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Modeling the Spirality of Cotton Knit Fabric Using Fuzzy Expert System

Md. Abdus Shahid*
Department of Textile Engineering, Dhaka University of Engineering and Technology (DUET), Gazipur 1700 Bangladesh
E-mail: te_shahid@yahoo.com
*Corresponding author

Md. Ismail Hossain
Department of Mechanical Engineering, University of Malaya (UM), Kuala Lumpur, Malaysia
E-mail: ismailpab@yahoo.com

Abstract

The purpose of this experiment is to model the spirality of single jersey cotton knit fabric by using fuzzy expert system. Spirality is a very common inherent problem in single jersey knit fabric and it has an obvious influence on both the aesthetic and functional performance of knitwear. Knitting stitch and yarn count are two important variables that strongly influence the spirality. So, these two parameters have been used as inputs and spirality has been treated as output for construction the fuzzy expert system. The model demonstrates the relationship and interactions between input and output variables in complex field of knit fabric manufacturing. The model developed in this research has been proved by trial data. The authors found that developed model can be applied to predict the spirality of knit fabric fruitfully.

Keywords: Spirality, Fuzzy expert system, Modeling, Cotton, Knit fabric

1. Introduction

Knit fabrics are very popular for apparel wears such as T-shirts, shirts, sweaters, blouses, underwear, casual wear, active wear and sportswear due to lower price (De Araujo and Smith, 1989) and their exclusive quality characteristics such as elasticity, drape, wrinkle resistance, comfort, softness and easy-care properties over woven fabrics as compared to woven fabrics (Jamshaid et al. 2013 and Degirmenci & Topalbekiroglu, 2010). However, besides all the advantages, knit fabrics, especially single structure cotton knit fabrics have quality problems like dimensional change and deformation because of their unbalanced structures (Araujo and Smith, 1989). The most important problem of the single jersey structure is spirality, which affects all the fabric and creates big problems (Marmarali, 2003) during the clothing stage as it affects the garment by displacing side seams, which causes an important quality problem (Chen et al. 2003 and

Nahala, 2013). Spirality arises from twist stress in the constituent yarns of plain fabric, causing all loops to distort and throwing the fabric wales and courses into an angular relationship other than 90^0. Inclination of loops within knitted structures is commonly known as spirality.

The literature review exposed that various studies have been reported on the factors affecting the spirality of cotton knit fabric including yarn type, yarn count, yarn twist (Kothari et al. 2011), yarn evenness, fabrics GSM, fabric wale and courses, knitting stitch length (Shahid et al. 2011), cover factor, tightness factors and relaxation treatment (Tao et al. 1997 and Higgins et al. 2003). Moreover, all these factors perform non-linearly. Hence, it is very difficult to create an exact model for the input-output variables by the mathematical and statistical models (Hossain et al. 2014 and Majumdar & Ghosh, 2008). Due to this reason, mathematical and statistical models frequently fail to capture the non-linear relationships between the input-output variables (Hatua et al. 2014). In this background, fuzzy logic is the most efficient predicting tool compared to traditional mathematical models, statistical regression models and empirical models as fuzzy logic carry out extremely well in non-linear and complex domains with smallest experimental data (Degirmenci & Topalbekiroglu, 2010 and Hossain et al. 2015). Unlike statistical regression model, fuzzy logic needs no information or afore mentioned appraisal of any mathematical models in advance (Ertugrul & Ucar, 2000). Furthermore, fuzzy logic is used to resolve problems in which descriptions of behavior and observations are imprecise, vague and uncertain (Majumdar & Ghosh, 2008 and Hossain et al. 2015). The main objective of this study is to develop a fuzzy logic model based on Mamdani approach for the prediction of spirality (SP) of single jersey cotton knit fabric as a function of yarn count (YC) and knitting stitch length (SL), which has not so far been mentioned in the published literature. So, it would be very essential tool for appaerel enterpreises for predicting the spirality of knit fabric.

2. Fabric preparation and spirality testing procedure to trial the model

In this investigation, total 16 cotton knit fabrics samples were made according to Table 1, knitting variables on Pailung single jersey circular knitting machine having 30 inches diameter, 20 gauges (needles/inch) and 90 yarn feeders.

Table 1. Knitting variables and their level

Process Parameters	Unit	Level			
		1	2	3	4
Stitch length (SL)	mm	2.7	2.75	2.8	2.85
Yarn count (YC)	Ne	24	26	28	30

The single jersey cotton knit fabric samples were subjected to bleached at 90^0C for 40 minutes in a sample dyeing machine using anti creasing agent (Kappavon CL 1 g/l), sequestering agent (Sirrix 2UD 0.5 g/l), wetting agent (Felosan NOF 1 g/l), soda ash (2.5 g/l), hydrogen peroxide 50% (1 g/l), stabilizing agent (0.2 g/l). Then the fabric samples were washed with proper rinsing and finally treated with acetic acid (1 g/l) and peroxide killing agent (0.2 g/l) for neutralizing and peroxide killing respectively. After bleaching, the fabric samples were dried in an open stenter and compacted properly.

After production, all the fabrics samples were conditioned firstly on a flat surface for at least 24 hours under standard textile testing conditions at relative humidity (65 ± 2) % and temperature (20 ± 2)°C. Then the manufactured fabrics samples were marked with two sets of marks in each direction (Length and width), a minimum of 50 cm apart and at a distance of approximately 3 cm from the edge (Shahid et al. 2010) according to Figure 1. No tension was applied to the samples during measuring spirality. Spirality was calculated according to the following equation:

$$\text{Spirality (\%)} = \frac{\text{Average distortion}}{\text{Average length}} \times 100$$

$$= \frac{\frac{d_1 + d_2}{2}}{\frac{D_1 + D_2}{2}} \times 100 \qquad (1)$$

Where, d_1, d_2 is displacement in bottom at left & right side respectively and D_1, D_2 is the height at left & right side respectively of the fabric.

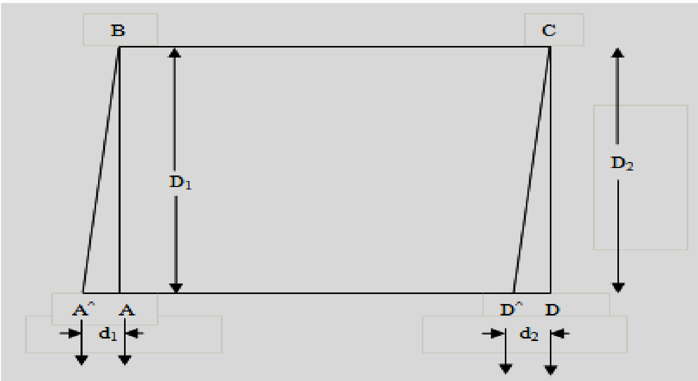

Figure 1. Spirality measurement (Nahla, 2013 and Shahid et al. 2011)

3. Fuzzy logic and its mechanism to predict the spirality

Fuzzy logic expert system is soft computing method based on fuzzy set theory established by Zadeh in 1965 (Hossain et al. 2015 and Zadeh, 1965). A basic fuzzy logic unit consists of a fuzzification interface, a fuzzy rule base, a decision making logic interface and a defuzzification interface as shown in Figure 2.

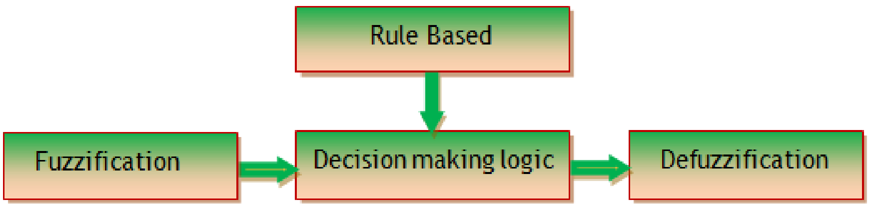

Figure 2. Basic component of a fuzzy logic expert system (Gopal, 2012)

Fuzzification interface: Fuzzification module converts all numeric input and output variables into linguistic fuzzy sets within a range from 0 to 1 such as low, medium, high and so on by membership functions. Among various forms of membership functions, the triangle membership function is the simplest and most often used due to accuracy (Majumdar & Ghosh, 2008 and Hossain et al. 2015) which defined as follows:

$$\mu_A(x) = \begin{cases} \dfrac{x-L}{m-L}; & L \le x \le m \\ \dfrac{R-x}{R-m}; & m \le x \le R \\ 0; & otherwise \end{cases} \quad (2)$$

In this case, the edge of the variable's interval may be represented with linear S- shaped and Z-shaped membership functions described respectively as:

(i) S-shaped membership functions

$$\mu_S(x,a,b) = \begin{cases} 1; & x \le a \\ \dfrac{b-x}{b-a}; & a \le x \le b \\ 0; & x \ge b \end{cases} \quad (3)$$

(ii) Z-shaped membership functions

$$\mu_Z(x,a,b) = \begin{cases} 0; & x \le a \\ \dfrac{x-a}{b-a}; & a \le x \le b \\ 1; & x \ge b \end{cases} \quad (4)$$

In Eq (2-4), x is the input and output variables, a, b, and c are the coefficient of membership functions for the explained input and output variables.

Rule based: The fuzzy rules which are deduced from expert knowledge and experience are commonly expressed by if-then statements that relate the input variables in the antecedent part and output variables in the subsequent part (Hossain et al. 2015 and Huang & Yu, 1999). The fuzzy rule base can be divided into two classes, namely the Mamdani and Sugeno (Haghighat et al. 2014).

Mamdani rules: Both of the antecedent and consequence parts are in fuzzy set form.
Sugeno rules : The antecedent part is in the form of a fuzzy set and the consequence part is made up by a linear equation or constant.

As an expression, when a fuzzy model with two inputs and one output involves, then development of fuzzy inference rules can be presented as follows:

Mamdani rule: If x_1 is A_1 and x_2 is A_2, then y is C_1;
Sugeno rule : If x_1 is A_1 and x_2 is A_2, then $y = b_0 + b_1x_1 + b_2x_2$;
Where x_1, x_2, and y are linguistic variables, A_1, A_2, and C_1 are the consequent fuzzy numbers that represent the linguistic states and b_0, b_1, and b_2 are linear equation parameters

Decision making logic interface: The decision making logic interface executes a fundamental role in the fuzzy modeling because of its aptitude like human decision making and deduce control actions by using an inference mechanism. Among various fuzzy inference mechanisms, usually Mamdani max-min is used because it pledges a linear interpolation of the output between the rules (Hossain et al. 2015 and Huang & Yu, 1999). For instance, in case of two-inputs and single-output fuzzy inference system, it can be shown in Figure 3 as below:

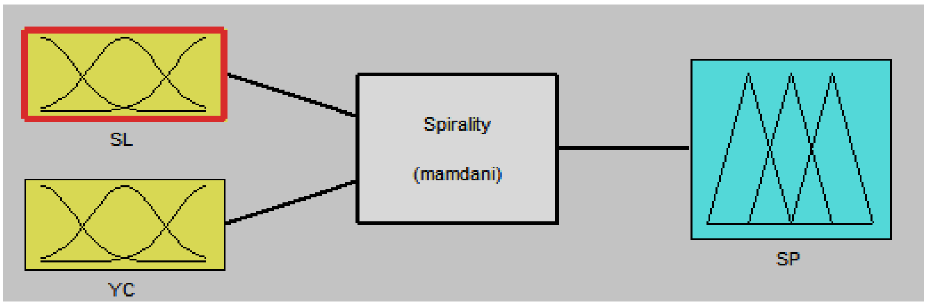

Figure 3. Fuzzy inference mechanisms

Where, SL (stitch length) and YC (yarn count) are inputs side and SP (spirality) is output side (Majumdar & Ghosh, 2008 and Hossain et al. 2015).

Defuzzification interface: Defuzzification is the final unit of fuzzy modelling. Basically, the conclusion of the decision making logic is the fuzzy output. The objective of the defuzzification is to convert the fuzzy output into a precise crisp value (Hatua et al. 2014 and Haghighat et al. 2014). The conversion of fuzzy output into a non-fuzzy value z can be expressed in the following form:

$$z = \frac{\sum_{i=1}^{n} \mu_i * (b_i)}{\sum_{i=1}^{n} \mu_i} \qquad (5)$$

Where b_i is the position of the singleton and μ_i is the membership function of i rule.

Fuzzy logic prediction model development: For construction of fuzzy logic expert model, two knitting variables such as knitting stitch length (SL) and yarn count (YC) have been used as input variables and spirality of knit fabrics as output variable. Stitch length (SL) is the length of yarn in a knitted loop and yarn count (YC) is a numerical expression of yarn that indicate fineness or coarseness of yarn. These knitting variables have been exclusively selected as they influence the fabric spirality significantly. A fuzzy logic toolbox from MATLAB (version 7.10) was used to develop the proposed fuzzy model of spirality. For fuzzification, four convenient linguistic fuzzy sets namely very low (VL), low (L), medium (M) and high (H) were chosen for input variables SL and YC in such a way that they are evenly spaced and cover up the full input places. Nine output fuzzy sets namely very very low (VVL), very low (VL), low (L), low medium (LM), medium (M), high medium (HM), high (H), very high (VH), and very high (VVH) were deemed for fabric spirality (SP), in order that the fuzzy logic system can map little modification in spirality with the changes in input variables. In this investigation, the triangular shaped membership functions are used for input-output variables because of their accuracy. In addition, a Mamdani max-min inference mechanism and the center of gravity defuzzification method (Hossain et al. 2015 and Haghighat et al. 2014) have been applied in this work. Fuzzifications of the used factors are made by aid following functions:

$$SL(i_1) = \begin{cases} i_1; & 2.7 \le i_1 \le 2.85 \\ 0; & otherwise \end{cases} \tag{6}$$

$$YC(i_2) = \begin{cases} i_2; & 24 \le i_2 \le 30 \\ 0; & otherwise \end{cases} \tag{7}$$

$$SP(o_1) = \begin{cases} o_1; & 3.29 \le o_1 \le 7.21 \\ 0; & otherwise \end{cases} \tag{8}$$

Where i_1, and i_2 are the first (SL) and second (YC) input variables respectively and o_1 is the output variable (SP) showing in Eq (6-8).

Prototype triangular fuzzy sets for the fuzzy variables, namely, stitch length (SL) & yarn count (YC) and spirality (SP) have been developed using fuzzy tool box from MATLAB. The membership values obtained from the above formula are shown in the Figures 4-6. Further, fuzzy rules are the heart of fuzzy model. In this investigation, a total of 16 fuzzy rules have been formed based on expert knowledge and previous experience. Some of the rules are presented in Table 2.

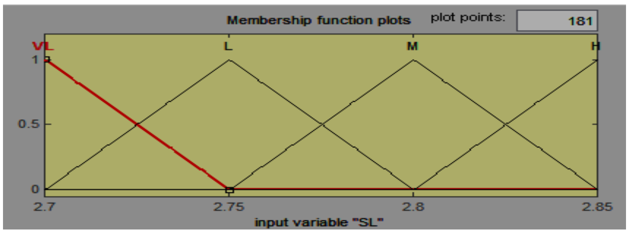

Figure 4. Membership function of SL

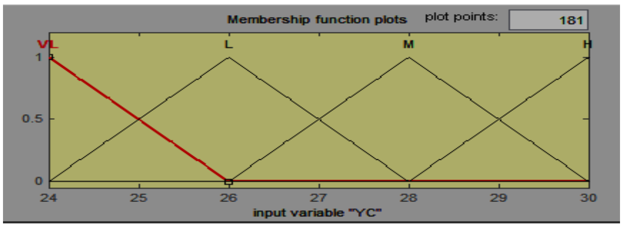

Figure 5. Membership function of YC

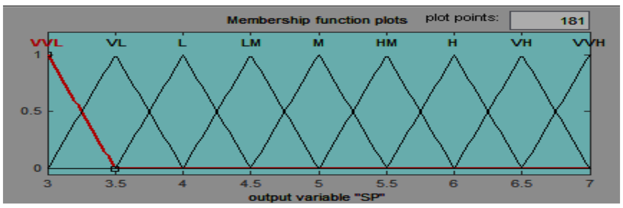

Figure 6. Membership function of SP

Table 2. Fuzzy rules

Rules no.	Input variables SL	YC	Output variable SP
1	VL	VL	VVL
2	L	VL	LM
…..	…..	…..	…..
10	L	M	HM
11	M	M	H
…..	…..	…..	…..
15	M	H	VH
16	H	H	VVH

Prediction accuracy investigation: The prediction accuracy of the developed model has been investigated according to the global prediction error such as Mean Relative Error *(MRE)* and coefficient of determination (R^2). The formulations of those accuracy measures are given below:

$$\mathrm{MRE} = \frac{1}{\mathrm{N}} \sum_{i=1}^{i=N} \left(\frac{\left| E_a - E_p \right|}{E_a} \times 100 \right) \tag{9}$$

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{i=N} (E_a - E_p)^2}{\sum_{I=1}^{I=N} (E_a - E_M)^2} \right) \tag{10}$$

Where, E_a = Actual result, E_p = Predicted result, E_M = Mean value, N = Number of pattern.

The coefficient of determinations (R^2) compares the accuracy of the model to the accuracy of a trivial benchmark model. The Mean Relative Error (*MRE*) gives the deviation between the predicted and experimental values and it is required to reach zero (Hossain et al. 2015).

4. Fuzzy model for prediction the spirality

The diagram of the fuzzy logic prediction model operation has been shown in Figure 7. For simple expression, just one fuzzy rule out of sixteen has been publicized in the picture. As per this rule, if knitting stitch length (SL) is low (L) and yarn count (YC) is medium (M), then output fabric spirality (SP) will be high medium (HM). For example, if input SL is 2.75 mm and YC is 28/1Ne, then fuzzy output SP is 5.5%.

Using MATLAB fuzzy toolbox the fuzzy surface plot was developed as shown in Figure 8. The image explains the relationship between knitting stitch length (SL) and yarn counts (YC) on the input side and spirality (SP) on the output side. The surface plot shown in Figure 8 exposes the impact of stitch length (SL) and yarn count (YC) on spirality (SP).

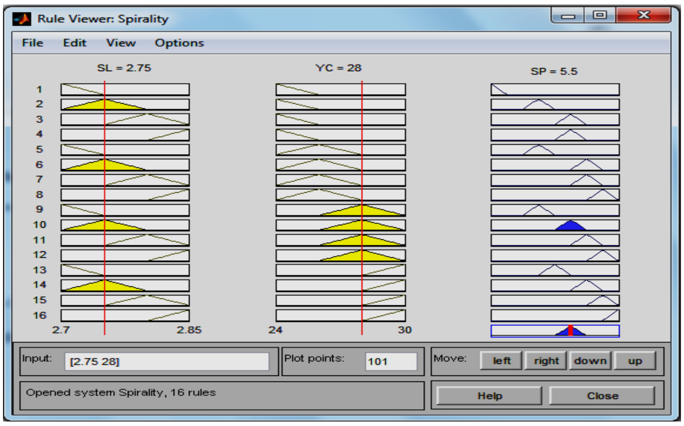

Figure 7. Fuzzy rule viewer

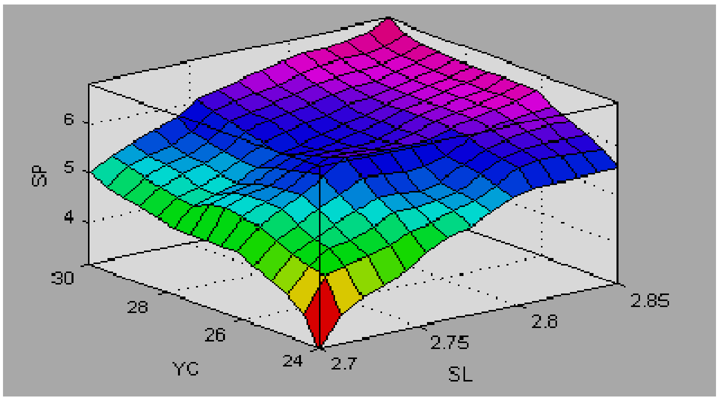

Figure 8. Surface plot of FIS

5. Analysis of experimental results

Effects of stitch length and yarn count on spirality have been depicted in Figure 9. It is clear from Figure 9 that fabric spirality increases radically with the increasing in knitting stitch length from 2.7 mm to 2.85 mm and vice versa. Approximately, the spirality increases 70 % with an increasing of 6% in knitting stitch length. The reason for an increase in spirality with increasing in stitch length is probably due to decreasing number of loops per unit area which indicates the looseness of fabrics, leading to higher fabric spirality. In contrast, the effect of yarn count on the spirality is not more profound as compared to stitch length as shown in Figure 9. Figure shows that fabric spirality increases normally with the increasing in yarn count and vice versa. Approximately, the spirality increases 46% with an increasing of 25% in yarn count. It is fairly coherent that the lower linear density of yarns of higher count would result in lower fabric GSM, which leads to higher fabric spirality. However, it was found from Figure 9 that knitting stitch length and yarn counts have burly relations on fabric spirality.

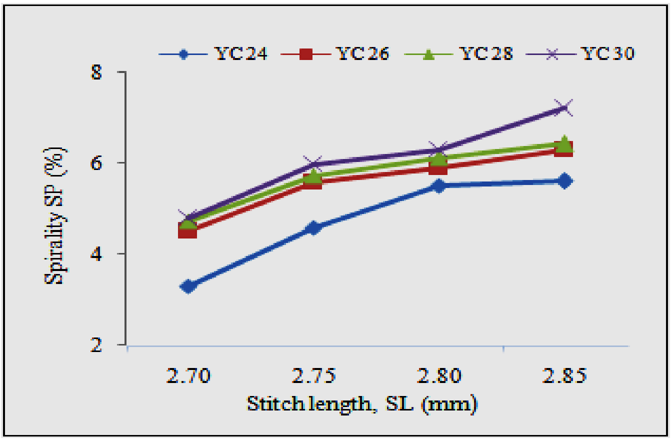

Figure 9. Effects of stitch length and yarn count on spirality

From this investigation, it is distinctly examined that knitting stitch length has the greatest effect on spirality when compared to yarn count. Hence, it is very essential to sustain optimal level of knitting parameter in the knitting process as to attain desired spirality.

Validation of the prediction model: The developed prediction models have been verified by trial data. Prediction was done using the fuzzy logic rule viewer. The data of 9 samples out of 16 knitted samples were used for the confirmation of the built model. The results from the constructed fuzzy model were then compared with the experimental results as shown in Table 3.

Table 3. Comparison of actual and predicted values of spirality

No.	Stitch length (mm)	Yarn count (Ne)	Actual spirality	Fuzzy model Predicted spirality	Relative error%
1	2.7	24	3.29	3.15	4.26
2	2.8	24	5.51	5.5	0.18
3	2.85	24	5.6	5.5	1.79
4	2.7	26	4.52	4.5	0.44
5	2.85	26	6.3	6.5	3.17
6	2.75	28	5.73	5.5	4.01
7	2.85	28	6.43	6.5	1.09
8	2.7	30	4.8	5	4.17
9	2.85	30	7.21	6.85	4.99
Mean relative Error (%)					2.41
Correlation coefficient (R)					0.991

Figure10 shows the correlations between the experimental values of spirality and that predicted by the fuzzy logic expert model. The correlation coefficient (R) and mean relative error between the actual spirality and that predicted by the fuzzy logic expert model were found to be 0.991 and 2.41%, respectively. All the results revealed good prediction ability and accuracy of the built model.

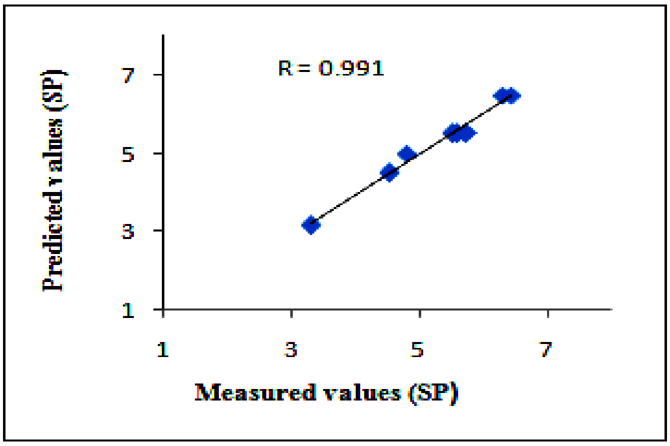

Figure 10. Correlation between actual and predicted value by fuzzy logic model.

6. Conclusions and future work

In this study, fuzzy logic has been developed for modeling the spirality of single jersey cotton knit fabric. The developed model was created by taking the knitting stitch length (SL) and yarn count (YC) as input variables and fabric spirality (SP) as output variable. The developed model present an outstanding perceptive about the interaction between knitting process variables and their effects on the fabric spirality. It has been found that knitting stitch length (SL) has maximum effects on the fabric spirality (SP) than that of yarn count (YC). Lastly, the model derived in this research has been verified from the experimental data. The correlation coefficient (R) and mean relative error (MRE) were found to be 0.991 and 2.41%, respectively between the actual fabric spirality and that predicted spirality by fuzzy logic developed model. From this study, it has been found that the fuzzy logic model has the ability to predict fabric spirality.

Future study will be carried out considering more than two input variables for predicting the spirality. Composition of yarn (fiber type), yarn twist etc. will be considered as input variables.

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Fuzzy Formulas and Fuzzy Dependency

Nedžad Dukić *
Fac. of Science, Dept. of Mathematics, University of Sarajevo, Bosnia and Herzegovina
E-mail: ndukic@pmf.unsa.ba
*Corresponding author

Fatih Destović
Faculty of Educational Sciences, University of Sarajevo, Bosnia and Herzegovina
E-mail: dekanatpf@pf.unsa.ba

Haris Dukić
Faculty of Science, Department of Mathematics, University of Sarajevo, Bosnia and Herzegovina
E-mail: hdukic@pmf.unsa.ba

Abstract

Problem with fuzzy relations databases is that procedures of executing logical consequence from given fuzzy functional dependencies set in context of given axiomatisation is very complex. There is no algorithm, which would choose corresponding axioms in given step that will lead to solution; also there is no global strategy that gives valid results that are independent from fuzzy functional dependencies set in fuzzy relation database model. This problem, in this paper, will be solved with formulas in fuzzy logic. Instead of fuzzy functional dependencies we will use fuzzy formulas.

Keywords: Fuzzy functional dependencies, Fuzzy logic, Fuzzy relational database, Fuzzy formula.

1. Introduction

In this paper we consider a relational database (Codd’s model, [2]) and a new type of functional dependency [10], called fuzzy functional dependency (ffd). In the Codd’s model of relational databases, all the information in it, have to involve precisely defined values (atomic). In real word applications, data are often partially known or ambiguous. The Codd's model of relational databases does not deal with this information. But, in real-life problems, the data associated are often imprecise, or non-deterministic. All real data cannot be precise because of their fuzzy nature. Consequently, for comparing such data, classical logic is not appropriate.

The other way of considering this imprecise information is the involving of fuzzy value to the domain of an attribute [9]. Fuzzy set theory and fuzzy logic provide a mathematical framework to deal with the imprecise information in fuzzy relational databases. In a fuzzy set each element of the set has an associated degree of membership. The degree of membership is a real number between zero and one and measures the extent to which an element is in a fuzzy set [5].

The most important concept in relational databases is that of the functional dependency of one set of attributes upon another. A functional dependency is a property of the semantics or the meaning of the attributes. The searching for functional dependencies is based on such computations for which equalities of data (attribute values) are tested. But the classical equality relation is not appropriate for comparing data of fuzzy nature.

Our primary aim in this paper is to establish a connection between the theory of fuzzy functional dependency and one fragment of fuzzy logic. So it will be shown that if relation r satisfies a fuzzy functional dependence then its truth value of the belonging fuzzy formula is greater than 0.5 and vice verse.

2. Preliminaries

In this section we give some basic preliminaries on fuzzy set theory [9].
Let $U = \{u_1, u_2 ,..., u_n\}$ be a universe of discourse.

Definition 2.1 A fuzzy set A in the universe of discourse U is characterized by the membership function μ_A given by

$$\mu_A: U \rightarrow [0; 1].$$

The membership function for a fuzzy set of U takes values from the closed interval [0,1]. A fuzzy set A is defined as the set of ordered pairs

$$A = \{ (u, \mu_A(u)) : u \in U\},$$

where $\mu_A(u)$ is the grade of membership of element u in the set A. The greater the amount of $\mu_A(u)$, the greater is the truth of the statement that 'the element u belongs to the set A'.

Definition 2.2 If A and B are two fuzzy sets of the universe U, then

$A \subseteq B$ iff $\forall x \in U$; $\mu_A(x) \leq \mu_B(x)$,
$B \supseteq A$ iff $A \subseteq B$,
$A = B$ iff $\forall x \in U$, $\mu_A(x) = \mu_B(x)$,
$Com(A) = \{(x; 1 - \mu_A(x)): x \in U\}$
$A \cap B = \{(x, \min \{\mu_A(x), \mu_B(x),\}) : x \in U\}$
$A \cup B = \{(x, \max \{\mu_A(x), \mu_B(x),\}) : x \in U\}$.

The basic mathematics used in Codd's relational model is relations and their properties.

Definition 2.3 A fuzzy relation $R(X \times X)$ is said to be

1) reflexive : iff $\forall x \in X$, $\mu_R(x,x) = 1$
2) symmetric : iff $\forall x_1,x_2 \in X$, $\mu_R(x_1,x_2) = \mu_R(x_2,x_1)$

A fuzzy relation is said to be a *fuzzy tolerance* relation if it is reflexive and symmetric.

3. Fuzzy logic

Fuzzy logic is based on the concepts of fuzzy sets and symbolic logic. Logic operators of conjunction, disjunction and negation are defined as follows,

a) $x_1 \wedge x_2 = min(x_1, x_2)$
b) $x_1 \vee x_2 = max(x_1, x_2)$
c) $\neg x = 1 - x$

where $x_i(i = 1, 2,..., n)$ variable in [0, 1]; see [8].
In fuzzy logic, the truth value of a formula can assume any value in the interval [0, 1] and is used to indicate the degree of truth represented by the formula.

Definition 3.1 (Satisfiability in fuzzy logic) A formula f $\in$ S, where S is a set of a fuzzy formulas is said to satisfy in interpretation I, if truth value of a formula T(f) $\geq$ 0,5 under I. An interpretation I is said to be false if T(f) $<$ 0,5.
A formula is said to be unsatisfiable if it is false for every interpretation of it.

Definition 3.2. Let $D_1 : L_1 \vee D_1'$ and $D_2 : L_2 \vee D_2'$ be two disjuncts , and L_1 and L_2 , contra pair of literals i.e. $L_2 : \neg L_1$ and let D_1' and D_2' do not contain any such pair. Then , disjunct $D_1' \vee D_2'$ is said to be resolvent disjuncts D_1 and D_2 with the key word L_1.

4. Fuzzy functional dependency (FFD) and f-Armstrong's axioms

The most important concept in relational databases is that of the functional dependency of one set of attributes upon another. A functional dependency is a property of the semantics or the meaning of the attributes. The searching for functional dependencies is based on such computations for which equalities of data (attribute values) are tested. But the classical equality relation is not appropriate for comparing data of fuzzy nature. In practical and in many real applications, there is a genuine need of thinking about integrity constraints with fuzzy theoretical concepts.

Functional dependency $X \rightarrow Y$, in classical database states that if $t_1 [X] = t_2 [X]$ then must be $t_1 [Y] = t_2 [Y]$. There is no clear way of checking whether two imprecise values are equal. Therefore the definiton of functional dependency have to be extended namely to be generalized and this generalization version of functional dependency is said to be the fuzzy functional dependency (FFD).

There are several way in corrected definition of fuzzy functional dependency [1,4,6,7]. One of the important definition for fuzzy functional dependences was presented in paper [10]. In that paper firstly was defined α-equality of two tuples in relation.

Let *X* be a set, the universe and *R* be a fuzzy tolerance relation on *X*. Consider a choice parameter $\alpha \in [0,1]$. see.[10]

Definition 4.1 $(\alpha)_R$ -nearer or α-nearer elements. Two elements $x_1,x_2 \in R$ are said to be $(\alpha)_R$ -nearer (or α-nearer, in short) if

$$\mu_R (x_1,x_2) \geq \alpha$$

We denote this by the notation $x_1 N_{(\alpha)R} x_2$.

The following results are straightforward.

Proposition 4.1
1) *∀x □ X, and ∀□ □* [0, 1], *x is* (□)$_R$ *-nearer to itself in X, i.e., x* $N_{(□)R}$ *x.*
2) *If* x_1 *is* (□)$_R$ *-nearer to* x_2 *in X, then* x_2 *is also* (□)$_R$*-nearer to* x_1 *in X, i.e.,*

$x_1\ N_{(□)R}\ x_2$ □ $x_2\ N_{(□)R}\ x_1$

3) *Suppose that* 0 □ □$_2$ □ □$_1$ □ 1. *Then* $x_1\ N_{(□1)R}\ x_2$ □ $x_1\ N_{(□2)R}\ x_2$.

Definition 4.2: (□)$_R$ -equality or □-equality. Two elements x_1,x_2 □ X are said to be (□)$_R$ -equal (or □-equal) if

1) either $x_1\ N_{(□)R}\ x_2$ or
2) □ y_1,y_2 ,..., y_r □ X such that {$x_1\ N_{(□)R}\ y_1$, .., $y_{r-1}\ N_{(□)R}\ y_r$} and $y_r\ N_{(□)R}\ x_2$.

This equality is denoted by the notation $x_1\ E_{(□)R}\ x_2$.

The following results follow from the results in Proposition 4.1

Proposition 4.2

1) ∀x □ X, and ∀□ □ [0, 1], xE(□)$_R$x.
2) x_1E(□)$_R$$x_2$ □ x_2E(□)$_R$$x_1$
3) Suppose that 0 □ □$_2$ □ □$_1$ □ 1. Then $x_1\ E_{(□1)R}\ x_2$ □ $x_1\ E_{(□2)R}\ x_2$.

Definition 4.3 □(□)$_R$ relation on X. The crisp relation □(□)$_R$ on X is defined as :

For x_1,x_2 □ X , x_1□ (□)$_R$$x_2$, if $x_1E_{(□)R}x_2$.

Proposition 4.3 The relation □(□)$_R$ R defined on X is an equivalence relation.

Definition 4.4 Two tuples t_1 [X] and t_2 [X] are said to be (□)$_R$-equal if t_1 [x_i] $E_{(□)Ri}$ t_2 [x_i] ∀i = 1,2,..., k. We denote the equality by the notation t_1 [X] $□_{(□)R}$ t_2 [X].

Definition 4.5. The conformance of attribute set X for any two tuples t_i and t_j present in relation instance R is given as

t_i [X] $□_{(□)R}$ t_j [X] = min {t_i [A_k] $□_{(□)R}$ t_j [A_k]}, A_k □ X

Definition 4.6. Fuzzy Functional Dependency. Let X, Y □ **R** = {$A_1,A_2,...,A_n$}. Choose a parameter □ □ [0, 1] and propose a fuzzy tolerance relation R. A fuzzy functional dependency (FFD) X $□_{(□)R}$ Y is said to exist if, whenever t_1 [X] $□_{(□)R}$ t_2 [X], it is also the case that t_1 [Y] $□_{(□)R}$ t_2 [Y].

For any value of the choice parameter □ in [0,1], the following propositions are true.

Proposition 4.4 (f-Armstrong's Axioms)
(A1) If Y □ X, then X $□_{(□)R}$ Y . (f-Reflexive rule)
(A2) If X $□_{(□)R}$ Y , then XZ $□_{(□)R}$ Y Z. (f-Augmentation rule)
(A3) If X $□_{(□)R}$ Y and Y $□_{(□)R}$ Z then X $□_{(□)R}$ Z. (f-Transitive rule)
(A4) If X $□_{(□)R}$ Y Z, then X $□_{(□)R}$ Y . (f-Decomposition rule)
(A5) If X $□_{(□)R}$ Y and X $□_{(□)R}$ Z, then X $□_{(□)R}$ Y Z. (f-Union rule)
(A6) If X $□_{(□)R}$ Y and WY $□_{(□)R}$ Z, then WX $□_{(□)R}$ Z.
(A7) If X $□_{(□)R}$ Y , then XZ $□_{(□)R}$ Y .see [10]

5. Fuzzy functional dependency and fuzzy formulas

In this section we establish a connection between fuzzy logic and the theory of fuzzy functional dependencies.We give a way to interpret fuzzy functional dependencies as formulas in fuzzy logic. For a set of fuzzy dependencies **F** and single fuzzy functional dependency **f**, we show that **F** implies **f** as fuzzy functional dependencies if and only if **F** implies **f** under the logic interpretation.

The correspondence between fuzzy functional dependencies and fuzzy formulas is direct. Let X □ $_{(□)R}$ Y be a fuzzy functional dependency where X = {A_1, A_2,...,A_m } and Y = {B_1, B_2,...,B_n. } The corresponding logical formula is

(A_1 □ A_2 □,..., □ A_m) □ (B_1 □ B_2 □ ,...,□ B_n).

Let *r* be a fuzzy relation over schema *R* with exactly two tuples. A fuzzy relation *r* can be used to define a truth assignment, for attributes in *R* when they are considered as fuzzy variables.

Definition 5.1. Let R = {A1,A2,...,Am } be a relation schema and let r = {t_1, t_2} be a two tuple relation on R. The truth assignment for r, denoted ir, is the function from R to [0, 1] defined by

$i_r(A_k)$ { > **[0.5, 1] if** ti [**A**k] □(□)$_R$ tj [**A**k], is true □ □ **[0, 1],**
□ **[0.5, 1] if** ti [**A**k] □(□)$_R$ tj [**A**k] is not true.

The following theorem enables equivalence between fuzzy functional dependence and fuzzy formulas. Therefore by this theorem the mentioned equivalence will be proved taking for the fuzzy formulas [3] the following:

X □ Y = max(1 – X, Y)

Theorem 5.1. Let X □ Y be a FFD over relation scheme R and let r be a relation on R with two tuples. An FFD X□ Y is satisfed by relation r if and only if X □ Y is satisfed under the truth assignments i_r.

Proof. Let us assume, as frst, that relation *r* satisfes FFD *X* □ *Y* i.e. suppose whenever

t_1 [*X*] □$_{(□)R}$ t_2 [*X*] is true, t_1 [*Y*] □$_{(□)R}$ t_2 [*Y*]is also true,

where is X = {A_1,A_2,...,A_m} and Y = {B_1,B_2,...,B_n}.

Let us assume to the contrary that the assertion that assignments

F: (A_1□ A_2 □,...,□ A_m) □ (B_1□ B_2 □,...,□ B_n)

is false under interpretation i_r *':*

Then it follows that in interpretation i_r *'* that i_r *'* (*F*) □ 0.5, and respectively

i_r *'* (*F*) = i_r *'* ((A_1□ A_2 □,...,□ A_m) □ (B_1□ B_2 □,...,□ B_n))=

= max(min(1- i_r ' (A_1), 1 - i_r ' (A_2),..., 1 - i_r ' (A_m)), min(i_r ' (B_1), i_r ' (B_2),..., i_r ' (B_n)))□ 0.5.

Then we have

i_r *'* (*F*) = □i_r *'* (A_i) > 0.5, □ i = 1,2,...,m,

i_r *'* (B_j) □ 0.5, □ j = 1,2,...,n

If i_r *'* (A_i) > 0.5, □ i = 1,2,...,m is valid then according to Defnition 5.1 t_1 [A_k] □$_{(□)R}$ t_2 [A_k] is true.

Based on the defnition 4.5 we have

$t_i\ [X]\ \square_{(\square)R}\ \ t_j\ [X] = min\ \{t_i\ [A_k]\ \square_{(\square)R}\ \ t_j\ [A_k]\}$, $A_k\ \square\ X$.

Now, we have $t_i\ [X]\ \square_{(\square)R}\ \ t_j\ [X]$ is true .

Because of the assumption that FFD is satisfed, we have

$t_1\ [Y]\ \square_{(\square)R}\ \ t_2\ [Y] = min\ \{t_1\ [B_k]\ \square_{(\square)R}\ \ t_2\ [B_k]\}$

This implies that $t_1\ [B_j]\ \square_{(\square)R}\ \ t_2\ [B_j]$ is true for $\square$ j = 1,2,...,n. So it follows that i_r ' (Bj) > 0.5, which is contrary to i_r ' (Bj) $\square$ 0.5.

Now we prove the converse of theorem. Assume that F satisfes in interpretation i_r ' .

Then i_r '(F) = max(min(1- i_r '(A_1), 1 - i_r '(A_2),..., 1- i_r '(A_m)), min(i_r '(B_1); i_r '(B_2),..., i_r '(B_n)) > 0.5, which results in

i) i_r ' ($A_1\ \square\ A_2\ \square$,..., $\square\ A_m$) $\square$ 0.5 or

ii) i_r ' ($B_1\ \square\ B_2\ \square$,...,$\square\ B_n$) > 0.5.

Suppose i) is valid, then i_r '($A_1\ \square\ A_2\ \square$,..., $\square\ A_m$) = min(i_r '(A_1), i_r '(A_2),..., i_r '(A_m)); hence i_r ' (Aj) $\square$ 0.5 for some j in {1, 2,...,*m*}, from which it follows that $t_i\ [A_j]\ \square_{(\square)R}\ \ t_j\ [A_j]$ is not true for some j in {1, 2,...,*m*}. Then t_i [X] $\square_{(\square)R}$ t_j [X] is not true.

From this it follows that the relation satisfes FFD $X\ \square_{\ (\square)R}\ \ Y$.

Suppose ii) is valid i.e. i_r '($B_1\ \square\ B_2\ \square$,...,$\square\ B_n$) > 0.5 for each *i* =1,2,..., *n*. Then min(i_r '(B_1), i_r ' (B_2),..., i_r '(B_n)) > 0.5, and i_r ' (Bi) > 0.5 for each *i* = 1,2,..., *n* from which it follows that $t_1\ [B_i]\ \square_{(\square)R}\ \ t_2\ [B_i]$ is true. Hence it follows that *r* satisfes FFD $X\ \square_{\ (\square)R}\ \ Y$.

In the following theorem we are going to show that if the relation *r* satisfes a set of fuzzy functional dependencies F and does not satisfy the dependency $X\ \square_{\ (\square)R}\ \ Y$ then there exists two tuples sub-relations of the relation *r,* which satisfes all the fuzzy functional dependencies from set F and does not satisfy the dependency $X\ \square_{\ (\square)R}\ \ Y$.

Theorem 5.2. Let X $\square_{\ (\square)R}$ Y be an FFD over the scheme R; and {A_1, A_2,...,A_m} = X $\square$ R; and {B_1, B_2,...,B_n} $\square$ R; and let F be a set of FFDs over R. Then the following is true:

1) F $\square$ X $\square_{\ (\square)R}$ Y if and only if

2) F $\square$ X $\square_{\ (\square)R}$ Y in the world of two tuple relations.

Proof. Obviously 1) implies 2). Now we show that 2) implies 1).

Let us assume that 2) does not imply 1). In that case there is some relation *r* satisfed by all the fuzzy functional dependencies from F, that does not satisfy the dependency $X\ \square_{\ (\square)R}\ \ Y$. This means that there exists elements t_1 and t_2 in *r*, therefore whenever $t_i\ [X]\ \square_{(\square)R}\ \ t_j\ [X]$, *it is true,* $t_i\ [Y\ ]\ \square_{(\square)R}\ \ t_j\ [Y\ ]$ is not true.

Let be $r^* = \{t_1, t_2\}$. It is obvious that r^* satisfes all the FFDs from F, but does not satisfy the dependency $X\ \square_{\ (\square)R}\ \ Y$. This is shown by following.

Lemma 5.1. Let r be a relation, let F be a set of FFDs on R, and let X $\square_{\ (\square)R}$ Y be a single FFD on R. If relation r satisfes all the FFDs from the set F and violates fuzzy dependency X $\square_{\ (\square)R}$ Y , then some two tuple sub-relation r* of r satisfes F and violates X $\square_{\ (\square)R}$ Y .

This is a contradiction of our assumption.

Theorem 5.3. Let X $\square_{\ (\square)R}$ Y be an FFD over relation scheme R and let F be a set of FFDs over R. Then F implies X $\square_{\ (\square)R}$ Y in the world of two tuple relations, if and only if F implies X $\square$ Y when FFDs are interpreted as fuzzy formulas.

Proof. Assume that *ir* : R □ [0,1] is an interpretation where every formula is satisfed, which is generated by FFDs from set F, and let a formula which is generated by the dependency *X* □ *Y* be false. Let us consider *Z* = *{A* □R : *ir*(*A*) > 0.5*}*.
Let r_z be a fuzzy relation instance with two tuples t_1 and t_2 as shown in Table. 1. We choose the set *{*a, b*}* as the domain of each attributes in R, where *a* = a_1,..., a_p, *b* = b_1,..., b_q for (*p* □ 1, *q* □ 1). Let t_i [*A*] □$_{(□)R}$ t_j [*A*] is true for any attribute set *A* in r_z,

Table 1. The fuzzy relation instance r_z

	Attributes of *Z*	other attributes
t_1	a,...,a	a,...,a
t_2	a,...,a	b,...b

Namely r_z = *{*t_1, t_2*}* where t_1 = *a,...,a* for each attribute A from R, and let t_2 be defined as
t_2 = □a,...,a, R □Z
b,...,b, R □ Z

We will prove that relation r_z defined in such way satisfies each fuzzy functional dependency from F. To be able to prove this, let *U* □ $_{(□)R}$ *V* be any fuzzy functional dependency from F for which t_i [*U*] □$_{(□)R}$ t_j [*u*] *is true.*
Due to the definition t_1 and t_2 = *a*,..., *a* for each attribute A from *U*, we have Let t_i [*A*] □ $_{(□)R}$ t_j [*A*] is true. This means that i_r(*A*) > 0.5 for each *A* from *U*. There fore *U* □ *Z*, i.e.

i_r ' (*U*) > 0.5. (*)

If t_i [*A*] □$_{(□)R}$ t_j [*A*] does not hold, then t_1 = *a*,..., *a* and t_2 = *b*,..., *b* for some attribute *A* from *V*. From this we have that *A* does not belong set *Z*, and therefore i_r(*A*) < 0.5, and also i_r(*V*) < 0.5.
Based on this and (*) we have by the Kleens-Diens implication

i_r(*U* □ *V*) = max(i_r(1- *U*), i_r(*V*)) □ 0.5

which is a contradiction of our first assumption.
Now we prove that r_z does not satisfy the fuzzy functional dependency *X* □ $_{(□)R}$ *Y*.
By assumption the fuzzy formula is false in the interpretation i_r and hence i_r(*X*) > 0.5 and

i_r(*Y*) □ 0.5 *(**)*

Let assume that

t_i [*X*] □$_{(□)R}$ t_j [*X*] is true.

If t_i [*Y*] □$_{(□)R}$ t_j [*Y*] is true, then Y □ *Z*, namely i_r(*Bj*) > 0.5 for each *j* =1, 2,..., *n; Bj* □*Y*.
This implies that i_r(*Y*) > 0.5, which is a contradiction of (**).
Now we prove the converse. Assume the converse is false, i.e. that it does not hold that from the set of FFDs F follows FFD *X* □ $_{(□)R}$ *Y* . Then here exist two tuples relation *r* = *{t*, *t**}* which satisfy each FFDs from *F*, but does not satisfy FFD *X* □ $_{(□)R}$ *Y*.
By the above mentioned description the interpretation i_r is defined by the relation *r*, formulas U_1 □ U_2 □ ,...,□ U_p □ V_1 □ V_2 □ ,..., □ V_q, for *U* □ *V* from *F* and formula

X_1 □ X_2 □ ,...,□ X_m □ Y_1 □ Y_2 □ ,..., □ Y_n .

Let us now prove that the following holds

i) i_r(U_1 □ U_2 □ ,...,□ U_p *)* □ *(*V_1 □ V_2 □ ,..., □ V_q)) > 0.5 and
ii) *ir*((X_1 □ X_2 □ ,...,□ X_m) □ (Y_1 □ Y_2 □ ,..., □ Y_n)) □ 0.5.

If i) is false then i_r(*Ui*) > 0.5 and i_r(*Vj*) □ 0.5, namely

$t[P]$ □$_{(□)R}$ $t_*[P]$ is true.

for each *P* from *U* and

$t[Q]$ □$_{(□)R}$ $t_*[Q]$ is not true,

for some *Q* from *V*.

This first inequality implies that $t[U]$ □$_{(□)R}$ $t_*[U]$ is true, and the second implies that $t[V]$ □$_{(□)R}$ $t_*[V]$ is not true. Therefore these taken together is a contradiction with starting assumption that *r* satisfies each fuzzy functional dependencies from *F*. Therefore i) is true.

Suppose ii) is false, then

iii) $i_r(X_i)$ □ 0.5 or

iv) $i_r(Y_j) > 0.5$.

If iii) holds, then $t[A_i]$ □$_{(□)R}$ $t_*[A_i]$ is not true, for some $j = 1,2,..., m$, A_i □X, and from these $t[X]$ □$_{(□)R}$ $t_*[X]$ is not true. It is obvious that *r* satisfies the fuzzy functional dependency X □ $_{(□)R}$ Y, which is a contradiction of the opening assumption.

If iv) holds then $t[B]$ □$_{(□)R}$ $t_*[B]$ is true for each $j = 1,2,..., n$; B_j □Y and from this $t[Y]$ □$_{(□)R}$ $t_*[Y]$ is true. Therefore we conclude that *r* satisfies the fuzzy functional dependency X□ $_{(□)R}$ Y , which is also a contradiction of our opening assumption.

The right proved theorems enable the application of resolution rules in fuzzy logic as the rule of inference on calculation of fuzzy functional dependencies.

Example 5.1. Let R = *{Name,Intelligence,Capability,Job,Success}* be a relation scheme, and let Γ = $\{A_1A_2$ □ $_{(□1)R}$ A_3, A_2 □ $_{(□2)R}$ A_4, A_3,A_4□ $_{(□3)R}$ $A_5\}$
be set a FFDs over scheme R, where is noted by *A_1 -Name, A_2 - Intelligence, A_3 - Capability, A_4 - Job, A_5 - Success*.

Prove that holds

Γ □ A_1A_2 □ $_{(□)R}$ A_5

where is □=*min(min*(□$_1$,□$_2$),□$_3$)

Lets prove in two ways that this examples holds , using following

a) Calculus of fuzzy functional dependences

b) The resolution principle in fuzzy logic.

1) A_2 □ $_{(□2)R}$ A_4 (hypothesis)
2) A_1A_2 □ $_{(□2)R}$ A_1A_4 (**A2** , 1)
3) $A_4 \subseteq A_1A_4$
4) A_1A_4 □ $_{(□2)R}$ A_4 (**A1** , 3))
5) A_1A_2 □ $_{(□2)R}$ A_4 (**A3**, 2), 4))
6) A_1A_2 □ $_{(□1)R}$ A_3 (hypothesis)
7) A_1A_2 □ $_{min(□1□2)R}$ A_3A_4 (A5, 5),6))
8) A_3A_4 □ $_{(□3)R}$ A_5 (hypothesis)
9) A_1A_2 □ $_{(□)R}$ A_5 (A3, 7),8))

where is □=*min(min*(□$_1$,□$_2$),□$_3$)

c) According to the previous theorems it is enough to prove that hold □□ A_1 □ A_2 □ A_5
Let' s assert , as first , to FFDs the corresponding formulas :

$A_1A_2 \rightarrow_{(\alpha_1)R} A_3$	F1: $(A_1 \wedge A_2) \rightarrow A_3$
$A_2 \rightarrow_{(\alpha_2)R} A_4$	F2: $A_2 \rightarrow A_4$
$A_3A_4 \rightarrow_{(\alpha_3)R} A_5$	F3: $(A_3 \wedge A_4) \rightarrow A_5$

According to the definition logical consequence and already said mentioned, it is enought to show that

F: $F_1 \wedge F_2 \wedge F_3 \wedge \neg G$

unsatisfiable, where is G: $(A_1 \wedge A_2) \rightarrow A_5$.

To be able to apply a rule of resolution, it is needed, at first transform F in conjuctive normal form so to get a set F*, as a represent of F.

$F^* = \{ \neg A_1 \vee \neg A_2 \vee A_3, \neg A_2 \vee A_4, \neg A_3 \vee \neg A_4 \vee A_5, A_1, A_2, \neg A_5 \}$

The following set of disjunct show resolvent inference**.**

1) $\neg A_1 \vee \neg A_2 \vee A_3$ (element from F*)
2) $\neg A_3 \vee \neg A_4 \vee A_5$ (element from F*)
3) $\neg A_1 \vee \neg A_2 \vee \neg A_4 \vee A_5$ (Resolvent 1) and 2))
4) $\neg A_5$ (element from F*)
5) $\neg A_1 \vee \neg A_2 \vee \neg A_4$ (Resolvent 3) and 4))
6) $\neg A_2 \vee A_4$ (element from F*)
7) $\neg A_1 \vee \neg A_2$ (Resolvent 5) and 6))
8) A_2 (element from F*)
9) $\neg A_1$ (Resolvent 7) and 8))
10) A_1 (element from F*)
11) $\min(A_1, \neg A_1) \leq 0.5$ $\min(A_1, \neg A_1) \leq 0.5$ (Resolvent 9) and 10)

6. Conclusion

In this paper we proved the equivalence between the theory of the fuzzy functional dependencies (at α - level of choice) and the one fragment theory of fuzzy logic. We introduced the defnition of truth assignment of attributes in the relation *r* over the relation scheme *R.* Based on this, definition of fuzzy functional dependencies (at α - level of choice) was attached to the fuzzy formula and we proved that if the relation *r* satisfies the fuzzy functional dependencies then the fuzzy formula is satisfied in the given interpretation and vice verse. The equivalence between the set of the fuzzy functional dependencies and the fuzzy formulas was proved as well.

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