading to rational results. This provides ability for any individual to develop actuarial models for various real-life prediction problems. It is possible to make predictions either by crisp logic mathematical formulations or fuzzy logic expert views with a set of linguistic rules.

An important question is whether the predictions of human experts are more reliable than mathematical models? Experts make their predictions on the basis of the same evidence as for the mathematical foundations, but additionally they consider the usefulness of the linguistic data in the form of vague statements in the adjustment of the final model. Such vague information cannot be digested by crisp logic. Fuzzy modeling by experts considering vague information is more successful than mathematical models, which are valid for ideal cases under the validity of a set of assumptions. Among the most important problems are natural phenomena predictions, because they have the following properties.

(1) Even the best mathematical models are not reliable,

(2) The best results seem reasonable predictions, but somewhat unsafe, and therefore, as mentioned in the previous section the FoS is imported to make the results more dependable.

In order to move understanding towards a deeper and broader grasp of complexity, the emergent meanings need to be neither stable nor unstable, that is stable enough to rely upon them when generating hypotheses, concepts, and emotional attitudes, and unstable enough not to allow these concepts and attitudes to harden and become dogmas and addictions. In other words, after scientific thinking, meanings need to be fuzzy (flexible), ready to immediately respond to the changes continuously occurring in each of the countless dimensions of reality.

7. Education and uncertainty

Education is a terminology that is used to enlighten others through a sequence of systematic courses that include basic concepts, which are expected to provide for the students a vivid domain of idea creation by pondering on some phenomena. Of course, in such as training, the rational thinking is the core of creative and free opinion. Besides, education has three main facets that should contribute interactively for fruitful and even emotionally stable end purposes. Figure 2 indicates these three ingredients in their interactive courses.

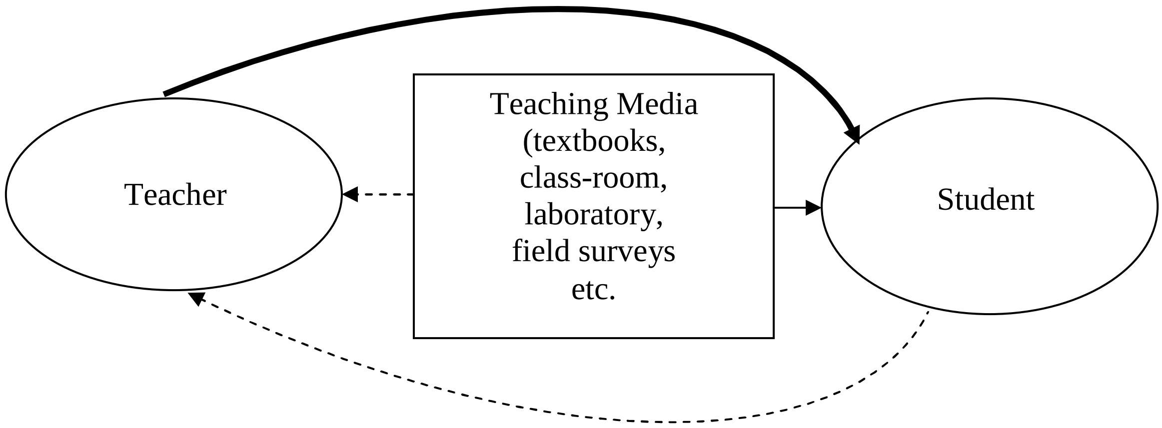

Figure 2. Education system parts

It is obvious from this figure that education does not mean knowledge influx from teacher to student only. In an effective education system, there are instances when sudden and rather unexpectedly knowledge flows either from the student to the teacher or new knowledge emergence through mutual discussion in the midst of fuzzy logic (linguistic) domain. Unfortunately, in many parts of the world and especially in the developing countries, classical educational systems provide crisp and elusive knowledge leading to locally valid certificate with mass production. The defective points in any traditional education system can be specified as follows,

1) There is a tough authority of teaches who are directed according to a set of state or traditional rules, which does not give freedom of creative reasoning or thinking. In such a system, logic means any answer to any question in black or white. This is classical logical attitude towards the problem solving,
2) Teaching media, which can be referred to as educational gadgets, may become indispensable organs and they are exploited in a crisp and rather

dogmatic manner without improvement throughout years. In non-native English speaking communities, such devices may easily become show offs for affecting the attention of learners only rather than basic educational concepts,

3) There are expectations of ready answers to questions in textbook style of information, which are only jointly shared by different learners and teachers alike without uncertainty,
4) Scientific concepts are provided in a crisp manner as if there is only one way of thinking and solving the problems with scientific certainty,
5) Assumptions, hypotheses and idealizations are the common means for mind to grasp the natural phenomena, and therefore, any scientific conclusion or equation is valid under certain circumstances.

In a modern and innovative educational system almost all the concepts must be provided with uncertainty flexibility especially at the higher educational systems. It is fixed from the long history of science by experience that not only freedom of thinking, but also suspicion from scientific conclusions should be incorporated for better advancements. The very word of suspicion leads to expectation and even viewing scientific knowledge as uncertain. Hence, the basic points in a modern and innovative educational system, the following points must be considered which are contrary to classical or traditional education.

1) Traditional and classical elements must be minimized and even dismissed from an innovative education system. The authorizable teacher is the one who is regerded as knowledgeable (crisp logic), and especially, who has the ability of knowledge and information giving only,
2) The teacher should not be completely dependent on educational gadgets, and the students through discussions and questions should try to force the teacher on the margins of the presented material for more information,
3) It should be kept in mind that each scientific conclusion is subject to uncertainty, fuzziness and suspicion, and hence, to further refinements leading to innovative ideas and modifications,
4) Especially, the fuzzy logical principles and philosophical basis must be kept in the education agenda by the teachers so that each student can grasp and approach the problem with his/her abilities,
5) At higher educational level, scientific thinking must be geared towards the fuzziness and falsifiability of the conclusions or theories rather than exactness and verifiability.

After the consideration of these points collectively, it is possible to conclude that modern and innovative educational training should include philosophical thinking and then logical trimmings which imply fuzziness in the scientific training. This implies that the conclusions are acceptable with a certain degree of belief that is not completely certain. The graduates must be confident that there is still domain to make creative inventions and scientific discoveries. Otherwise, a classical logic and traditional educational system with the certainty principles does not leave any room for future development, and consequently, graduates from such a system may hold only the certificate and dogmatic knowledge. However, with the advancement of time in their later ages, they may be frustrated that the knowledge they obtained during their education were not certain, but fuzzy.

8. Fuzzy logic education

In natural sciences rather than numbers, qualitative descriptions are dominant at initial information in any reconnaissance study with descriptive linguistic explanations. Ordinary people thinks in a fuzzy manner because they do not have proper terminology or concrete scientific laws for the descriptions and modeling of the phenomenon concerned. This indicates the effectiveness and naturality of fuzzy logic, which is linguistic in content, but connective between different categories at the background. In order to distinguish between the classical Aristotelian and fuzzy logics, let us consider the statement that "one variable (output) is directly proportional with another variable (input)". Such a proposal gives a crisp logical relationship between two variables, which implies that as the input increases, output variable also increases. It is not possible to clearly identify from this statement the following points,

1) Whether the increase is linear or nonlinear?
2) What is the validity domain of both variables?
3) What are the sub-domains of each variable?

In any research, these are significant questions that need proper answers. In the classical scientific educational systems, these points can be objectively identified by measurements and observations. However, herein the very word of observation must be closely examined and its meaning must be explained again linguistically. Measurements need instruments suitable for the study. However, observations may be achieved by human senses and put into words accordingly. Observations are especially significant sources of information in natural sciences. It is rather impossible for a naturalist to set forward logical statements about the phenomenon concerned prior to making effective observations and measurements. In natural sciences, each case has its special and different features that may not be repeated completely. Hence, right at the beginning, it is known that different cases will have common specifications, features, trends and descriptions, but even so, there are also be dissimilar features. The dissimilarities make the comparison or deduction of information on more than two cases to have fuzzy behaviors. This is tantamount to saying that natural patterns at different sites are dissimilar to a certain degree of content. For instance, globally two different sites of igneous rocks might have the same rock types, say, granite, diorite and gabbro, but it is not possible to insist that each rock type has the same degree of membership in these sites. From the classical logic point of view, these two sites are identical to each other without any further detailed specifications. However, the geologist is not convinced fully that they are identical, because whatever the circumstances, there are uncertainties linguistically which are fuzzy in content. It is possible to ask what is the hardness of the granite in different sites? In general, they will have hardness but not at equal degrees, and hence the variation in the hardness can be categorized relatively as "low", "medium", or "high", which allows the entrance of the fuzzy concepts into the assessments. Similar to the word of "proportionality" in the above proposition, "hardness" in the description of the same rock category, cannot be distinctive in sub-categorization. This leads to the general rule that in any logical assessment, sub-categories are significant, and it is possible to deduce that the more (the finer) the categories, the better is the description.

In fuzzy logic, the fundamental significance is not the sub-categorization, but the relationship between them. So far, one can summarize that for fuzzy assessment of any phenomenon the following steps are a priori necessity.

1) Identify the variables for the description of the phenomenon at hand, such as the input and output variables,
2) Sub-categorize the variables through adjectives such as “low”, “medium”, “high”, “warm”, “more”, etc.,
3) State proposals between the sub-categorization of at least two variables (inputs and output), which must include the logical connections in “IT . . . THEN . . .” forms.

Many scientists are not familiar or do not prefer to apply mathematical rules in their preliminary works, and therefore, most of the information are in the form of rather vague statements. This is the main reason why especially in natural and social sciences the fuzzy logic rule is preferable. It is possible to state that in every walk of daily life individuals unconciously use fuzzy concepts, but this paper gives a formal forum for the fuzzy logic ingredients into an innovative education system.

Fuzzy logic approach provides a way of identifying vague relationships between different variables that play role in the causal of a certain phenomenon. In fact, the mathematical equations either through analytical, statistical or probabilistic approaches might lead to such relations, but they are in concrete form attached with numbers, where non-numerical effects cannot be taken into consideration.

Each fuzzy proposition can be thought of consisting two parts as before and after the word THEN. The part before THEN is the antecedent segment, and after THEN it is the consequent segment. These pair-wise fuzzy logical statements can be generalized into triple-wise, quadruple-wise, etc. propositions with care. It is stressed, herein, that in any innovative educational system, a systematic must be given to the students, so that they can tackle any problem with logical solutions prior to any quantitative investigation. Correct logical statements empower the students with quantitative solutions only after the availability of numerical data. In classical education systems, students are given already cooked equations or algorithms without logical steps, and hence, they become formula, equation, procedure, algorithm, and certainty addicted. Whenever they are confronted with a different problem than what they have learnt in the classrooms, then they still expect ready answers from the previous crisp information arena in their mind. Had it been that they are trained with fuzzy logical thinking and self confident logical and rational solutions, they will be eager to attack any problem even the ones that are not directly in their domain of specification, but still in their personal interest.

Another comparison of classical and fuzzy logic propositions can be effectively observed and grasped on the basis of Cartesian coordinate system display. Let us assume that output, O, increases with input, I. This is a crisp logical statement, which may be rendered into a formal proposition as " IF I increases THEN O increases"and there are no adjectives(fuzzy words) in this statement (Figure 3). Non-existence of adjectives is the main difference from fuzzy propositions. Any classical statement does not tell whether the relationship is linear or non linear.

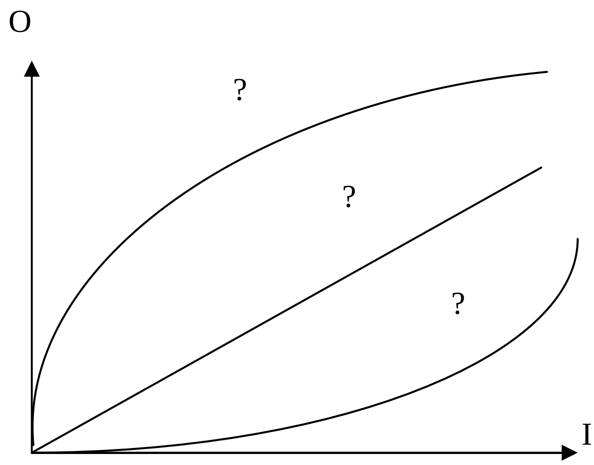

Figure 3. Classical logic domain

On the other hand, in the case of fuzzy logic categorization, I and O can be specified at least by three adjectives, which implies that the two axes on the Cartesian coordinate system can be considered as three divisions, which are shown in Figure 4 for different proportions of each adjective. Herein, h, m, s and w letters imply adjectives of "high", "medium", "small", and "weak", respectively.

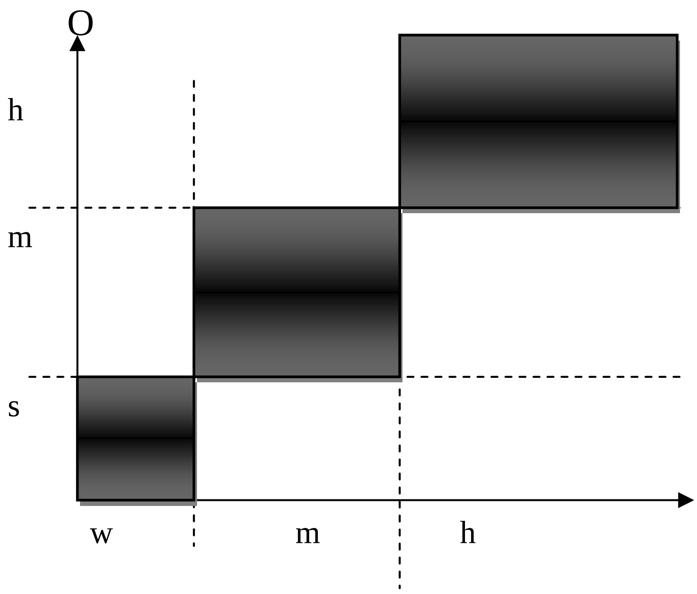

Figure 4. Fuzzy logic domains

Comparison of these four domains with Figure 3 indicates that after the fuzzy partitioning there are now nine sub-domains each corresponding to a specific relationship between the two variables. Hence, more detailed and logical interpretations can be done with ease. There are three sources of information for the identification of valid sub-domains for the problem at hand. These are,

1) Logical deductions, which may be completely work of a non-specialist in the subject,
2) Expert deductions with specific knowledge on the problem from the previous similar or the same studies,
3) Data deductions provided that there are measurements or records of previous similar problems.

In many classical educational systems the students are trained with concentration on the third point whereas the first and second steps are grossly overlooked. Many techniques are thought concerning the third point, especially in engineering and physical sciences through the scatter diagrams and consequent curve fitting procedure by the well known least squares technique without noticing that this technique has many restrictive assumptions.

In the innovative educational systems, perhaps, the last point must be left to students more than the two first ones, which constitutes the fundamentals of creative thinking. Logical deductions should furnish the basis for tackling any problem. If the history of science is reviewed properly, it is possible to see that most of the famous scientists became successful outside of their proper trainings. This indicates that, classical and systematic educational training renders the thinking capability of students into moulds with definite boundaries. For instance, if asked to many students all over the world about the Newton's law, the ready answer will be as F = ma, or force equals the multiplication of mass by acceleration. Such minds cannot be creative but dogmatic, mechanic or robotic, because given the two of the variables (F, m and a) the student will be able to calculate the third one. Such an approach is a nonsense and nuisance for the prosperity of scientific atmosphere. This is exactly what the third step is in the above explanation. Rather than the formulation, if someone states the Newton's law as saying that the force is directly proportional with acceleration, then he/she has exploited dependence on logical principles to a certain extent. The same saying can be put into a formal form as "IF acceleration increases THEN force increases". This statement does not tell anything about the rate of increase. Let us consider the same law from the fuzzy logic point of view by sub-categorizing the force and acceleration into three categories as" low", "moderate" and "high". Consequently, consideration of acceleration as input and force as output variables, there will be 9 sub-domains as in Figure 4 but not all of these sub-domains will be valid logically. Rational and logical thinking without any expertise or data exposition will invalidate 6 of these sub-domains which are, (low acceleration-moderate force), (low acceleration-high force), (moderate acceleration-low force), (moderate acceleration-high force), (high acceleration-moderate force), and finally, (high acceleration-moderate force). Hence, three logical statements remain, namely, (low acceleration-low force), (moderate acceleration-moderate force), and (high acceleration-high force). Furthermore, the graphical representations of these three sub-domains are already shown in Figure 4 with black shaded sub-domain. In Figure 4, black and white sub-domains are still indicators of classical logical taste, which can be further fuzzified for complete description of fuzzy logic assessments. One of the fuzzy logic properties is that there must not be sharp (crisp) boundaries between sub-domains. This brings still another question as “adjective domains” and what are representatives of “adjectives”. The logical answer is that any adjective domain has degrees of representativeness by each adjective. The most representative point in each sub-domain is in the centroid of the area. Such an approach leads to the gray areas in the sub-domains depending on the degree of membership of the adjective attachment to the sub-domain. Hence, the blackness fades away towards the edges and even there appears overlapping between the adjacent sub-domains as shown in Figure 5.

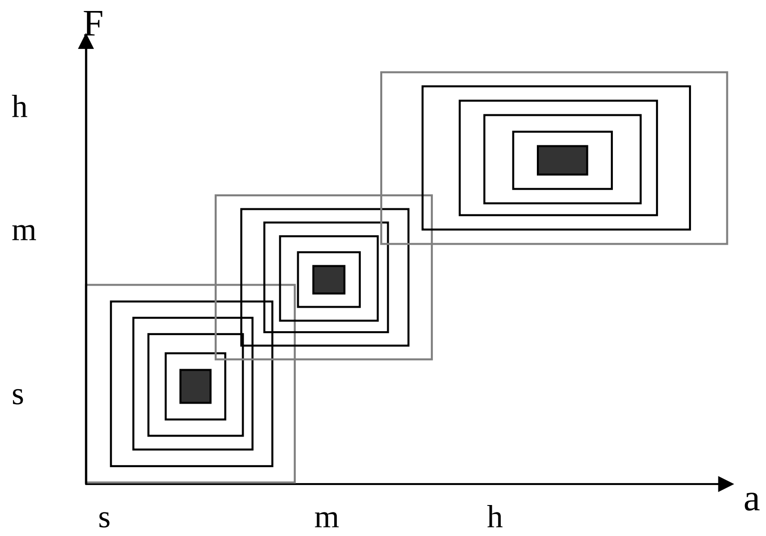

Figure 5. Fuzzy partitioning of sub-domains

9. Innovative education

In an effective innovation education system for scientifically productive results, the following implementations are necessary. It is assumed at this stage that not only the students are considered for the improvement but also the staff members must be ready to undertake these implementations.

1) General behavior of the phenomenon considered must be explained from different points of view on a rather philosophical level, which indicates the significance of language in the planning and tackling of the problem. This step exposes the significance of language structural and grammatical features in a scientific thinking procedure,
2) During the presentation and definition stages of the problem, by all means the students’ contribution must be encouraged through various related questions and views. Accordingly, rather than the unique view and the style

of the teacher, the topic is rendered to be the common mental property of the student group. It may not be possible to guarantee 100 % agreement between the individuals, but at least a common and majority agreement is created. The students' ability is not at the same level, and consequently, there will remain fuzzy uncertainties at the minds of some students. This is also useful, because it will give further room for discussion among them after the formal classroom sittings,

3) The causative effects on the problem must be identified with all possible detail and verbal attachments to variables. Subsequently, the verbal variables must be ordered mentally in the best possible manner according their significance in the problem at hand. This stage may be considered as dismantling of joint causative effects into individual effects,
4) Among the causative effects, a single variable of interest is depicted as the subject of the problem, and hence there are causative and subject variables. As a first stage, it is necessary to consider the classical logical relationships between these variables. These relationships are quite primitive and indicate direct or inverse proportionality. Initially there is a list of logical proportionality relationships, which may be further exploited for the refinement of the problem solution,
5) Sub-categorization of each variable with at least two and preferably three or more classes. This is the stage where the variable names are attached with suitable adjectives. In this manner, each classical variable is rendered into fuzzy variable with various sub-categories,
6) Logical propositions including premises among the sub-categories of causative variables are constituted, and subsequently, each one of these premises is attached with sensible, rational and logical consequent parts of the subject variable. In this manner, the linguistic structure of innovative education by fuzzy logic principles is complete,
7) In order to assure the understanding of the students, it is useful to give a common homework and to request the solution of problem with their individual abilities and linguistic background.

It is possible to conclude that the innovative training through fuzzy logical ingredients is completely linguistic in character, which gives basic principles of learning and discussing the fuzzy remains from the complete solution. In this manner, information and knowledge are transferred from teacher to student or vice versa. Furthermore, fuzzy logic training does not include any mathematical formulations or restrictive assumptions. This implies that in educational systems the mathematical concepts are not the preliminary requests. It should be stated herein that any statement, which insists that the more the mathematics, the better is the research, is mistaken, because the creative education takes place at institutions where the philosophical discussions and consequent logical regularizations are plenty.

10. Conclusions

Present education systems are rather classical with extensive dependence on crisp and blueprint type of information. In many institutions almost spoon fed knowledge and information loadings on fresh brains are given without creative or functional productivities. This is perhaps one of the main reasons why in many institutions all over the world, analytical and especially creative thinking capabilities are not advanced sufficently. It is easy to mention about the quality of students, but the view taken in this paper is that the quality of staff member should also be improved. In developing countries, it is thought most often that the quality control can be improved through the improvement of students' quality only, which is a defective approach, since highly qualified staff members may lead to improvements in students' quality whereas the reverse is not true. In classical educational systems, more than basic logical propositions, formulations and determinism are mentioned for the solution of problems. Especially, in natural and social and even in engineering and physical sciences almost each case study is different from other cases even though they may be close to each other. Therefore, determinism or crisp information systems cannot be sufficient for the description of phenomena concerned.

It is stated in this paper that rather than crisp information and solution techniques, as a first step in any innovative education system, fuzzy logic fundamentals must be provided to the students, because it is the natural logic, which has been forgotten unfortunately, due to continuous classical (crisp, Aristotalian) logic training in education institutions. For instance, prior to any equation proposition or verification by data, fuzzy logic concepts may lead to general solution of the problem concerned with philosophical background and logical rule. In a fuzzy logic education training the causes of a phenomenon must be identified as variables and then these variables are considered as sub-categories, which are then combined together through logic propositions to each other.

The main conclusions are that the scientific knowledge cannot be completely varifiable or falsifiable but rather it is always fuzzifiable which provides potantiality for further reseraches. As a general conclusion of this paper, it is suggested that the science and any related attribute to it is never completely verifiably or falsifiable, but always fuzzifiable and hence further developments in the form of prescience, traditional science and occasional revolutionary science will be in view for all times, spaces and societies.

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Fuzzy rule based optimization in machining of FRP composites

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Abstract

Fiber Reinforced Plastic (FRP) (also fiber reinforced polymer) is a composite material made of a polymer matrix reinforced with fibers. The fibers are usually glass fiber, carbon, or aramid, while the polymer is usually an epoxy, vinylester or polyester thermosetting plastic. FRPs are commonly used in the aerospace, automotive, marine, and construction industries. Fiber reinforced plastics are best suited for any design problem that demands weight savings, precision engineering, finite tolerances, and the simplification of parts in both production and operation.
With the upcoming usage of fiber reinforced polymer (FRP) composites in various areas of applications, machining of these materials has become a major concern for the manufacturing industries. The current knowledge and state of art of machining FRP composites, unfortunately, is seemed inadequate for its optimal economic utilization. This paper presents an optimization study made on machining of randomly oriented glass fiber reinforced (GFRP) polymer composite rods with different process environment. An expert system based on fuzzy rule based modeling approach combined with Taguchi's robust design philosophy has been adopted to evaluate the optimal process parameters thereby satisfying conflicting requirements of material removal rate (MRR) and surface roughness of the machined composite product. Effectiveness of the proposed model has been illustrated in this reporting.

Keywords: Fiber reinforced polymer (FRP), fuzzy rule, Taguchi's robust design

1. Introduction: Prior state of art and problem formulation

FRP (Fiber Reinforced Polymer) Composite is a mixture of materials that include special polymers reinforced with fibers designed to carry loads much stronger than a regular plastic or ordinary fiberglass. Other materials include resins, fillers and additives. Polymer is plastic, and fibers are added to reinforce it. Each material has its own purpose and contribution to the strength and durability of the product being manufactured. When combined in the proper manner, the result is the best in strength, light weight as well as cost effectiveness.

FRP Composite itself has many benefits over other common materials:

- It has a high strength to weight ratio
- It does not contract or expand due to temperature changes
- It does not rust or absorb water
- It is non-flammable
- It does not conduct electricity
- It is generally chemical resistant

Santhanakrishnam et al. (1988) carried out face turning trials on glass fiber reinforced polymers (GFRP), carbon fiber reinforced polymers (CFRP) and kevlar fiber reinforced polymers (KFRP) cylindrical tubes to study their machined surfaces for possible application as friction surfaces. The surface roughness obtained and the observed morphology of the machined surfaces of fiber reinforced polymer (FRP) composites were compared. The mechanisms of material removal and tool wear were also discussed. The cutting forces encountered during machining of composites were also reported. El-Sonbaty et al. (2004) investigated the influence of cutting speed, feed, drill size and fiber volume fraction on the thrust force, torque and surface roughness in drilling processes of fiber reinforced epoxy composite materials. Davim and Mata (2005) presented an optimization study of surface roughness in turning FRPs tubes manufacturing by filament winding and hand lay-up, using polycrystalline diamond cutting tools. Optimal cutting parameters were identified to obtain a certain surface roughness (*Ra* and *Rt/R*max), corresponding to international dimensional precision (ISO) IT7 and IT8 in the FRP work pieces, using multiple analysis regression (MRA). Additionally, the optimal material removal rates were identified. Mohan et al. (2005) outlined the Taguchi optimization methodology, which is applied to optimize cutting parameters in drilling of glass fiber reinforced composite (GFRC) material followed by Analysis of Variance (ANOVA); to study the effect of process parameters on machining process. The drilling parameters and specimen parameters evaluated were speed, feed rate, drill size and specimen thickness. A series of experiments were conducted to relate the cutting parameters and material parameters on the cutting thrust and torque. An orthogonal array, signal-to-noise ratio were employed to analyze the influence of these parameters on cutting force and torque during drilling. Analysis of the Taguchi method indicated that among the all-significant parameters, speed and drill size were found to impose more significant influence on cutting thrust than the specimen thickness and the feed rate. Davim and Mata (2005) studied on the machinability in turning processes of fiber reinforced polymers (FRPs) using polycrystalline diamond cutting tools. A statistical technique, using orthogonal arrays and ANOVA, was employed to investigate the influence of cutting parameters on specific cutting pressure and surface roughness. The objective was to evaluate the machinability of these materials as a function of manufacturing process (filament winding and hand lay-up). A new machinability index was proposed by the authors. Jawali et al. (2006) fabricated a series of short glass fiber-reinforced nylon 6 composites with different weight ratios of glass contents by melt mixing. The fabricated nylon 6 composites have been characterized for physical-mechanical properties such as specific gravity, tensile properties, and wear resistance. A marginal improvement in tensile strength and tensile modulus was observed with increase in high modulus fiber. Wear resistance was increased with the increase in rigid glass fiber content in the nylon matrix. The dimensional stability of the composite was found improved with the increase in fiber content. The acoustic behavior of these composites was measured using acoustic emission technique. The surface morphological behavior of the composites was investigated by scanning electron microscopy (SEM). Bagci and Işık (2006) carried out orthogonal cutting tests on unidirectional glass fiber reinforced polymers (GFRP), using cermet tools. During the tests, the depth of cut, feed rate, cutting speed was varied, whereas the cutting direction was held parallel to the fiber orientation. Turning experiments were designed based on statistical three level full factorial experimental designs. An artificial neural network (ANN) and response surface (RS) model were developed to predict surface roughness on the turned part surface. In the development of predictive models, cutting parameters of cutting speed, depth of cut and feed rate were considered as model variables. The required data for predictive models were obtained by conducting a series of turning test and measuring the surface roughness data. Good agreement was observed between the predictive models results and the experimental measurements. The ANN and RSM models for GFRPs turned part surfaces were compared with each other for accuracy and computational cost. Palanikumar et al. (2006) attempted to assess the influence of machining parameters on the machining of GFRP composites. Full factorial design of experiments concept was used for experimentation. The machining experiments were conducted on all geared lathe using coated cermet tool inserts with two level of factors. The factors considered were cutting speed, work piece fiber orientation angle, depth of cut and feed rate. A procedure was developed to assess and optimize the chosen factors to attain minimum surface roughness by incorporating: (i) response table and response graph; (ii) normal probability plot; (iii) interaction graphs; (iv) Analysis of Variance (ANOVA) technique. Palanikumar et al. (2006) discussed the application of the Taguchi method with fuzzy logic to optimize the machining parameters for machining of GFRP composites with multiple characteristics. A multi-response performance index (MRPI) was used for optimization. The machining parameters viz., work piece (fiber orientation), cutting speed, feed rate, depth of cut and machining time were optimized with consideration of multiple performance characteristics viz., metal removal rate, tool wear, and surface roughness. The results from confirmation runs indicated that the determined optimal combination of machining parameters improved the performance of the machining process. Palanikumar et al. (2006) developed a mathematical model to predict the surface roughness of machined glass fiber reinforced polymer (GFRP) work piece using regression analysis and analysis of variance (ANOVA) in order to study the main and interaction effects of machining parameters, viz., cutting speed, work piece fiber orientation angle, depth of cut, and feed rate. The adequacy of the developed model was verified by calculating the correlation coefficient. This model could be effectively used to predict the surface roughness of the machined GFRP components. Davim and Mata (2007) investigated the machinability in turning processes of glass fiber reinforced plastics (GFRPs) manufactured by hand lay-up. A plan of experiments was performed on controlled machining with cutting parameters prefixed in work piece.

A statistical technique, using orthogonal arrays and analysis of variance (ANOVA), were employed to know the influence of cutting parameters on specific cutting pressure and surface roughness. The objective was to evaluate the machinability of these materials in function of cutting tool (polycrystalline diamond and cemented carbide tools). A new machinability index has been proposed by the authors. Palanikumar and Davim (2007) derived a mathematical model to predict the tool wear on the machining of GFRP composites using regression analysis and analysis of variance (ANOVA) in order to study the main and interaction effects of machining parameters, viz., cutting speed, feed rate, depth of cut and work piece fiber orientation angle. The adequacy of the developed model was verified by using coefficient of determination and residual analysis. This model could be effectively used to predict the tool wear on machining GFRP components within the ranges of variables studied. The influences of different parameters in machining GFRP composite were also analyzed. Palanikumar (2007) attempted to model the surface roughness through response surface method (RSM) in machining GFRP composites. Four factors five level central composite, rotatable design matrix was employed to carry out the experimental investigation. Analysis of Variance (ANOVA) was used to check the validity of the model. For finding the significant parameters student’s t-test was used. Also, an analysis of the influences of the entire individual input machining parameters on the response were carried out and presented in this study. Karnik et al. (2008) presented application of artificial neural network (ANN) model to study the machinability aspects of unreinforced polyetheretherketone (PEEK), reinforced polyetheretherketone with 30% of carbon fibers (PEEK CF 30) and 30% of glass fibers (PEEK GF 30) machining. A multilayer feed forward ANN was employed to study the effect of parameters such as tool material, work material, cutting speed and feed rate on two aspects of machinability, namely, power and specific cutting pressure. The input-output patterns required for training were obtained from the experiments planned through full factorial design. The analysis reveals that minimum power results from a combination of lower values of cutting speed and feed rate for all work-tool combinations. However, higher values of feed rate were required to achieve minimum specific cutting pressure. The investigation results exhibited that, K10 tool provided better machinability for PEEK and PEEK CF 30 materials, while PCD tool was found preferable for PEEK GF 30 material. Palanikumar (2008) discussed the use of Taguchi and response surface methodologies for minimizing the surface roughness in machining glass fiber reinforced (GFRP) plastics with a polycrystalline diamond (PCD) tool. The experiments were conducted using Taguchi’s experimental design technique. The cutting parameters used were cutting speed, feed and depth of cut. The effect of cutting parameters on surface roughness was evaluated and the optimum cutting condition for minimizing the surface roughness was determined. A second-order model was established between the cutting parameters and surface roughness using response surface methodology. The experimental results revealed that the most significant machining parameter for surface roughness was feed followed by cutting speed. Basheera et al. (2008) presented an experimental work on the analysis of machined surface quality on Al/SiCp composites leading to an artificial neural network-based (ANN) model to predict the surface roughness. The predicted roughness of machined surfaces based on the ANN model was found to be in very good agreement with the unexposed experimental data set. Palanikumar et al. (2008) presented a study of influence of cutting parameters on surface roughness parameters such as Ra, Rt, Rq, Rp and R3z in turning of glass fiber reinforced composite materials. Empirical models were developed to correlate the machining parameters with surface roughness. Analysis of experimental results was carried out through area graphs and three-dimensional surface plots. Palanikumar (2008) discussed the use of fuzzy logic for modeling machining parameters in machining glass fiber reinforced plastics by poly-crystalline diamond tool. The Taguchi method was used for conducting the experiments, which in turn reduced the number of experiments. An orthogonal array was used to investigate the machining process. The cutting parameters selected were cutting speed, feed, and depth of cut. The output responses considered for the investigation were surface roughness parameters such as arithmetic average height (Ra) and maximum height of the profile (Rt). Fuzzy rule based models were developed for correlating cutting parameters with surface roughness parameters. The model predicted values and measured values were fairly close to each other. The confirmation test results proved the fact that the developed models were effectively representing the surface roughness parameters Ra and Rt in machining of GFRP composites. Davim et al. (2009) reported on the better understanding of the machinability of PA 66 polyamide with and without 30% glass fiber reinforcing, when precision turning at different feed rates and using four distinct tool materials. The findings indicated that the radial force component presented highest values, followed by the cutting and feed forces. The PCD tool gave the lowest force values associated with best surface finish, followed by the ISO grade K15 uncoated carbide tool with chip breaker when machining reinforced polyamide. Continuous coiled micro-chips were produced, irrespectively of the cutting parameters and tool material employed. Palanikumar and Davim (2009) attempted to assess the factors influencing tool wear on the machining of GFRP composites. The factors considered were cutting speed, fibre orientation angle, depth of cut and feed rate. A procedure was developed to assess and optimize the chosen factors to attain minimum tool wear by incorporating (i) response table and effect graph; (ii) normal probability plot; (iii) interaction graphs; (iv) Analysis of Variance (ANOVA) technique. The results indicated that cutting speed is a factor, which had greater influence on tool flank wear, followed by feed rate. Also the determined optimal conditions reduced the tool flank wear on the machining of GFRP composites within the ranges of parameters studied. Kilickap (2010) investigated the influence of the cutting parameters, such as cutting speed and feed rate, and point angle on delamination produced when drilling a GFRP composite. The damage generated associated with drilling GFRP composites were observed, both at the entrance and the exit during the drilling. The author obtained optimum cutting parameters for minimizing delamination at drilling of GFRP composites. This paper presented the application of Taguchi method and Analysis of Variance (ANOVA) for minimization of delamination influenced by drilling parameters and drill point angle. The optimum drilling parameter combination was obtained by using the analysis of signal-to-noise ratio. The conclusion revealed that feed rate and cutting speed were the most influential factor on the delamination, respectively. The best results of the delamination were obtained at lower cutting speeds and feed rates. Kini and Chincholkar (2010) studied the effect of varying machining parameters in turning on surface roughness and material removal rate (MRR) for $\pm 30^0$ filament wound glass fiber reinforced polymers (GFRP) in turning operations using coated tungsten carbide inserts under dry cutting conditions. The authors described the development of an empirical model for turning GFRP utilizing factorial experiments. Second order predictive model covering speed, feed, depth of cut and tool nose radius was developed at 95% confidence interval for surface roughness and material removal rate. Hussain et al.

(2010) studied on development of a surface roughness prediction model for the machining of GFRP pipes using Response Surface Methodology (RSM). Experiments were conducted through the established Taguchi's Design of Experiments (DOE) on an all geared lathe using carbide (K20) tool. The cutting parameters considered were cutting speed, feed, depth of cut, and work piece (fiber orientation). A second order mathematical model in terms of cutting parameters was developed using RSM. The effect of different parameters on surface roughness was also analyzed. Hussain et al. (2011) studied of machinability of GFRP composite tubes of different fiber orientation angle vary from 30^0 to 90^0. Machining studies were carried out on an all geared lathe using three different cutting tools: namely Carbide (K-20), Cubic Boron Nitride (CBN) and Poly-Crystalline Diamond (PCD). Experiments were conducted based on the established Taguchi's Design of Experiments (DOE) L_{25} orthogonal array on an all geared lathe. The cutting parameters considered were cutting speed, feed, depth of cut, and work piece (fiber orientation). The performances of the cutting tools were evaluated by measuring surface roughness (Ra) and Cutting force (Fz). A second order mathematical model in terms of cutting parameters was developed using RSM. Sait et al. (2008) presented a new approach for optimizing the machining parameters on turning glass-fiber reinforced polymer (GFRP) pipes. Optimization of machining parameters was done by an analysis called desirability function analysis. Based on Taguchi's L_{18} orthogonal array, turning experiments were conducted for filament wound and hand layup GFRP pipes using K20 grade cemented carbide cutting tool. The machining parameters such as cutting velocity, feed rate and depth of cut were optimized by multi-response considerations namely surface roughness, flank wear, crater wear and machining force. A composite desirability value was obtained for the multi-responses using individual desirability values from the desirability function analysis. Based on composite desirability value, the optimum levels of parameters were identified, and significant contribution of parameters was determined by ANOVA. Thus, the application of desirability function analysis in Taguchi technique proved to be an effective tool for optimizing the machining parameters of GFRP pipes.

Literature depicts that efforts have been made by previous researchers in understanding various aspects of composite machining. Machinability aspects with a variety of tool-work material combination have been addressed and well documented in literature. Issues of tool wear, surface roughness, and involvement of cutting forces have been investigated as well. Predictive models haves also been developed using regression modeling, response surface modeling as well as neural network. Optimization aspects have been attempted but to a limited extent.

In parametric optimization, Taguchi method has been found extensive application as it explores statistically designed experiments (orthogonal array) and the concept of signal-to-noise (SN) ratio. The approach is advantageous from economic point of view as it requires well balanced (limited number of experiments) experimental runs resulting reliable prediction outcome. Moreover, Taguchi approach follows optimal search at discrete levels of process parameters in the prescribed domain which can easily be adjusted in the experimental setup. The limitation of the traditional Taguchi approach is the incapability in addressing optimization issues of multiple conflicting objectives. Desirability function was reported (Sait et al., 2008) to combine multiples responses into overall desirability value which was finally optimized by Taguchi method.

However, these optimization approaches were based on the assumption of negligible response correlation; while in practical situation definitely some correlations exist among output responses. Secondly, uncertainty arises in assigning individual response priority weights. Degree of importance of individual responses is represented by the priority weights decided by the decision maker which may vary according to individual's discretion. These create uncertainty, vagueness in the solution. To avoid this fuzzy logic has come into picture. Rajasekaran et al. (2011) attempted to develop a fuzzy model to predict the cutting force thereby cutting power and specific cutting force in machining CFRP composites. The developed models offered satisfactory performance on comparison with the experimental results and hence these models could be effectively used to predict cutting forces in machining of carbon fiber-reinforced plastic composites. In order to bypass various shortcomings of aforesaid traditional optimization approaches, fuzzy linguistic reasoning has been adopted in the present work. Using Fuzzy Inference System (FIS) multiple responses (objectives) have been aggregated into a single quality index: Multi-Performance Characteristic Index (MPCI) which has been finally optimized by Taguchi method. The study demonstrates a case study on selecting an optimal process environment for GFRP composite machining (turning) in which conflicting requirements of (i) material removal rate (MRR in the process) and (ii) surface roughness of the machined product have been satisfied simultaneously. As material removal rate is directly related to productivity and product surface roughness dictates the aspect of product quality; the present problem is reduced to a situation of quality-productivity optimization. It is felt that there must be an optimal compatible balance between quality and productivity.

2. Experimentation

The present study has been done through the following plan of experiments.

[1] Checking and preparing the centre lathe ready for performing the machining operation.
[2] Cutting GFRP bars and performing initial turning operation in lathe to get desired dimension ($\phi 50 \times 150$) of the work pieces.
[3] Calculating weight of each specimen by the high precision digital balance meter before machining.
[4] Performing straight turning operation on specimens in various cutting environments involving various combinations of process control parameters like spindle speed, feed and depth of cut.
[5] Calculating weight of each machined GFRP bars again by the digital balance meter.
[6] Calculating MRR of the process for each experimental run.
[7] Measuring surface roughness (R_a) of the machined surface for each experimental run.

Fiber Reinforced Polyester composite has been selected as work piece material. The specifications of the work piece material are shown in Table 1. Carbide tool (K20) has been has been used for this investigation. In the present study, spindle speed (N, rpm), feed rate (f, mm/min) and depth of cut (d, mm), have been selected as design factors while other parameters have been assumed to be constant over the experimental domain. The process variables (design factors) with their values at different levels have been listed in Table 2. It is known that the selection of the values of the variables is limited by the capacity of the machine used in the experimentation as well as the recommended specifications for different work piece and tool material combinations. Therefore, three levels have been selected for each of the aforesaid three factors. In the present investigation, Taguchi's L_9 orthogonal array (OA) design (without factorial interaction) has been considered for experimentation (Table 3). The machine used for turning is PINACHO manually operated lathe. The surface roughness parameters have been measured using the stylus-type profilometer, Talysurf (Taylor Hobson, Surtronic 3+). The definitions of surface roughness average (R_a), selected in the present study, along with MRR selected in the present study have been given below. The values of measured roughness parameter (average of trials) R_a along with material removal rate (MRR) has been shown in Table 4.

R_a (arithmetic average height)

Roughness average R_a is the arithmetic average of the absolute values of the roughness profile ordinates. R_a is the arithmetic mean roughness value from the amounts of all profile values.

$$R_a = \frac{1}{l}\int_0^l \left|Z(X)\right| dx \tag{1}$$

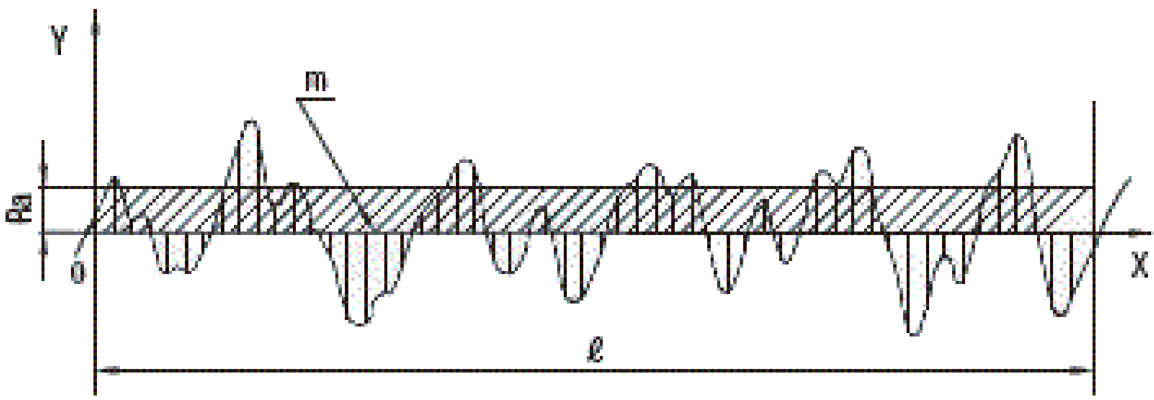

Figure 1. Measurement of R_a

Material Removal Rate (MRR)

Material removal rate (MRR) has been calculated from the difference in weights of the work pieces before and after experiment.

$$MRR = \frac{W_i - W_f}{\rho . t_m} \left(mm^3 / \min \right) \tag{2}$$

Here, W_i is the initial weight of the work piece in gm

W_f is the final weight of the work piece in gm

ρ is the density of work material (2 gm/cm^3 for GFRP polyester) and
t_m is the machining time in minute.

3. Fuzzy Inference System (FIS)

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The mapping then provides a basis from which decisions can be made, or patterns discerned. The process of fuzzy inference involves the following elements: Membership Functions, Logical Operations, and If-THEN Rules. Most commonly two types of fuzzy inference systems can be implemented: *Mamdani* type and *Sugeno* type. These two types of inference systems vary somewhat in the way outputs are determined [30-43].

Fuzzy inference systems have been successfully applied in fields such as automatic control, data classification, decision analysis, expert systems, and computer vision. Because of its multidisciplinary nature, fuzzy inference systems are associated with a number of names, such as fuzzy-rule-based systems, fuzzy expert systems, fuzzy modeling, fuzzy associative memory, fuzzy logic controllers, and simply (and ambiguously) fuzzy systems.

Mamdani's fuzzy inference method is the most commonly viewed fuzzy methodology. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed in 1975 by Ebrahim Mamdani (Mamdani, 1976; 1977) as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators.

Mamdani type inference expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. It is possible, and in many cases much more efficient, to use a single spike as the output membership functions rather than a distributed fuzzy set. This type of output is sometimes known as a singleton output membership function, and it can be thought of as a pre-defuzzified fuzzy set. It enhances the efficiency of the defuzzification process because it greatly simplifies the computation required by the more general Mamdani method, which finds the centroid of a two-dimensional function. Rather than integrating across the two-dimensional function to find the centroid, weighted average of a few data points is used. Sugeno-type systems support this type of model. In general, Sugeno-type systems can be used to model any inference system in which the output membership functions are either linear or constant. The basic structure of FIS is shown in the following diagram (Fig. 2). The fuzzy inference process has been described below in Fig. 3.

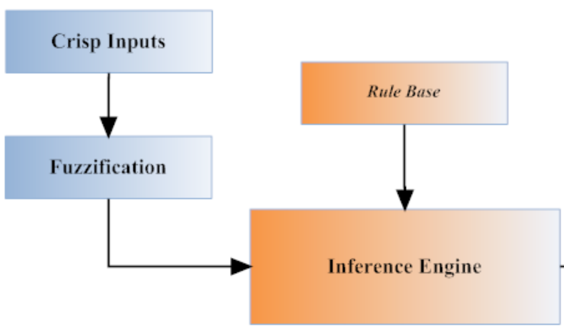

Figure 2. Basic structure of FIS

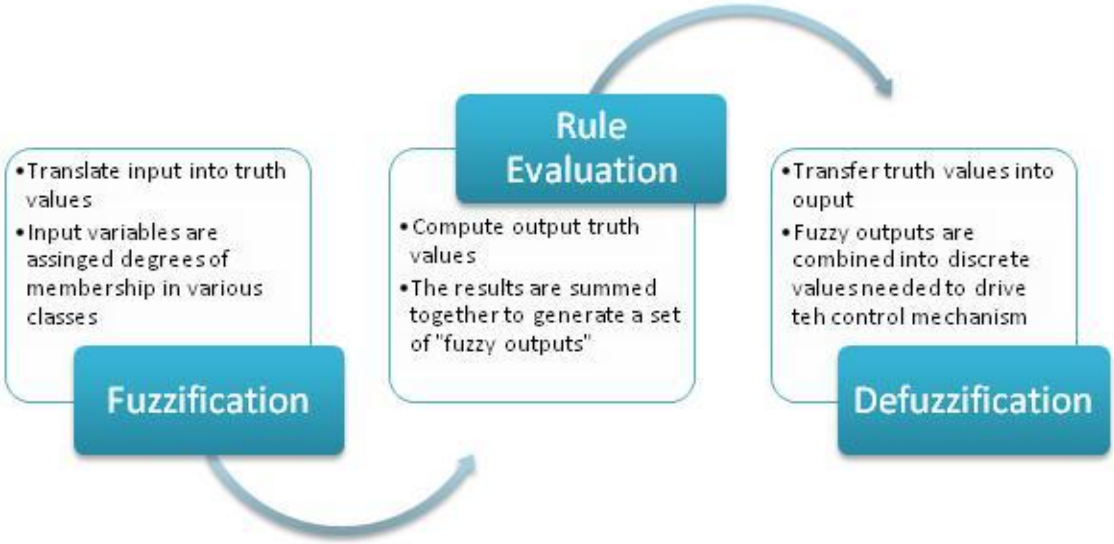

Figure 3. Operation of fuzzy inference system

4. Parametric optimization: Results and discussions

Experimental data (corresponding to Table 4) have been converted into corresponding SN ratios using Eqs 3-4. For surface roughness parameter R_a, a Lower-the-Better (LB) criterion and for MRR, a Higher-the-Better (HB) criterion has been selected.
The SN ratio with a Lower-the-Better (LB) characteristic can be expressed as:

$$\eta_{ij} = -10\log\left(\frac{1}{n}\sum_{j=1}^{n} y_{ij}^2\right) \quad (3)$$

The SN ratio with a Higher-the-Better (HB) characteristic can be expressed as:

$$\eta_{ij} = -10\log\left(\frac{1}{n}\sum_{j=1}^{n} \frac{1}{y_{ij}^2}\right) \quad (4)$$

Here, y_{ij} is the ith experiment at the jth test, n is the total number of the tests.

Computed SN ratios have been furnished in Table 4. These SN ratios have then been normalized (Table 5) based on Higher-the-Better (HB) criteria using Eq. 5.
For Higher-the-Better (HB) criterion, the normalized data can be expressed as:

$$x_i(k) = \frac{y_i(k) - \min y_i(k)}{\max y_i(k) - \min y_i(k)} \quad (5)$$

Here, $x_i(k)$ is the value of the response (SN ratio) k for the i^{th} experiment, $\min y_i(k)$ is the smallest value of $y_i(k)$ for the k^{th} response (SN ratio), and $\max y_i(k)$ is the largest value of $y_i(k)$ for the k^{th} response (SN ratio).

Normalized SN ratios (Table 5) of the responses (R_a and MRR) have been fed as inputs in Fuzzy Inference System (FIS) (Fig. 4). FIS explores fuzzy rule base (Table 6). The output of the fuzzy inference system has been defined as MPCI (Table 7). This Multi-Performance Characteristic Index (MPCI) has been finally optimized by using Taguchi methodology. Higher- the- Better (HB) criterion has been used for optimizing (maximizing) the MPCI (Eq. 5).

In calculating MPCI in FIS system, various membership functions (MFs) (Fig. 5-6) have been assigned to the input variables: (i) normalized SN ratio of MRR and (ii) normalized SN ratio of R_a. The selected membership functions for input variables are given below.

$MRR_{\text{Normalized SN ratio}}$: "Low", "Medium" and "High".
$R_{a\ \text{Normalized SN ratio}}$: "Low", "Medium" and "High"

Five membership functions have been selected for MPCI: "Very Small", "Small", "Medium", "Large", and "Very Large" (Fig. 7). Nine fuzzy rules (Table 6) have been explored for fuzzy reasoning (Fig. 8). Fuzzy logic converts linguistic inputs into linguistic output. Linguistic output is again converted to numeric values (MPCI) by defuzzification method. Numeric values of MPCIs have been tabulated in Table 7 with corresponding SN ratio. SN ratios of MPCIs have been calculated using Higher-the-Better (HB) criterion. Fig. 9 represents optimal parametric combination (N_3 f_2 d_2). Optimal result has been validated by satisfactory confirmatory test. Predicted value of SN ratio of MPCI has been found 1.92944 (higher than all entries of SN ratios in Table 7). In confirmatory experiment the value came 1.8913. So, it can be concluded that quality and productivity have improved using the said optimal setting. Table 8 represents mean values table of MPCIs. The degree of influence of various factors on MPCI can be estimated from this table. It shows that spindle speed is the most significant factor on influencing MPCIs followed by depth of cut and feed rate.

4. Conclusions

In this study, fuzzy rule based expert system has been adopted using two input variables with single output i.e. MPCI. By this way a multi-response optimization problem has been converted into an equivalent single objective optimization problem which has been further solved by Taguchi philosophy. The proposed procedure is simple, effective in developing a robust, versatile and flexible mass production process. Response correlations need not to be revealed and eliminated. In the proposed model it is not required to assign individual response weights. FIS can efficiently take care of these aspects into its internal hierarchy. Degree of influence of various process control factors can be investigated easily. Accuracy in prediction of the model analysis can be subsequently increased by assigning adequate fuzzy rules as well as by increasing number of membership functions in the fuzzy inference system. This approach can be recommended for continuous quality improvement and off-line quality control of a process/product in any manufacturing/ production environment.

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Table 1. Specifications of work material

Resin used	Polyester resin
Fiber orientation	Random
Method of preparation	Hand moulding method
Composition	75:25 (Resin: Fiber)
Weight percentage of hardener	5%
Density	2 gm/cm^3

Table 2. Machining parameters (domain of experiments)

Parameters	Notation and Unit	Level Values		
		Level 1	Level 2	Level 3
Spindle Speed	N (RPM)	530	860	1400
Feed Rate	f (mm/rev)	0.298	0.308	0.331
Depth of cut	d (mm)	3.0	4.0	5.0

Table 3. Design of experiments (L_9 orthogonal array)

Sl. No.	Factor setting (coded form)		
	N	f	d
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

Table 4. Experimental data, corresponding SN ratios and computed MPCI

Sl. No.	MRR (mm^3/min)	Ra (µm)	SN Ratio of MRR (dB)	SN Ratio of Ra (dB)
1	13767.7	5.1333	82.7773	-14.2079
2	13843.4	5.8533	82.8248	-15.3480
3	18525.7	5.9933	85.3555	-15.5533
4	30763.1	5.2933	89.7606	-14.4745
5	27686.8	4.9533	88.8454	-13.8979
6	14648.2	4.5400	83.3157	-13.1411
7	37492.5	5.0200	91.4789	-14.0141
8	28794.2	5.2800	89.1861	-14.4527
9	35762.1	5.2066	91.0685	-14.3311

Table 5. Normalized SN ratios

Sl. No.	Normalized SN Ratio of MRR	Normalized SN Ratio of Ra
1	0.00000	0.55774
2	0.00500	0.08510
3	0.29660	0.00000
4	0.80260	0.44722
5	0.69750	0.68260
6	0.06230	1.00000
7	1.00000	0.63080
8	0.73660	0.45626
9	0.95280	0.50671

Table 6. Fuzzy rule matrix

Rule No.	IF Normalized SN Ratio of MRR is:	AND Normalized SN Ratio of Ra is:	THEN MPCI is:
1	LOW	LOW	VERY SMALL
2	MEDIUM	LOW	SMALL
3	HIGH	LOW	MEDIUM
4	LOW	MEDIUM	SMALL
5	MEDIUM	MEDIUM	MEDIUM
6	HIGH	MEDIUM	LARGE
7	LOW	HIGH	MEDIUM
8	MEDIUM	HIGH	LARGE
9	HIGH	HIGH	VERY LARGE

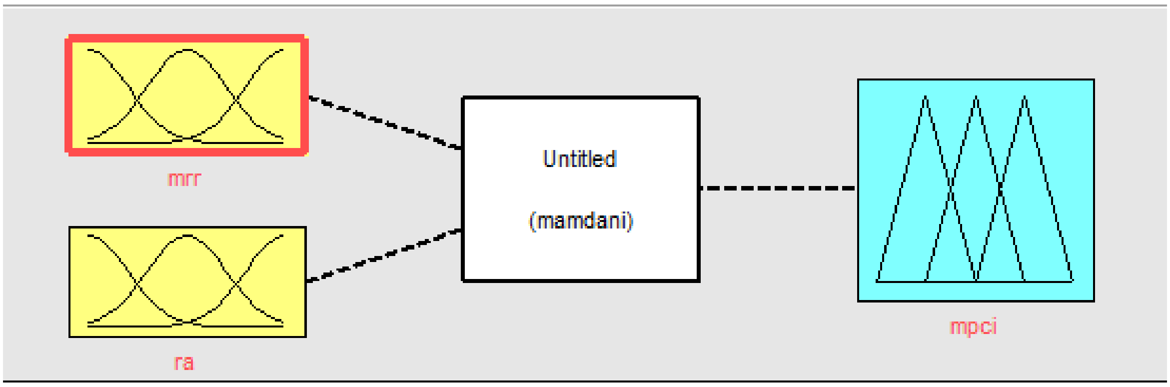

Figure 4. Proposed fuzzy inference system

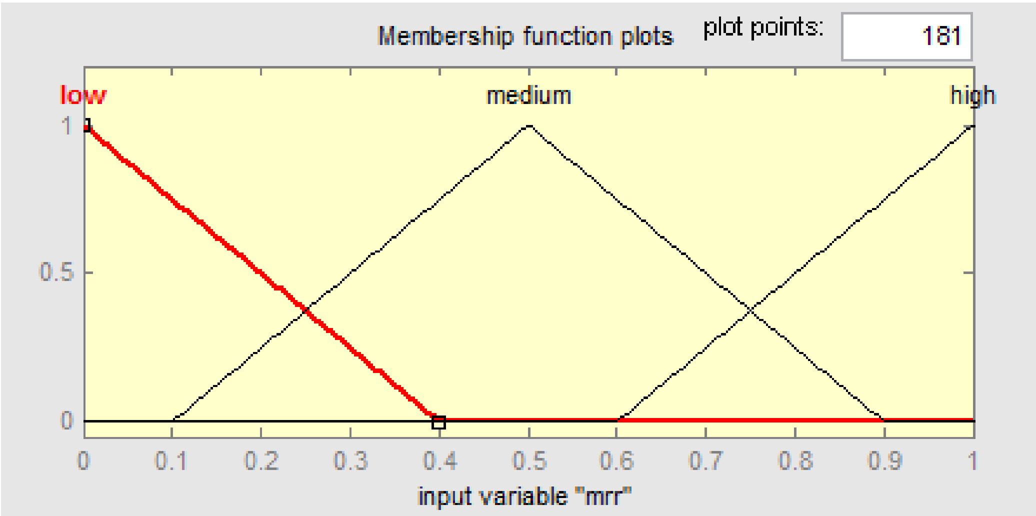

Figure 5. Membership functions (MFs) for MRR (normalized SN ratio of MRR)

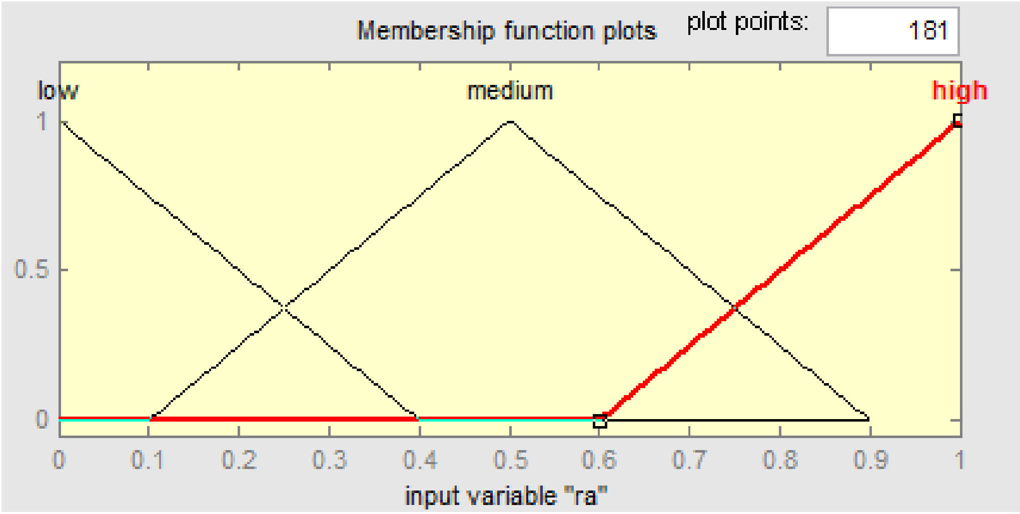

Figure 6. Membership functions (MFs) for Ra (normalized SN ratio of Ra)

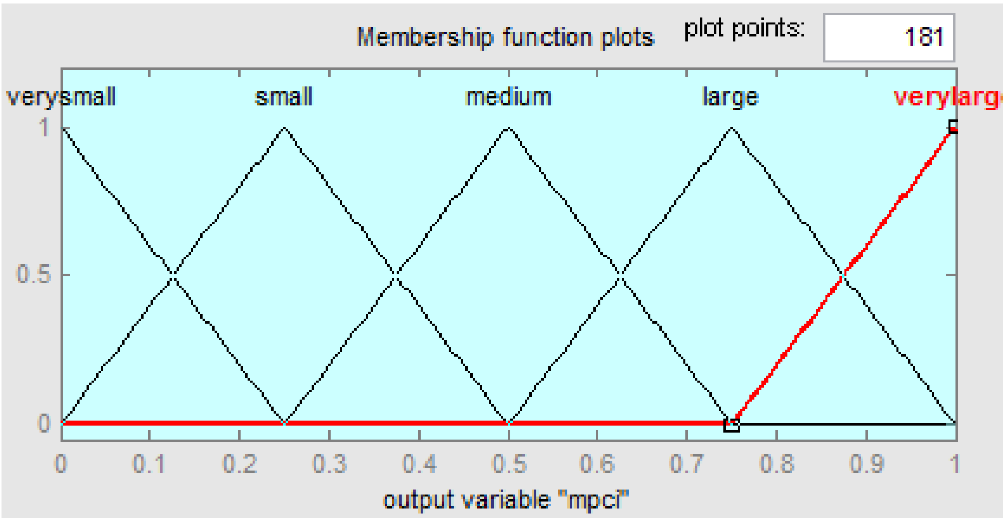

Figure 7. Membership functions (MFs) for MPCI

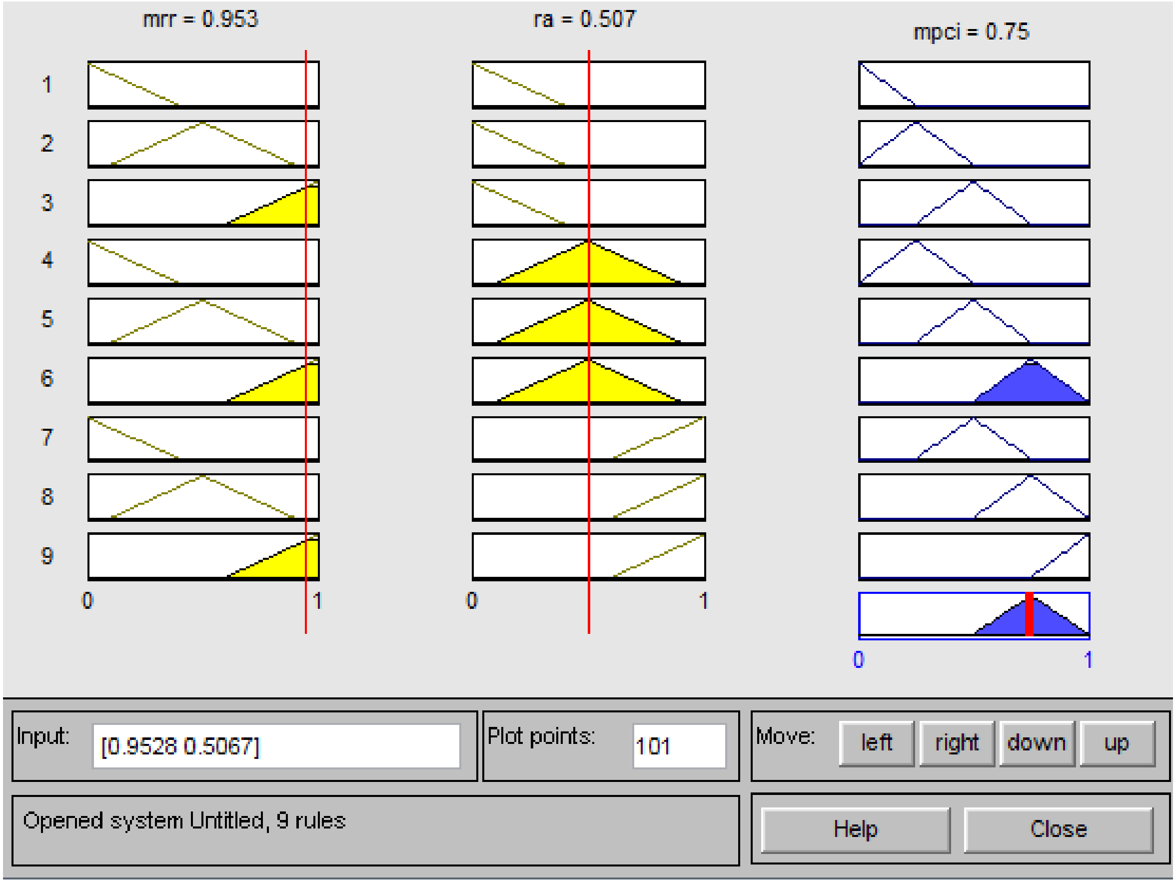

Figure 8. Fuzzy reasoning rule base

Table 7. Computed MPCI and corresponding SN ratio

Sl. No.	MPCI	SN Ratio of MPCI (dB)
1	0.250	-12.0412
2	0.836	-1.5559
3	0.238	-12.4685
4	0.666	-3.5305
5	0.594	-4.5243
6	0.500	-6.0206
7	0.752	-2.4756
8	0.614	-4.2366
9	0.750	-2.4988

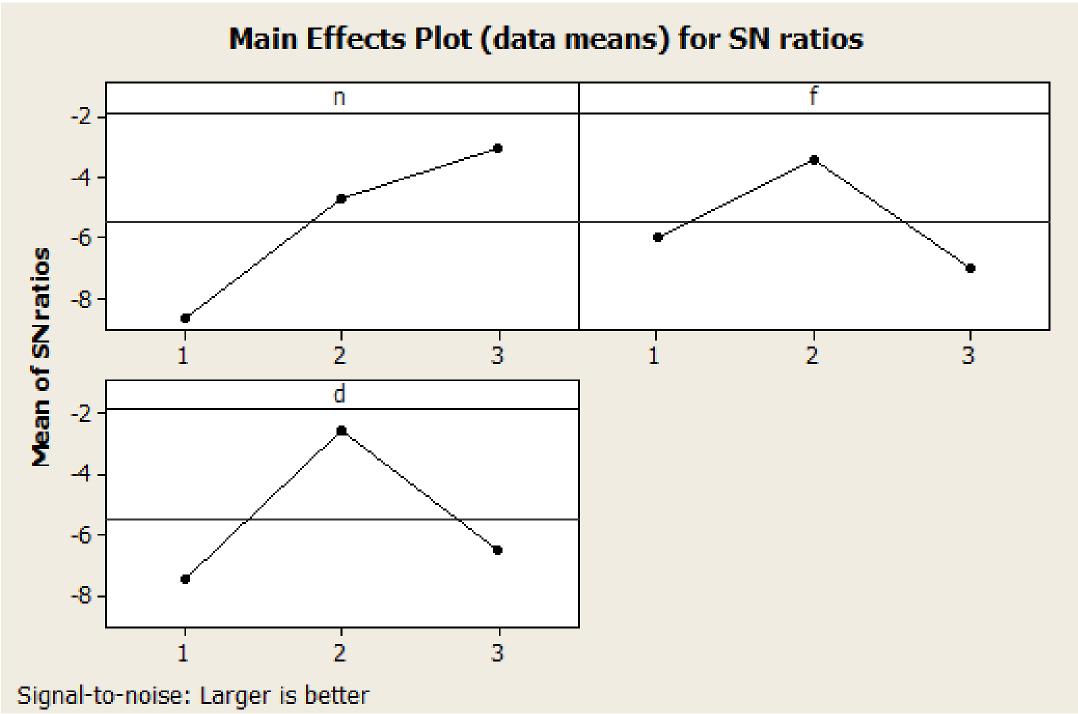

Figure 9. Evaluation of optimal setting (SN ratio plot of MPCIs)

Table 8. Mean response table (SN ratio of MPCIs)

Level	N	f	d
1	-8.689	-6.016	-7.433
2	-4.692	-3.439	-2.528
3	-3.070	-6.996	-6.489
Delta (max.-min.)	5.618	3.557	4.904
Rank	1	3	2