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# An Application of Type 2 Fuzzy Time Series Model

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**Abstract**

Classical and fuzzy methods which are used to model the time series are encountered in many areas of life. The tendency to model with fuzzy logic increases when the classical analyses are inadequate, unsatisfying and when the assumptions that classical model needs are not met. In the fuzzy time series analysis, which allows working ith uncertainties, the Type 2 models which include more information in calculations can be used in the literature. Furthermore, the applicability of a model to different data set is important. Therefore, In this study, Type 2 fuzzy time series model developed by Huarng and Yu (2005) was handled, and it was fitted in the gold price data in Turkey. The forecast results were obtained and the weaknesses of Type 2 fuzzy time series model were also examined.

**Keywords:** Fuzzy time series, intersection, gold prices, forecasting, Type 1, Type 2, union

## 1. Introduction

The goal of the time series modeling is to study the historical data and make a forecast about the future values of the data. Therefore, it has applications in many fields such as business economics, finance, science and engineering. In addition to conventional time series models, models developed with fuzzy logic are also useful alternatives. Indeed, the interest on the fuzzy time series models is increasing recently and it has been considered as an alternative to the conventional time series models. One of the reasons of this attention is fuzzy time series models can work with linguistic variables and does not require the assumptions needed by classical methods. It is also possible to obtain satisfactory forecasting results for time series with a small sample size. One other advantage of the fuzzy time series is that it does not require complex computations as conventional methods do [1, 2]. In addition, Type 2 fuzzy time series models are also highly preferred to better analyze by including more information in calculations.

The fuzzy set theory was first introduced by Zadeh [3]. Then, the description of the fuzzy time series and its definitions were firstly introduced by Song and Chissom [4,5].

In Song and Chissom [4, 5] a first order fuzzy time series forecasting model was put forward. In a first order fuzzy time series model, an observation is only affected by previous observation. Such a model is constructed based on only this relation between sequential observations. Chen [6] suggested simpler fuzzy time series model by dealing with the computational complexity problem of the initial model proposed by Song and Chissom [4,5]. Chen [6] proposed an approach based on fuzzy logic group relation tables to reduce the computational complexity. Although calculations in first order fuzzy time series approach proposed by Chen [6] is easy, there are some drawbacks in this approach. Later on, various fuzzy time series forecasting models have been proposed in the literature in order to obtain better results by getting over these drawbacks of the method [7].

Huarng [8] extended Chen's [6] model by adding intuitional information. Huarng [9] proposed two novel approaches which are based on the average and the distribution. Yu [10] suggested that weighting repeated fuzzy relations instead of handling them only once would produce better forecasting results. In another study, Yu [11] adjusted the lengths of intervals determined during the early stages of forecasting, when the fuzzy relationships are formulated. Cheng et al. [12] employed an adaptive expectation model in fuzzy time series.

Zadeh introduced Type 2 fuzzy sets by extending the Type 1 fuzzy sets by using extending principle [2]. In the Type 1 fuzzy sets, the membership degree is considered a crisp value, while in the Type 2 fuzzy sets as a fuzzy set. Huarng and Yu [1] discussed that many classic and Type 1 fuzzy time series models use a single variable, and some of the observations about this variable are used for forecasting. An example of this situation is the use of one of these to forecast, even though the stock index variable has many values, such as in average, closing, high, low. They argued that if unused variables were included in the model, forecast values could be improved [1]. For this reason, the Type 2 fuzzy time series model is successful in forecasting.
In addition, the applicability of a model and its ability to work in different data is important. In this study, the Type 2 fuzzy time series model developed by Huarng and Yu [1] fitted Turkey's gold price daily data from 01/07/2015 to 01/07/2016 and the steps of the method are described in detail. The RMSE values are calculated to evaluate the forecasting performance.

The remainder of this study is organized as follows: Section 2 gives basic definitions of fuzzy time series. Definitions about Type 2 fuzzy sets are mentioned in Section 3. Section 4 presents Huarng and Yu's model algorithm. Section 5 gives applications of the mentioned approaches to two real-world time series. Finally, the last section evaluating the results concludes the paper.

## 2. Basic definition of fuzzy time series

The basic definitions of fuzzy time series can be summarized as follows [3, 4]:

***Definition 1.*** *Let* $Y_t$ $(t = ...,0,1,2,...)$, *a subset of* $R^1$, *be the universe of discourse on which fuzzy sets* $f_i(t)$ $(i = 1,2,...)$ *are defined and* $F(t)$ *is the collection of* $f_i(t)$. *Then* $F(t)$ *is called a fuzzy time series on* $Y_t$ $(t = ...,0,1,2,...)$.

***Definition 2.*** *Suppose* $F(t)$ *is caused by* $F(t-1)$ *only, i.e.,* $F(t-1) \to F(t)$. *Then this relation can be expressed as* $F(t) = F(t-1) \circ R(t,t-1)$ *where* $R(t,t-1)$ *is a fuzzy relationship between* $F(t-1)$ *and* $F(t)$, *and "*$\circ$*" represents an operator.* $F(t) = F(t-1) \circ R(t,t-1)$ *is called the first-order model of* $F(t)$.

***Definition.3.*** *Let* $F(t-1) = A_i$ *and* $F(t) = A_j$. *The relationship between* $F(t)$ *and* $F(t-1)$ *is defined as* $A_i \to A_j$ *where* $A_i$ *is called the LHS (left hand side) and* $A_j$ *the RHS (right hand side) of the FLR (fuzzy logic relationships).*

***Definition 4.*** *For each* $F(t)$, *if only the* $f_i(t)$ *with the maximum value is used for forecasting,* $F(t)$ *is called an interval fuzzy time series.*

***Definition 5.*** *Suppose there are the following FLR*$_s$

$A_i \to A_{j1}$

$A_i \to A_{j2}$

...

$A_i \to A_{jl}$

*Following Chen's model, these FLR*$_s$ *can ben grouped into an FLRG as*

$A_i \to A_{j1}, A_{j2}, ..., A_{jl}$

***Definition 2.3.*** *Suppose* $F(t) = A_i$ *is caused by* $F(t-1) = A_j$, *then the fuzzy logical relationship is defined as* $A_i \to A_j$.

### 3. Type 2 fuzzy sets

A simple description of Type 2 fuzzy sets is given as follows [1]:
*Suppose there is a fuzzy sets for "approximate 2", as in Figure 1. We have a crisp degree of membership value of 1 for* $x = 2$ *and a crisp degree of membership value of 0.5 for* $x = 1$. *Based on these Type 2 fuzzy sets, there can be a fuzzy sets for any degree of membership. Huang and Yu used Chen's first-degree fuzzy time series model [5] for the Type 1 model. For example, there is a triangular fuzzy set* $(0.4, 0.5, 0.6)$ *as a degree of membership for* $x = 1$ *in Figure 2. In other words, for the same* $x$ *there can be multiple degrees of membersip.*

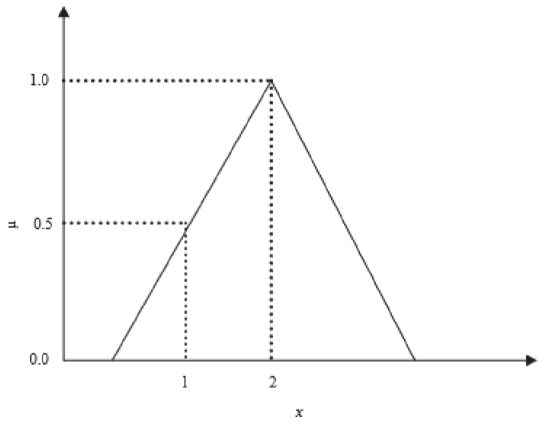


**Figure 1.** A Type 1 fuzzy set

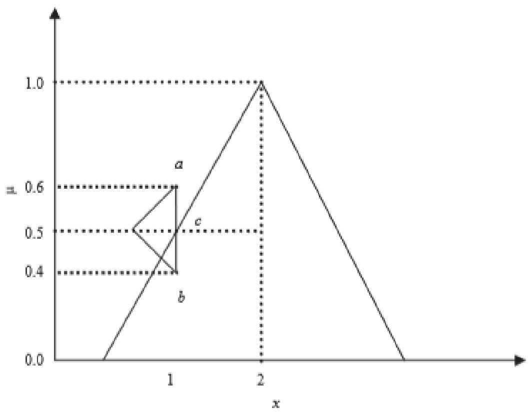


**Figure 2.** A Type 2 fuzzy set

To apply this concept, Huarng and Yu [1] adopt more observation for forecasting in each

time slot. In addition to the opening values of TAIEX (Taiwan Stock Exchange Capitalization Weighted Stock Index) data, they used high and low values.

### 4. Huarng and Yu's type 2 fuzzy time series model

Huarng and Yu [1] have suggested a method that provides a convenient calculation for Type 2 fuzzy time series analysis. This method first calculates the parameters by using the Type 2 fuzzy time series model and then forecasting by using the Type 2 model by including Type 2 observations in the calculations. Some of the details of the method are given below regarding the study of Huarng and Yu. Huarng and Yu's study [1] can be examined.

***Definition 6.*** *The Type 2 fuzzy time series model is an extended version of the Type 1 model. This model uses fuzzy relationships created by the Type 1 model. Operators are used to incorporate fuzzy relationships obtained from Type 1 and Type 2 observations into calculations. Type 2 forecasts are calculated from these fuzzy relationships.*

***Definition 7.*** *Union ($\vee$) and intersection ($\wedge$) operators are used to calculate the relationship between two FLRGs. The operators have shown below, $\cup$ is the union and $\cap$ is the intersection operator in the classical set theory.*

$$\vee(LHS_d, LHS_e) = RHS_D \cup RHS_e$$

$$\wedge(LHS_d, LHS_e) = RHS_D \cap RHS_e$$

***Definition 8.*** *For multiple FLRG, union and intersection operators are defined as follows.* $\vee_m(LHS_c, LHS_d, LHS_e, ...) = \vee...(\vee(\vee(LHS_c, LHS_d), LHS_e), ...)$

$$\wedge_m(LHS_c, LHS_d, LHS_e, ...) = \wedge...(\wedge(\wedge(LHS_c, LHS_d), LHS_e), ...)$$

***Definition 9.*** *If* $\vee_m(LHS_c, LHS_d, LHS_e, ...) = \varnothing$ *then* $\vee_m(LHS_c, LHS_d, LHS_e, ...) = LHS_x$. *If* $\vee_m(LHS_c, LHS_d, LHS_e, ...) = \varnothing$ *then* $\vee_m(LHS_c, LHS_d, LHS_e, ...) = LHS_x$. *Where* $LHS_x$ *obtained from the FLRG established by Type 1 observations.*

An algorithm for Type 2 fuzzy time series model is summarized follows:

1. Determine a Type 1 fuzzy time series model.
2. Select a variable and Type 1 observations.
3. Apply the Type 1 model to the Type 1 observations and obtain FLRGs.
4. Pick Type 2 observations.
5. Map out-of-sample observations to FLRGs and obtain forecasts.
6. Apply operators to the FLRGs for all the observations.
7. Defuzzify the forecasts.
8. Calculate forecasts for Type 2 model.

## 5. Application of Huarng and Yu's model

The implementation steps of the Type 2 fuzzy time series model developed by Huarng and Yu [1] are shown below detailed below.

*Step 1: Determine a Type 1 fuzzy time series model*

Huarng and Yu have chosen Chen's first-order fuzzy time series model [6] because of its facility of calculation and good forecasting results.

*Step 2: Select a variable and Type 1 observations*

In this study, gold price data were used. The opening values of the daily data between July,1,2015 and July,1,2016 were used for the Type 1 model. However, high and low values of gold prices were included in the calculations for the Type 2 model and forecasts were obtained. Values of gold opening values between July,1,2015 and April,29,2016 were used for parameter estimation. The opening, high and low value between May,2,2016 and July,1,2016 were used for forecasting.

*Step 3: Apply the Type 1 model to the Type 1 observations and obtain FLRGs.*

*Step 3-1: Define the universe of discourse and the intervals.*

The universe of discourse indicated by $U$ was defined as $U=[D_{\min}-D_1, D_{\max}+D_2]$. Where $D_{\min}$ and $D_{\max}$ the minimum and maximun values of the data respectively. As it can be understood from the Table 1 $D_{\min}=1053{,}7$ and $D_{\max}=1355{,}6$. $D_1$ and $D_2$ are the two proper positive numbers. The universe of discourse is selected to cover the entire data set. $D_1=3{,}7$ and $D_2=4{,}4$ is selected and thus the universe of discourse is determined as $U=[1050, 1360]$. And then U is divided into 40 intervals that give the best forecasting result.

$u_1=[1050,1057.8]$, $u_2=[1057.8,1065.5], \ldots, u_{40}=[1352.3,1360]$

*Step 3-2: Define the fuzzy sets*

Fuzzy sets $A_i$ are defined by intervals $u_1, u_2, ..., u_{40}$ in the following.

$A_1 = 1/u_1 + 0.5/u_2 + ... + 0/u_{40}$

…

$A_{57} = 0/u_1 + 0/u_2 + ... + 1/u_{40}$

Where, the denominator of fuzzy sets $A_i$ denote intervals, and the numbers in the numerator are $u_i$'s membership between 0 and 1 to fuzzy sets $A_i$.

*Step 3-3: Fuzzify the observations*

All observation values to be used for parameter estimation are fuzzified. Fuzzifying is done according to the largest membership degree. If the highest degree of membership is seen in $A_k$, the fuzzy value of the data becomes $A_k$. For example, the data for July,01,2015 can be examined. The value of the data is 1173.1, which is a value in the intervals $u_{16}$. The maximum membership degree of the interval is fuzzified as $A_{16}$ because it is in $A_{16}$. The maximum membership level of the $u_{16}$ sub-range is $A_{16}$, therfore it is fuzzified as $A_{16}$. The fuzzified values of the data are in Table 1.

Table 1. Fuzzy values of observed data

| **Date** | **Crisp value** | **Fuzzy value** | **Date** | **Crisp value** | **Fuzzy value** | **Date** | **Crisp value** | **Fuzzy value** |
|---|---|---|---|---|---|---|---|---|
| **07/01/2015** | 1173.1 | $A_{16}$ | **10/12/2015** | 1155.9 | $A_{15}$ | **01/25/2016** | 1099.3 | $A_{9}$ |
| **07/02/2015** | 1166.4 | $A_{16}$ | **10/13/2015** | 1163.5 | $A_{16}$ | **01/26/2016** | 1115.1 | $A_{9}$ |
| **07/03/2015** | 1164.7 | $A_{15}$ | **10/14/2015** | 1172.4 | $A_{18}$ | **01/27/2016** | 1117.8 | $A_{10}$ |
| **07/06/2015** | 1163.9 | $A_{15}$ | **10/15/2015** | 1184.2 | $A_{18}$ | **01/28/2016** | 1125.4 | $A_{9}$ |
| **07/07/2015** | 1166.8 | $A_{16}$ | **10/16/2015** | 1182.5 | $A_{17}$ | **01/29/2016** | 1114.4 | $A_{9}$ |
| **07/08/2015** | 1152.6 | $A_{14}$ | **10/19/2015** | 1176.5 | $A_{16}$ | **02/01/2016** | 1116.7 | $A_{11}$ |
| **07/09/2015** | 1154.8 | $A_{14}$ | **10/20/2015** | 1168.7 | $A_{18}$ | **02/02/2016** | 1128.3 | $A_{11}$ |
| **07/10/2015** | 1160.1 | $A_{15}$ | **10/21/2015** | 1175.9 | $A_{15}$ | **02/03/2016** | 1128.5 | $A_{12}$ |
| **07/13/2015** | 1159 | $A_{15}$ | **10/22/2015** | 1166.2 | $A_{15}$ | **02/04/2016** | 1142.5 | $A_{14}$ |
| **07/14/2015** | 1157.2 | $A_{14}$ | **10/23/2015** | 1166.2 | $A_{15}$ | **02/05/2016** | 1155.6 | $A_{16}$ |
| **07/15/2015** | 1154 | $A_{14}$ | **10/26/2015** | 1165.1 | $A_{15}$ | **02/08/2016** | 1173.5 | $A_{18}$ |
| **07/16/2015** | 1146.2 | $A_{13}$ | **10/27/2015** | 1164.5 | $A_{17}$ | **02/09/2016** | 1188.7 | $A_{19}$ |
| **07/17/2015** | 1144 | $A_{13}$ | **10/28/2015** | 1175.1 | $A_{14}$ | **02/10/2016** | 1189.8 | $A_{21}$ |
| **07/20/2015** | 1130.4 | $A_{11}$ | **10/29/2015** | 1158 | $A_{14}$ | **02/11/2016** | 1205.6 | $A_{26}$ |
| **07/21/2015** | 1097.1 | $A_{7}$ | **10/30/2015** | 1147.5 | $A_{12}$ | **02/10/2016** | 1247.8 | $A_{24}$ |
| **07/22/2015** | 1093 | $A_{6}$ | **11/02/2015** | 1137.9 | $A_{11}$ | **02/14/2016** | 1235.7 | $A_{24}$ |
| **07/23/2015** | 1097.4 | $A_{7}$ | **11/03/2015** | 1133.3 | $A_{9}$ | **02/15/2016** | 1234 | $A_{24}$ |
| **07/24/2015** | 1078.6 | $A_{4}$ | **11/04/2015** | 1119 | $A_{8}$ | **02/16/2016** | 1233.1 | $A_{24}$ |
| **07/27/2015** | 1100 | $A_{7}$ | **11/05/2015** | 1107.9 | $A_{8}$ | **02/17/2016** | 1196.5 | $A_{19}$ |
| **07/28/2015** | 1097.2 | $A_{7}$ | **11/06/2015** | 1105.2 | $A_{5}$ | **02/18/2016** | 1209.6 | $A_{21}$ |
| **07/29/2015** | 1096.4 | $A_{6}$ | **11/09/2015** | 1088.6 | $A_{6}$ | **02/19/2016** | 1230.8 | $A_{24}$ |
| **07/30/2015** | 1096.1 | $A_{6}$ | **11/10/2015** | 1090.1 | $A_{6}$ | **02/22/2016** | 1226.1 | $A_{23}$ |
| **07/31/2015** | 1087.7 | $A_{5}$ | **11/11/2015** | 1091.4 | $A_{5}$ | **02/23/2016** | 1207.4 | $A_{21}$ |

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|---|---|---|---|---|---|---|---|---|
| **08/03/2015** | 1095.5 | $A_6$ | **11/12/2015** | 1086 | $A_5$ | **02/24/2016** | 1230 | $A_{24}$ |
| **08/04/2015** | 1085.1 | $A_5$ | **11/13/2015** | 1083.1 | $A_6$ | **02/25/2016** | 1239.4 | $A_{25}$ |
| **08/05/2015** | 1086.5 | $A_5$ | **11/16/2015** | 1092 | $A_4$ | **02/26/2016** | 1232.4 | $A_{24}$ |
| **08/06/2015** | 1084.4 | $A_5$ | **11/17/2015** | 1075.5 | $A_2$ | **02/29/2016** | 1218.6 | $A_{22}$ |
| **08/07/2015** | 1088.9 | $A_6$ | **11/18/2015** | 1064 | $A_4$ | **03/01/2016** | 1240.5 | $A_{25}$ |
| **08/10/2015** | 1093.2 | $A_6$ | **11/19/2015** | 1077.4 | $A_5$ | **03/02/2016** | 1232.8 | $A_{24}$ |
| **08/11/2015** | 1103.8 | $A_7$ | **11/20/2015** | 1082.9 | $A_3$ | **03/03/2016** | 1238.4 | $A_{25}$ |
| **08/12/2015** | 1106 | $A_8$ | **11/23/2015** | 1070 | $A_3$ | **03/04/2016** | 1263 | $A_{28}$ |
| **08/13/2015** | 1124.3 | $A_{10}$ | **11/24/2015** | 1073.2 | $A_3$ | **03/07/2016** | 1259.8 | $A_{28}$ |
| **08/14/2015** | 1113.9 | $A_9$ | **11/25/2015** | 1070.2 | $A_3$ | **03/08/2016** | 1268.7 | $A_{29}$ |
| **08/17/2015** | 1113.8 | $A_9$ | **11/26/2015** | 1070.1 | $A_3$ | **03/09/2016** | 1254.5 | $A_{27}$ |
| **08/18/2015** | 1118.2 | $A_9$ | **11/27/2015** | 1070.7 | $A_1$ | **03/10/2016** | 1250 | $A_{26}$ |
| **08/19/2015** | 1117 | $A_9$ | **11/30/2015** | 1056.5 | $A_2$ | **03/11/2016** | 1270 | $A_{29}$ |
| **08/20/2015** | 1132.8 | $A_{11}$ | **12/01/2015** | 1064.6 | $A_3$ | **03/14/2016** | 1256 | $A_{27}$ |
| **08/21/2015** | 1154.1 | $A_{14}$ | **12/02/2015** | 1068.7 | $A_1$ | **03/15/2016** | 1228.3 | $A_{24}$ |
| **08/24/2015** | 1164 | $A_{15}$ | **12/03/2015** | 1054.4 | $A_2$ | **03/16/2016** | 1229.5 | $A_{24}$ |
| **08/25/2015** | 1151.5 | $A_{14}$ | **12/04/2015** | 1063 | $A_5$ | **03/17/2016** | 1261.5 | $A_{28}$ |
| **08/26/2015** | 1139.2 | $A_{12}$ | **12/07/2015** | 1084.9 | $A_3$ | **03/18/2016** | 1257.1 | $A_{27}$ |
| **08/27/2015** | 1127.2 | $A_{10}$ | **12/08/2015** | 1071 | $A_4$ | **03/21/2016** | 1256.2 | $A_{27}$ |
| **08/28/2015** | 1125.3 | $A_{10}$ | **12/09/2015** | 1075.1 | $A_3$ | **03/22/2016** | 1244 | $A_{26}$ |
| **08/31/2015** | 1131.1 | $A_{11}$ | **12/10/2015** | 1073 | $A_3$ | **03/23/2016** | 1247.7 | $A_{26}$ |
| **09/01/2015** | 1133.5 | $A_{11}$ | **12/11/2015** | 1067.8 | $A_4$ | **03/24/2016** | 1214.2 | $A_{22}$ |
| **09/02/2015** | 1137.3 | $A_{12}$ | **12/14/2015** | 1074 | $A_2$ | **03/25/2016** | 1216.7 | $A_{22}$ |
| **09/03/2015** | 1130.3 | $A_{11}$ | **12/15/2015** | 1064.6 | $A_2$ | **03/28/2016** | 1211.5 | $A_{21}$ |
| **09/04/2015** | 1123.4 | $A_{10}$ | **12/16/2015** | 1064.8 | $A_3$ | **03/29/2016** | 1217.8 | $A_{22}$ |
| **09/06/2015** | 1122.3 | $A_{10}$ | **12/17/2015** | 1072.6 | $A_1$ | **03/30/2016** | 1241.4 | $A_{25}$ |
| **09/07/2015** | 1119.5 | $A_9$ | **12/18/2015** | 1053.7 | $A_3$ | **03/31/2016** | 1225.6 | $A_{23}$ |
| **09/08/2015** | 1119.8 | $A_{10}$ | **12/21/2015** | 1070.2 | $A_4$ | **04/01/2016** | 1232.3 | $A_{24}$ |
| **09/09/2015** | 1123.5 | $A_{10}$ | **12/22/2015** | 1078.6 | $A_4$ | **04/04/2016** | 1221.7 | $A_{23}$ |
| **09/10/2015** | 1107.3 | $A_8$ | **12/23/2015** | 1074.5 | $A_4$ | **04/05/2016** | 1215.4 | $A_{22}$ |
| **09/11/2015** | 1110.8 | $A_8$ | **12/24/2015** | 1074.3 | $A_4$ | **04/06/2016** | 1230.4 | $A_{24}$ |
| **09/14/2015** | 1107.8 | $A_8$ | **12/28/2015** | 1077.9 | $A_4$ | **04/07/2016** | 1224.8 | $A_{23}$ |
| **09/15/2015** | 1104.5 | $A_8$ | **12/29/2015** | 1079.1 | $A_3$ | **04/08/2016** | 1241 | $A_{25}$ |
| **09/16/2015** | 1109.8 | $A_8$ | **12/30/2015** | 1068.6 | $A_2$ | **04/11/2016** | 1241.5 | $A_{25}$ |
| **09/17/2015** | 1118.6 | $A_9$ | **12/31/2015** | 1061.9 | $A_2$ | **04/12/2016** | 1260 | $A_{28}$ |
| **09/18/2015** | 1128.3 | $A_{11}$ | **01/04/2016** | 1063.4 | $A_4$ | **04/13/2016** | 1256.5 | $A_{27}$ |
| **09/21/2015** | 1137.6 | $A_{12}$ | **01/05/2016** | 1075.6 | $A_5$ | **04/14/2016** | 1231.4 | $A_{24}$ |
| **09/22/2015** | 1132.6 | $A_{11}$ | **01/06/2016** | 1081.6 | $A_6$ | **04/15/2016** | 1225.7 | $A_{23}$ |
| **09/23/2015** | 1128.6 | $A_{11}$ | **01/07/2016** | 1091.6 | $A_8$ | **04/18/2016** | 1238.7 | $A_{25}$ |
| **09/24/2015** | 1133 | $A_{13}$ | **01/08/2016** | 1111.1 | $A_8$ | **04/19/2016** | 1231.6 | $A_{24}$ |
| **09/25/2015** | 1149.8 | $A_{11}$ | **01/11/2016** | 1105.7 | $A_6$ | **04/20/2016** | 1256.1 | $A_{27}$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **09/28/2015** | 1133.2 | $A_{11}$ | **01/12/2016** | 1093.8 | $A_5$ | **04/21/2016** | 1247.7 | $A_{26}$ |
| **09/29/2015** | 1132.2 | $A_{10}$ | **01/13/2016** | 1081.5 | $A_6$ | **04/22/2016** | 1248.5 | $A_{26}$ |
| **09/30/2015** | 1124.3 | $A_9$ | **01/14/2016** | 1091.5 | $A_5$ | **04/25/2016** | 1231.6 | $A_{24}$ |
| **10/01/2015** | 1115.2 | $A_9$ | **01/15/2016** | 1085 | $A_5$ | **04/26/2016** | 1239.8 | $A_{25}$ |
| **10/02/2015** | 1112.2 | $A_{12}$ | **01/17/2016** | 1088.6 | $A_6$ | **04/27/2016** | 1248.6 | $A_{26}$ |
| **10/05/2015** | 1137.1 | $A_{12}$ | **01/18/2016** | 1090.2 | $A_6$ | **04/28/2016** | 1245.9 | $A_{26}$ |
| **10/06/2015** | 1136.3 | $A_{13}$ | **01/19/2016** | 1089.8 | $A_7$ | **04/29/2016** | 1267.3 | $A_{29}$ |
| **10/07/2015** | 1147.6 | $A_{12}$ | **01/20/2016** | 1099.8 | $A_7$ | | | |
| **10/08/2015** | 1142.7 | $A_{14}$ | **01/21/2016** | 1103.6 | $A_7$ | | | |
| **10/09/2015** | 1151.6 | $A_{14}$ | **01/22/2016** | 1098 | $A_7$ | | | |

*Step 3-4: Establish the fuzzy relationships*

Fuzzy relationships between fuzzy values are determined. Since a first order model is used, the fuzzy value at the time t is affected by the fuzzy value at the time (t-1) and this relationship can be shown as $A_{(t-1)} \rightarrow A_t$ . The relationship between the fuzzy values starting from July,1,2015 and April,29,2016 is given below as an example. Fuzzy relationships (FLR) between all other values are determined in this way.

$A_{16} \rightarrow A_{16}$ , $A_{16} \rightarrow A_{15}$ , $A_{15} \rightarrow A_{15}$ , $A_{15} \rightarrow A_{16}$ , $A_{16} \rightarrow A_{14}$ , $A_{14} \rightarrow A_{14}$ , $A_{14} \rightarrow A_{15} \ldots$

All of the above mentioned fuzzy relationships are grouped. The grouping process is based on the left side of the relationship. In other words, the same fuzzy sets on the left side of the relationship to the same group is assigned. As Chen's model [6] recursive relationships are taken only once, the recurrent relationships to groups is repeated only once. For e
xample, all fuzzy relationships with left hand side is $A_{16}$ are as follows from Table 1.

$A_{16} \rightarrow A_{16}$ , $A_{16} \rightarrow A_{15}$ , $A_{16} \rightarrow A_{14}$ , $A_{16} \rightarrow A_{18}$ , $A_{16} \rightarrow A_{18}$ , $A_{16} \rightarrow A_{17}$ , $A_{16} \rightarrow A_{18}$

According to this fuzzy relationships, the FLRG is setted to the left hand side of the relationship with $A_{16}$ as follows.

$A_{16} \rightarrow A_{14}, A_{15}, A_{16}, A_{17}, A_{18}$

It should be noted that in this group, the recurring $A_{16} \rightarrow A_{18}$ relationships takes place only once in the group. Table 2 shows all fuzzy relationship groups (FLRG). These groups will be used to the forecast.

Tablo 2. Fuzzy logic relationship groups

| | | |
|---|---|---|
| $A_1 \rightarrow A_2, A_3$ | $A_{11} \rightarrow A_7, A_9, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}$ | $A_{21} \rightarrow A_{22}, A_{24}, A_{26}$ |
| $A_2 \rightarrow A_2, A_3, A_4, A_5$ | $A_{12} \rightarrow A_{10}, A_{11}, A_{12}, A_{13}, A_{14}$ | $A_{22} \rightarrow A_{21}, A_{22}, A_{24}, A_{25}$ |
| $A_3 \rightarrow A_1, A_2, A_3, A_4$ | $A_{13} \rightarrow A_{11}, A_{12}, A_{13}$ | $A_{23} \rightarrow A_{21}, A_{22}, A_{24}, A_{25}$ |
| $A_4 \rightarrow A_2, A_3, A_4, A_5, A_7$ | $A_{14} \rightarrow A_{12}, A_{13}, A_{14}, A_{15}, A_{16}$ | $A_{24} \rightarrow A_{19}, A_{22}, A_{23}, A_{24}, A_{25}, A_{27}, A_{28}$ |
| $A_6 \rightarrow A_4, A_5, A_6, A_7, A_8$ | $A_{15} \rightarrow A_{14}, A_{15}, A_{16}, A_{17}$ | $A_{25} \rightarrow A_{23}, A_{24}, A_{25}, A_{26}, A_{28}$ |
| $A_7 \rightarrow A_4, A_6, A_7, A_8, A_9$ | $A_{16} \rightarrow A_{14}, A_{15}, A_{16}, A_{17}, A_{18}$ | $A_{26} \rightarrow A_{22}, A_{24}, A_{26}, A_{29}$ |
| $A_8 \rightarrow A_5, A_6, A_8, A_9, A_{10}$ | $A_{17} \rightarrow A_{14}, A_{15}, A_{16}$ | $A_{27} \rightarrow A_{24}, A_{26}, A_{27}$ |
| $A_9 \rightarrow A_8, A_9, A_{10}, A_{11}, A_{12}$ | $A_{18} \rightarrow A_{17}, A_{18}, A_{19}$ | $A_{28} \rightarrow A_{27}, A_{28}, A_{29}$ |
| $A_{10} \rightarrow A_8, A_9, A_{10}, A_{11}$ | $A_{19} \rightarrow A_{21}$ | $A_{29} \rightarrow A_{27}$ |

*Step 4: Pick type 2 observation*

High (the highest price of the day) and Low (the highest price of the day) were used as Type 2 observations. The data are given in Table 3 with fuzzy values.

Table 3. Data for forecasting (with fuzzy values)

| Date | Closing | High | Low | Date | Closing | High | Low |
|---|---|---|---|---|---|---|---|
| **05/02/2016** | 1292.1 ( $A_{32}$ ) | 1304.4 ( $A_{33}$ ) | 1290.4 ( $A_{32}$ ) | **06/03/2016** | 1209.1 ( $A_{21}$ ) | 1209.1 ( $A_{26}$ ) | 1244.5 ( $A_{21}$ ) |
| **05/03/2016** | 1292.7 ( $A_{32}$ ) | 1301.5 ( $A_{33}$ ) | 1284.5 ( $A_{31}$ ) | **06/06/2016** | 1244.4 ( $A_{26}$ ) | 1244.4 ( $A_{26}$ ) | 1247.3 ( $A_{25}$ ) |
| **05/04/2016** | 1286.9 ( $A_{31}$ ) | 1290 ( $A_{31}$ ) | 1273 ( $A_{29}$ ) | **06/07/2016** | 1244 ( $A_{26}$ ) | 1244 ( $A_{26}$ ) | 1244.9 ( $A_{25}$ ) |
| **05/05/2016** | 1282.9 ( $A_{31}$ ) | 1286 ( $A_{31}$ ) | 1270.8 ( $A_{29}$ ) | **06/08/2016** | 1247.6 ( $A_{26}$ ) | 1247.6 ( $A_{28}$ ) | 1264 ( $A_{26}$ ) |
| **05/06/2016** | 1278.6 ( $A_{30}$ ) | 1295.6 ( $A_{32}$ ) | 1276.9 ( $A_{30}$ ) | **06/09/2016** | 1262.5 ( $A_{28}$ ) | 1262.5 ( $A_{29}$ ) | 1271.4 ( $A_{27}$ ) |
| **05/09/2016** | 1286.5 ( $A_{31}$ ) | 1286.5 ( $A_{31}$ ) | 1262.3 ( $A_{28}$ ) | **06/10/2016** | 1267.6 ( $A_{29}$ ) | 1267.6 ( $A_{30}$ ) | 1278 ( $A_{28}$ ) |
| **05/10/2016** | 1265.1 ( $A_{28}$ ) | 1267.4 ( $A_{29}$ ) | 1258.7 ( $A_{27}$ ) | **06/13/2016** | 1276.2 ( $A_{30}$ ) | 1276.2 ( $A_{31}$ ) | 1287.3 ( $A_{29}$ ) |
| **05/11/2016** | 1266.9 ( $A_{28}$ ) | 1279.2 ( $A_{30}$ ) | 1266.9 ( $A_{28}$ ) | **06/14/2016** | 1281.8 ( $A_{30}$ ) | 1281.8 ( $A_{31}$ ) | 1289.7 ( $A_{30}$ ) |
| **05/12/2016** | 1267 ( $A_{28}$ ) | 1281.2 ( $A_{30}$ ) | 1264 ( $A_{28}$ ) | **06/15/2016** | 1284.6 ( $A_{31}$ ) | 1284.6 ( $A_{32}$ ) | 1296.2 ( $A_{30}$ ) |
| **05/13/2016** | 1271.4 ( $A_{29}$ ) | 1275.7 ( $A_{30}$ ) | 1264.6 ( $A_{28}$ ) | **06/16/2016** | 1296.3 ( $A_{32}$ ) | 1296.3 ( $A_{35}$ ) | 1316.4 ( $A_{30}$ ) |
| **05/16/2016** | 1272.8 ( $A_{29}$ ) | 1287.8 ( $A_{31}$ ) | 1272.8 ( $A_{29}$ ) | **06/17/2016** | 1280.3( $A_{30}$ ) | 1280.3 ( $A_{33}$ ) | 1300.1 ( $A_{30}$ ) |
| **05/17/2016** | 1272.5 ( $A_{29}$ ) | 1281.6 ( $A_{30}$ ) | 1270.8 ( $A_{29}$ ) | **06/20/2016** | 1288.9 ( $A_{31}$ ) | 1288.9 ( $A_{32}$ ) | 1291.2 ( $A_{30}$ ) |
| **05/18/2016** | 1276.4 ( $A_{30}$ ) | 1276.4 ( $A_{30}$ ) | 1256.8 ( $A_{27}$ ) | **06/21/2016** | 1293.9 ( $A_{32}$ ) | 1293.9 ( $A_{32}$ ) | 1293.9 ( $A_{28}$ ) |
| **05/19/2016** | 1248 ( $A_{26}$ ) | 1255.5 ( $A_{27}$ ) | 1247.5 ( $A_{26}$ ) | **06/22/2016** | 1267 ( $A_{28}$ ) | 1267 ( $A_{29}$ ) | 1269.2 ( $A_{28}$ ) |
| **05/20/2016** | 1256.6 ( $A_{27}$ ) | 1256.6 ( $A_{27}$ ) | 1255.4 ( $A_{27}$ ) | **06/23/2016** | 1263.4 ( $A_{28}$ ) | 1263.4 ( $A_{29}$ ) | 1270.5 ( $A_{27}$ ) |
| **05/23/2016** | 1251.6 ( $A_{27}$ ) | 1251.6 ( $A_{27}$ ) | 1247.5 ( $A_{26}$ ) | **06/24/2016** | 1253.7 ( $A_{27}$ ) | 1253.7 ( $A_{40}$ ) | 1355.6 ( $A_{27}$ ) |
| **05/24/2016** | 1240 ( $A_{25}$ ) | 1240 ( $A_{25}$ ) | 1228.2 ( $A_{23}$ ) | **06/27/2016** | 1330 ( $A_{37}$ ) | 1330 ( $A_{37}$ ) | 1331.7 ( $A_{35}$ ) |
| **05/25/2016** | 1219.3 ( $A_{22}$ ) | 1219.3 ( $A_{22}$ ) | 1218.6 ( $A_{22}$ ) | **06/28/2016** | 1314.8 ( $A_{35}$ ) | 1314.8 ( $A_{35}$ ) | 1315.6 ( $A_{35}$ ) |
| **05/26/2016** | 1232.6 ( $A_{24}$ ) | 1232.6 ( $A_{24}$ ) | 1230.6 ( $A_{24}$ ) | **06/29/2016** | 1311.6 ( $A_{34}$ ) | 1311.6 ( $A_{36}$ ) | 1327.4 ( $A_{34}$ ) |
| **05/27/2016** | 1220.3 ( $A_{22}$ ) | 1223.2 ( $A_{23}$ ) | 1206.4 ( $A_{21}$ ) | **06/30/2016** | 1319 ( $A_{35}$ ) | 1319 ( $A_{37}$ ) | 1329.05 ( $A_{35}$ ) |
| **05/30/2016** | 1208.25 ( $A_{21}$ ) | 1213.15 ( $A_{22}$ ) | 1201.8 ( $A_{20}$ ) | **07/01/2016** | 1324.55 ( $A_{36}$ ) | 1324.55 ( $A_{38}$ ) | 1344.25 ( $A_{36}$ ) |
| **05/31/2016** | 1212.5 ( $A_{21}$ ) | 1217.6 ( $A_{22}$ ) | 1199 ( $A_{20}$ ) | | | | |
| **06/01/2016** | 1215.7 ( $A_{22}$ ) | 1219.8 ( $A_{22}$ ) | 1205.4 ( $A_{21}$ ) | | | | |
| **06/02/2016** | 1212.4 ( $A_{21}$ ) | 1217 ( $A_{22}$ ) | 1209.6 ( $A_{21}$ ) | | | | |

Step 5. Map-out-of sample observations to FLRSs and obtain forecasts

Fuzzy relationship groups are determined for fuzzified out-of-sample observations. The forecasts for fuzzy values are obtained by FLRG of the previous observation. For example, suppose $F(t-1)=A_1$, in this case the forecast for $F(t)$ by FLRG is $A_2, A_3$.

Step 6. Apply operators to the FLRGs for all the observations

Several forecasts for example are displayed in Table 4 and Table 5. In Table 4, the first column is the date, second column is the forecasts for closing, high and low values of the gold price data, the fourth column is intersection process, and finally the last column is as a result of the intersection process, respectively. In Table 5, similarly the first column is the date, the second column is the forecasts for closing, high and low values of the gold price data, the fourth column is union process, and finally the last column is as a result of the union process, respectively.

When the intersection or union operators are applied, it should be noted that Type 1 forecast is obtained as a Type 2 forecast if the result is $\varnothing$.

Table 4. Forecasts after $\wedge_m$

| Date | Type 1 forecasts | | $\wedge_m$ operators | Forecast after $\wedge_m$ |
|---|---|---|---|---|
| 05/16/2016 | Closing | $A_{29} \to A_{27}$ | $\wedge_m(A_{29}, A_{30}, A_{28}) = \{A_{27}\} \cap (\varnothing \cap \{A_{27}, A_{28}, A_{29}\}) = \{A_{27}\} \cap \varnothing = \varnothing$ | $A_{29}$ (forecast for Type 1) |
| | High | $A_{30} \to \varnothing$ | | |
| | Low | $A_{28} \to A_{27}, A_{28}, A_{29}$ | | |
| 05/17/2016 | Closing | $A_{29} \to A_{27}$ | $\wedge_m(A_{29}, A_{31}, A_{29}) = \{A_{27}\} \cap (\varnothing \cap \{A_{27}\}) = \{A_{27}\} \cap \varnothing = \varnothing$ | $A_{29}$ (forecast for Type 1) |
| | High | $A_{31} \to \varnothing$ | | |
| | Low | $A_{29} \to A_{27}$ | | |
| 05/18/2016 | Closing | $A_{29} \to A_{27}$ | $\wedge_m(A_{29}, A_{30}, A_{29}) = \{A_{27}\} \cap (\varnothing \cap \{A_{27}\}) = \{A_{27}\} \cap \varnothing = \varnothing$ | $A_{29}$ (forecast for Type 1) |
| | High | $A_{30} \to \varnothing$ | | |
| | Low | $A_{29} \to A_{27}$ | | |
| 05/19/2016 | Closing | $A_{26} \to A_{22}, A_{24}, A_{26}, A_{29}$ | $\wedge_m(A_{26}, A_{27}, A_{26}) = \{A_{22}, A_{24}, A_{26}, A_{29}\} \cap (\{A_{22}, A_{24}, A_{26}, A_{29}\} \cap \{A_{24}, A_{26}, A_{27}\})$ | $A_{24}, A_{26}$ |
| | High | $A_{27} \to A_{24}, A_{26}, A_{27}$ | $= \{A_{22}, A_{24}, A_{26}, A_{29}\} \cap \{A_{24}, A_{26}\} = \{A_{24}, A_{26}\}$ | |
| | Low | $A_{26} \to A_{22}, A_{24}, A_{26}, A_{29}$ | | |

Table 5. Forecasts after $\vee_m$

| Date | | Type 1 forecasts | $\vee_m$ operators | Forecast after $\vee_m$ |
|---|---|---|---|---|
| 05/16/2016 | Closing | $A_{29} \to A_{27}$ | $\vee_m(A_{29}, A_{30}, A_{28}) = \{A_{27}\} \cup (\varnothing \cup \{A_{27}, A_{28}, A_{29}\})$ | $A_{27}, A_{28}, A_{29}$ |
| | High | $A_{30} \to \varnothing$ | $= \{A_{27}\} \cup \{A_{27}, A_{28}, A_{29}\}$ | |
| | Low | $A_{28} \to A_{27}, A_{28}, A_{29}$ | $= \{A_{27}, A_{28}, A_{29}\}$ | |
| 05/17/2016 | Closing | $A_{29} \to A_{27}$ | $\vee_m(A_{29}, A_{31}, A_{29}) = \{A_{27}\} \cup (\varnothing \cup \{A_{27}\})$ | $A_{27}$ |
| | High | $A_{31} \to \varnothing$ | $= \{A_{27}\} \cup \{A_{27}\} = \{A_{27}\}$ | |
| | Low | $A_{29} \to A_{27}$ | | |
| 05/18/2016 | Closing | $A_{29} \to A_{27}$ | $\vee_m(A_{29}, A_{30}, A_{29}) = \{A_{27}\} \cup (\varnothing \cup \{A_{27}\})$ | $A_{27}$ |
| | High | $A_{30} \to \varnothing$ | $= \{A_{27}\} \cup \{A_{27}\} = \{A_{27}\}$ | |
| | Low | $A_{29} \to A_{27}$ | | |
| 05/19/2016 | Closing | $A_{26} \to A_{22}, A_{24}, A_{26}, A_{29}$ | $\vee_m(A_{26}, A_{27}, A_{26}) = \{A_{22}, A_{24}, A_{26}, A_{29}\} \cup (\{A_{22}, A_{24}, A_{26}, A_{29}\} \cup \{A_{24}, A_{26}, A_{27}\})$ | $A_{22}, A_{24}, A_{26}, A_{27}, A_{29}$ |
| | High | $A_{27} \to A_{24}, A_{26}, A_{27}$ | $= \{A_{22}, A_{24}, A_{26}, A_{29}\} \cap \{A_{22}, A_{24}, A_{26}, A_{27}, A_{29}\}$ | |
| | Low | $A_{26} \to A_{22}, A_{24}, A_{26}, A_{29}$ | $= \{A_{22}, A_{24}, A_{26}, A_{27}, A_{29}\}$ | |

Step7. Defuzzify the forecasts

For the defuzzification process, the midpoint method was chosen as in the Chen [6] model. Assuming the forecast result is $A_{q1}, A_{q2}, ..., A_{qj}$, the defuzzified forecast is the arithmetic mean of the $m_{q1}, m_{q2}, ..., m_{qj}$ s, which are the midpoints of the $u_{q1}, u_{q2}, ..., u_{qj}$ intervals. This arithmetic mean is calculated from (1).

$$defuzzification(t) = \frac{\sum_{z=1}^{j} m_{qz}}{j} \qquad (1)$$

For example, as the result of the intersection operator corresponding to the May,18,2016 data is $A_{29}$, its defuzzified value is calculated as:

$$defuzzification_{intersection}(18/16) = m_{29} = 1270.9$$

As the result of the union operator corresponding to the February,19,2016 data is $A_{24}, A_{26}$, its defuzzified value is calculated as:

$$defuzzification_{intersection}(19/16) = \frac{m_{24} + m_{26}}{2} = \frac{1232.1 + 1247.6}{2} = 1239.8$$

All forecast values are defuzzified in this way. The final Type 2 forecast values are obtained by averaging the fuzzified values obtained from the intersection and union operations. Table 6 shows the deffuzified forecast values.

Table 6. Forecasts of the golden Prices

| **Date** | **Type 1** | $\wedge_m$ | $\vee_m$ | **Type 2** | **Date** | **Type 1** | $\wedge_m$ | $\vee_m$ | **Type 2** |
|---|---|---|---|---|---|---|---|---|---|
| **05/02/2016** | - | - | - | - | **06/03/2016** | 1232.1 | 1224.4 | 1229 | 1237 |
| **05/03/2016** | 1294.1 | 1294.1 | 1294.1 | 1294.1 | **06/06/2016** | 1232.1 | 1232.1 | 1241.8 | 1241 |
| **05/04/2016** | 1294.1 | 1294.1 | 1294.1 | 1294.1 | **06/07/2016** | 1241.8 | 1239.9 | 1242.1 | 1241 |
| **05/05/2016** | 1286.4 | 1286.4 | 1255.4 | 1270.9 | **06/08/2016** | 1241.8 | 1239.9 | 1242.1 | 1259.3 |
| **05/06/2016** | 1286.4 | 1286.4 | 1255.4 | 1270.9 | **06/09/2016** | 1241.8 | 1270.9 | 1247.6 | 1254.6 |
| **05/09/2016** | 1278.6 | 1278.6 | 1278.6 | 1278.6 | **06/10/2016** | 1263.1 | 1255.4 | 1253.8 | 1267 |
| **05/10/2016** | 1286.4 | 1286.4 | 1263.1 | 1274.8 | **06/13/2016** | 1255.4 | 1270.9 | 1263.1 | 1267 |
| **05/11/2016** | 1263.1 | 1255.4 | 1253.8 | 1254.6 | **06/14/2016** | 1278.6 | 1278.6 | 1255.4 | 1278.6 |
| **05/12/2016** | 1263.1 | 1263.1 | 1263.1 | 1263.1 | **06/15/2016** | 1278.6 | 1278.6 | 1278.6 | 1286.4 |
| **05/13/2016** | 1263.1 | 1263.1 | 1263.1 | 1263.1 | **06/16/2016** | 1286.4 | 1286.4 | 1286.4 | 1294.1 |
| **05/16/2016** | 1255.4 | 1270.9 | 1263.1 | 1267 | **06/17/2016** | 1294.1 | 1294.1 | 1294.1 | 1278.6 |
| **05/17/2016** | 1255.4 | 1270.9 | 1255.4 | 1263.1 | **06/20/2016** | 1278.6 | 1278.6 | 1278.6 | 1286.4 |
| **05/18/2016** | 1255.4 | 1270.9 | 1255.4 | 1263.1 | **06/21/2016** | 1286.4 | 1286.4 | 1286.4 | 1278.6 |
| **05/19/2016** | 1278.6 | 1278.6 | 1245 | 1261.8 | **06/22/2016** | 1294.1 | 1294.1 | 1263.1 | 1259.3 |
| **05/20/2016** | 1241.8 | 1239.9 | 1244.5 | 1242.2 | **06/23/2016** | 1263.1 | 1255.4 | 1263.1 | 1254.6 |
| **05/23/2016** | 1245 | 1245 | 1245 | 1245 | **06/24/2016** | 1263.1 | 1255.4 | 1253.8 | 1250.2 |
| **05/24/2016** | 1245 | 1239.9 | 1244.5 | 1242.2 | **06/27/2016** | 1245 | 1255.4 | 1245 | 1332.9 |
| **05/25/2016** | 1241.4 | 1236 | 1233.2 | 1234.6 | **06/28/2016** | 1332.9 | 1332.9 | 1332.9 | 1317.4 |
| **05/26/2016** | 1224.4 | 1224.4 | 1224.4 | 1224.4 | **06/29/2016** | 1317.4 | 1317.4 | 1317.4 | 1309.6 |
| **05/27/2016** | 1232.1 | 1232.1 | 1232.1 | 1232.1 | **06/30/2016** | 1309.6 | 1309.6 | 1309.6 | 1317.4 |
| **05/30/2016** | 1224.4 | 1224.4 | 1229 | 1226.7 | **06/31/2016** | 1317.4 | 1317.4 | 1317.4 | 1237 |
| **05/31/2016** | 1232.1 | 1208.9 | 1229 | 1219 | | | | | |
| **06/01/2016** | 1232.1 | 1208.9 | 1229 | 1219 | | | | | |
| **06/02/2016** | 1224.4 | 1224.4 | 1229 | 1226.7 | | | | | |

The RMSE given in (2) is used to evaluate the forecasting performance. The RMSEs calculated from the forecast values for gold price data are given in Table 7 for

Type 1 model, after intersection operator, after union operator and Type 2 model. In addition, Table 7 shows the case results where recurrent relationships are included in and recurrent relationships are not included in the calculations.

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}\left(actual_t - defuzzification\left(t\right)\right)^2}{n}} \qquad (2)$$

Table 7. RMSE comparison for gold price data

| RMSE | Type 1 | $\wedge_m$ | $\vee_m$ | Type 2 |
|---|---|---|---|---|
| when recurring relationships are not included in the calculations | 18.382 | 15.942 | 17.975 | 15.696 |
| when recurring relationships are included in the calculations | 18.813 | 15.164 | 20.338 | 16.755 |

As can be seen from Table 7, when recurring relationships are not included in the calculations, Type 2 model has better forecasts result than the others. In Table 7, after the intersection operator result and type 2 model result are approximately the same. Similarly, after the union operator result and Type 2 model result are very close. We can conclude that the factor that improves the results of the Type 2 model is, in fact, the intersection operator. As the Type 2 model is the arithmetic mean of the results obtained from the intersection and union operators, it can be concluded that the factor that improves the results of the Type 2 model is actually the intersection operator.

When recurring relationships are included in calculations, RMSE of Type 2 model better than all results but only the after the intersection operator. It is also remarkable that the after union operator results are even worse than the Type 1 model results.

## 4. Conclusions and future work

This study examined Type 2 fuzzy time series model developed by Huarng and Yu (2005). It was fitted in the gold price data from 01/07/2015 to 01/07/2016 in Turkey. The part from 01/07/2015 to 29/04/2016 is used for parameter estimation, and 2-month data between 02/05/2016 and 01/07/206 is used for forecasting.

Type 2 model is an extended version of Type 1 model with extra observations. The Type 1 model was extended to the Type 2 model by adding the lowest and highest gold prices to the calculations. However, only the opening gold prices data were used for the Type 1 model.

From the results it is understood that the intersection operator acts as the frequency of the relationship recurrence for each observation. In addition, when the frequency of recurrence of the relationships for the entire data set is included better results are obtained. However, the union operator behaves in the opposite direction and confuses. Adding to this is the frequency of relations observed for the entire data set, the RMSE obtained is almost identical to the RMSE obtained from Chen Type 1 model.

Consequently, a further study of the union operator in the Type 2 model of Huarng and Yu [1] can contribute the better results.

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# An Integrated IF-AHP and IF-TOPSIS method for Neuroimaging Techniques Selection

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**Abstract**

Neuroergonomics is to provide the design of effective human machine systems by determining tendencies and characteristics of humans, and to try to understand brain. In neuroergonomics, it is important not to take a snapshot of the brain but to monitor it during the study. Neuroimaging techniques (NTs) are used to display the brain during the study. The NTs selection problem includes many qualitative criteria that decision makers have difficulties in making decision and they have uncertainty. The aim of this study is to evaluate the NTs by using AHP and TOPSIS methods based on Intuitionistic Fuzzy Set (IFS). IFS provides information on the membership, non-membership and hesitancy functions. It is useful tool to deal with uncertainty and fuzziness of complex problems. Because of these features and obtain to a more complete evaluation and more precise results, IF-AHP&TOPSIS method is appropriate for the problem of the NTs selection. In this regard, IF-AHP method is used to determine the criteria weights and then IF-TOPSIS method is conducted for ranking alternatives, the comparison analysis was performed using IF-VIKOR method. As a result of the literature review, it seen that there is no study about the evaluation of NTs before.

**Keywords:** Neuroergonomics, human factors, neuroimaging techniques, intuitionistic fuzzy AHP, intuitionistic fuzzy TOPSIS.

## 1. Introduction

Neuroergonomics is a multidisciplinary discipline that combines neuroscience, human factors, cognitive psychology and ergonomics with its brain structure and function in daily life. It allows research to better understand human behavior and performance in many different environments (Koç and Kokangül, 2018; Lees et al. 2010). Neuroergonomics uses neuroimaging techniques to understand brain structures, mechanisms and functions. Study fields conducted using neuroimaging techniques can be expressed in three groups. These; studies on mental workload measurement, studies on human errors and studies on adaptive automation. Roy and Sherrington (1890), in their study found that blood flow increased in the brain regions with the increase in electrical activities between nerve cells. This shows that when mental activity increases, blood flow occurs in the regions managing that activity in the brain. The prediction of human errors is very important for the efficiency of the employee. Gerhring et al. (1993) found that signals on the scalp peak after ~100–150 ms in the measurements taken by the electromyography when the people made mistakes, however these signals were not found in the correct answers. If we can identify and classify what kind of changes occur in the brain functions of people before making mistakes, it will be possible to prevent especially fatal accidents when neuroimaging devices are more portable and usable. Nowadays, people have developed automation systems for both efficiency and less error. However, it is not always possible to plan all the things in automation beforehand and to make an automation system suitable for all situations and closed to all errors. In adaptive automation systems, there is a division of labor between the operator and the automation system, but this is not static. In other words, it is dynamic and adaptable to the situation. When 100% human control and static automation are compared with adaptive automation, it was found that when using adaptive automation, operators have higher reliance and confidence as well as lower perceived workload. Because, operators should monitor the system continuously in automation systems and continuous attention increases the mental load. Some studies are conducted by using neuroimaging techniques:

During the flight control task on pilots, blood flow in the brain was measured using fMRI and TCD techniques. Moreover, blood flow was examined for the measurement of mental workload in the cerebral cortex (Peres et al. 2000). (Ayaz et al. 2012) used fNIRs imaging method in the measurement of mental workload for Airport tower operators with different levels of experience. The aim of the study is to determine strategies necessary for design of complex human-machine interface systems. In the study of Baldwin et al. 2012, electroencephalography (EEG) activity was recorded during implementation of tasks different as task sequence and difficulty level and the classification accuracy between tasks was examined with YSA. Causse et al. 2013, suggested that negative emotions and stress temporarily disrupt the decision-making process and made behavioral testing using fMRI technique. At the end of the study, it was determined that the brain was supported with the contribution of the emotion and reward circuit in the decision-making process. In the study of Takeda et al. 2014, the participants were asked to pass in front of the stores in the shopping center and the reaction of the people to the audial and visual changes in the stores was tried to be measured by using P300 technique. In the study, McKendrick, Ayaz, Parasuraman, 2014, tried to identify the relationship between education and increased work memory capacity and mental workload using fNIRs technique. In the study of Durantin et al. 2014, fNIRs method was used to estimate the mental workload during the simulated pilot task. Correa et al. 2014, monitored behavior performances during the simulated driving at different times of the day (morning 08: 00-evening 20:00) by using EEG technique. Giraudet et al. 2015, tried to determine behavioral and neural responses to a simulated air traffic control task in two different visual design areas by using the P300 technique. Mijović et al. 2015, used EEG and ERP technique to investigate the effect of micro breakage on the attention of a montage worker. Kosti et al. 2018, measured brain responses of software engineers to code comprehension during the understanding and investigation of code by using the EEG technique. The neural correlations of the subjective difficulty were determined during code comprehension. In the study of Hou and Lu, 2018, the emotional arousal in the process of understanding traffic signs was tried to be measured by ERP technique.

Neuroimaging techniques vary in terms of many criteria. These techniques also include good and bad directions. The aim of the study is to evaluate these techniques in terms of conflicting criteria. In accordance with this purpose, Intuitionistic Fuzzy AHP and Intuitionistic Fuzzy TOPSIS method (IF-AHP & TOPSIS), which are Multi-criteria decision-making method, are integrated and used in this study for suitability of problem structure. The most important factor in using AHP method in the selection process is to include qualitative and quantitative evaluation of criteria. With this method, problems that appear to be complex can be displayed in a hierarchical structure extending from the specified main objective to the sub-criteria. TOPSIS method is a multi-criteria decision-making method in which the solution is created by moving the shortest distance to the positive ideal solution and the longest distance from the negative ideal solution. IFSs are used to come up with complex problems that involve ambiguity in determining the preference values of the criteria considered. IFS distinguishes the classical fuzzy class because it can represent the degree of membership, the degree of non-membership and the degree of hesitancy functions, respectively, with three degrees of membership. However, triangular fuzzy numbers and trapezoidal fuzzy numbers do not have this property, and each represents a degree of membership that is clear in the unit range (Xu et. al. 2014). For this reason, IF-AHP & TOPSIS method has been applied to the evaluation of neuroimaging techniques problem in order to achieve a more complete evaluation and more precise results in coping with uncertainty and blurriness.

IF-AHP and IF-TOPSIS are used in advanced manufacturing technology by (Maldonado-Macias et al. 2014), for supplier selection by (Rouyendegh, 2014), for partner selection by (Büyüközkan and Güleryüz, 2016). IF-AHP method is used for mud selection by (Sadiq and Tesfemariam, 2009), risk evaluation by (Wang and Sun, 2012), human capital ranking (Abdullah and Najib, 2014), vendor selection by (Kaur, 2014), method proposal (Xu and Liao, 2014; Dutta and Guha, 2015; Ren et al. 2016), sustainable energy (Abdullah and Najib, 2016), logistics outsourcing by (Tavana et al. 2016), healthcare evaluation by (Otay et al. 2017) together Data Envelopment Analysis, sustainable development by (Wang and Xu, 2017). IF-AHP and IF-MOORA methods are used for new product selection by (Atalay and Can, 2018). IF-VIKOR method is used for robot selection (Devi, 2011), system selection (Ying-Yu and De-Jian, 2011), supplier selection (Chatterjee et al. 2013), faculty selection (Park et al. 2013), personnel selection (Wan et al. 2013), multi response problems (Peng et al. 2015), Plant location selection (Gupta et al. 2016), portfolio selection (Mousavi et al. 2016), pattern recognition (Mishra and Rani, 2017), material handling (Lou and Wang, 2017). IF-AHP and IF-VIKOR are used for selection hazardous waste carriers (Büyüközkan et al. 2019). As a result of the literature review, it seen that there is no study about the evaluation of neuroimaging techniques. We aim to contribute to literature with this study.

The rest of this paper is organized as follows. Next section presents steps of Integrated IF-AHP and TOPSIS method. Section Application presents the case study evaluation of neuroimaging techniques problem using integrated method. Section Comparison Analysis presents comparison analysis with IF-TOPSIS and IF-VIKOR. Final section concludes the paper by discussing key findings.

## 2. Methodology

Neuroergonomics is a discipline that seeks an answer to these questions "what are the cognitive boundaries of the human, what are the factors that form the mental workload of the employee, how this mental load can be reduced, how to ensure the efficiency in human-machine interaction, how to reduce employee errors, how to select personnel with cognitive characteristics to improve work efficiency or what are the works that personnel can work in terms of its cognitive characteristics? The increasing studies in the field of neuroergonomics may be explained by the following two reasons: Decrease in labor-intensive jobs and increase in jobs using technology-intensive cognition; development of undamaged neuroimaging techniques. By using neuroimaging techniques, it is possible to monitor which parts of the brain perform which functions during the activity of the individual and how the brain reacts to the changes in the working process. Neuroimaging techniques differ by criteria such as the measured material, device portability, the amount of restriction of the person, damage status, spatial and temporal resolution. In this study, neuroimaging techniques were evaluated using IF-AHP and IF-TOPSIS methods among multi-criteria decision-making methods for more than one criterion that contradict each other. This section presents the methodology used in the paper for selection the best neuroimaging technique problem. Following the introduction of IFS, the methodology and the steps of the integrated IF-AHP and IF-TOPSIS with GDM will be explained.

### *Neuroimaging Techniques*

MRI is a neuroimaging technique that does not use ionized radiation and that may obtain a three-dimensional image as well as that provides advantages such as taking sections of any region of the brain (deep inner region). It is used in many clini-cal and scientific applications in the field of psychiatry.
A relatively recent MRI technique called diffusion tensor imaging (DTI) is a tech-nique that uses MRI to target the diffusion of water molecules in axons forming the white

substance in the brain and allowing the calculation of fractional anisotropy (FA) (Mehta and Parasuraman, 2013).

fMRI is an undamaged neuroimaging technique used in measurement and map-ping of brain activity (Bandettini et al. 1992; Kwong et al. 1992). This method was used in studies such as neuroscience, psychiatry, pre-operative planning, and the response of the patient to the treatment or research on drug efficiency. The image in fMRI is obtained based on the magnetic properties between oxygenated hemoglobin (oxyhemoglobin) in the blood and non- oxygenated hemoglobin (deoxyhemoglobin) (Wise and Preston, 2010; Buxton and Frank, 1997). The technique is a method that does not require injection of a radioisotope or other pharmacological agent and is not very cost-effective (Ogawa et al. 1990; Wierenga and Bondi, 2007; Kim et al. 2010; Richards and Berninger, 2008).

fNIRs is a functional imaging method that allow direct or indirect measurement of brain activity by measuring changes in blood flow in the anterior part of the brain. It can be used in technical, biophysics, neurobiology, physics, psychology, engineer-ing and informatics. The operating mechanism is based on the measurement of changes in oxygenated and non-oxygenated hemoglobin concentrations in the blood through the rays (Lin et al. 2018). This method using the light to measure its effec-tiveness in the cerebral cortex (shell), is transformed into a hair band for use in every-day life. It has higher temporal resolution than fMRI.

EEG is a laboratory method that reflects the functional status of the brain rather than the structural features of the brain. Brain activity is measured by frequencies such as theta and alpha. This technique may also be used in operational environ-ments such as flight, air traffic control, road and rail transportation.

ERP is one of the techniques used to study the cognitive functions of the brain, based on the measurement of the electrophysiological markers of the brain.
TMS and tDCS are undamaged techniques that allow the temporary inhibition or activation of specific brain regions, thus examines the causal role of different brain regions in various cognitive functions (Walsh and Pascual-Leone, 2005; Mehta and Parasuraman, 2013).

In the study performed by Mehta and Parasuraman (2013), the techniques used in neuroimaging and the criteria and values to be taken into consideration according to them are given in Figure 1 and Table 1.

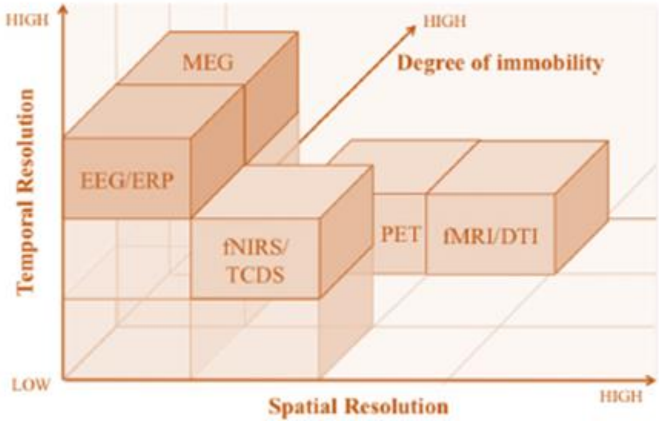


**Figure 1.** Comparison of some features of neuroimaging techniques (Mehta and Parasuraman, 2013)

In Figure 1, some neuroimaging techniques were compared for temporal, spatial resolution and inactivity. For example; the temporal resolution of EEG and ERP techniques and the spatial resolution of fMRI technique are higher than fNIRS and TCDS techniques. fNIRs portability is higher than fMRI technique.

**Table 1**. Neuroimaging techniques and features (Mehta and Parasuraman, 2013)

| Method | Measures | Portability | Cost | Spatial res. | Temporal res. |
|---|---|---|---|---|---|
| MRG | Grey matter volume | None | High | High | NA |
| DTI | White matter integrity | None | High | High | NA |
| fMRG | Relative blood oxygenation | None | High | High | Low |
| fNIRs | Oxyhemoglobin and deoxyhemoglobin | High | Low | Moderate | Low |
| EEG | Summated post-synaptic electrical activity | Moderate | Low | Low | High |
| ERP | Stimulus or response-related electrical activity | Moderate | Low | Low | High |
| TMS | Brain activation or inhibition | Low | Moderate | High | High |
| tDCS | Brain activation or inhibition | High | Very low | Low | Low |

In Table 1, unit/item, portability, costs, spatial and temporal resolutions measured according to the techniques were evaluated. For example; MRG technique measures gray matter volume. It is a technique with immovable, high cost, high spatial resolution and without temporal resolution.

### *Integrated IF-AHP and TOPSIS method*

This section first presents the basic definitions and notations of IFS, most of which are taken from Atanassov's study (Atanassov, 1999). In a finite set of X, IFS S can be stated as: $S = \{\langle x, \mu_s(x), v_s(x) \in \mathrm{X} \rangle\}$.

Here, $0 \le \mu_s(x) + v_s(x) \le 1$, $\mu_s(x), v_s(x) : X \to [0,1]$ is the membership function and the non-membership function respectively, so that, $0 \le \mu_s(x) + v_s(x) \le 1$.

In IFS, there is another parameter $\pi(x)$, called the intuitionistic fuzzy index or ‘‘hesitation degree” that checks if $x$ belongs to S, $\pi_s = 1 - \mu_s(x) - \nu_s(x)$. Here, for every $x \in X : 0 \leq \pi_s(x) \leq 1$. As $\pi_s(x)$ becomes smaller, the certainty of the knowledge about x becomes higher. As $\pi_s(x)$ gets higher, then the knowledge about x becomes less certain.

In this study, both IF-AHP and IF-TOPSIS techniques for GDM are proposed together for solving the neuroimaging techniques evaluation problem. The integrated IF-AHP and IF-TOPSIS consists of 11 main steps that are described briefly as follows (Büyüközkan and Güleryüz, 2016):

**Step 1:** Define objective, criteria, their sub-criteria and associated solution alternatives for the decision-making problem and then construct the hierarchy of the considered problem.

**Step 2:** Design and select the evaluation scale of IFS with AHP.
Table 2 shows the definition of linguistic terms and their equivalent forms in terms of IFS within the nine AHP linguistic scale and their intermediate values.

**Table 2**. Conversion of the AHP and TOPSIS preference into IFS (Abdullah & Najib, 2014a; Büyüközkan and Güleryüz, 2016)

| Definition of linguistic terms | 1-9 | IFS | Reciprocal IFS |
|---|---|---|---|
| **Equally important (EI)** | 1 | (0.02, 0.18,0.80) | (0.02, 0.18,0.80) |
| **Intermediate value (IV1)** | 2 | (0.06, 0.23,0.70) | (0.23, 0.06, 0.70) |
| **Moderately more important (MI)** | 3 | (0.13, 0.27,0.60) | (0.27, 0.13, 0.60) |
| **Intermediate value (IV2)** | 4 | (0.22, 0.28,0.50) | (0.28, 0.22, 0.50) |
| **Strongly more important (SI)** | 5 | (0.33, 0.27,0.40) | (0.27, 0.33, 0.40) |
| **Intermediate value (IV3)** | 6 | (0.47, 0.23,0.30) | (0.23, 0.47, 0.30) |
| **Very strong more important (VSI)** | 7 | (0.62, 0.18,0.20) | (0.18, 0.62, 0.20) |
| **Intermediate value (IV4)** | 8 | (0.80, 0.10, 0.10) | (0.10, 0.80, 0.10) |
| **Extremely more important (EMI)** | 9 | (1, 0,0) | (0, 1,0) |

**Step 3:** Establish the weights of DMs.
Importance degrees of these DMs are considered by using the IFS linguistic terms, as can be seen from Table 3. In this approach DMs’ importance degrees may change according to their experience and knowledge about the subject. In the light of these, let $D_k = [\mu_k, \nu_k, \pi_k]$ be an IF number as the rating of kth DM. Accordingly, the weight of kth DM can be calculated with Eq. (1):

$$\lambda_k = \frac{\left(\mu_k + \pi_k\left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)}{\sum_{k=1}^{l}\left(\mu_{k+}\pi_k\left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)} \tag{1}$$

**Table 3**. Linguistic terms for DMs' importance degrees (Boran et al., 2009; Büyüközkan and Güleryüz, 2016)

| Definition of linguistic terms | IFS |
|---|---|
| **Very important (VI)** | (0.90, 0.05, 0.05) |
| **Important (I)** | (0.75, 0.20, 0.05) |
| **Medium (M)** | (0.50, 0.40, 0.10) |
| **Unimportant (UNIMP)** | (0.25, 0.60, 0.15) |
| **Very Unimportant (VUNIMP)** | (0.10, 0.80, 0.10) |

**Step 4:** Determine DMs' intuitionistic preference relations. In this step first, the pair-wise comparison between each criterion and sub-criterion are obtained and then the intuitionistic preference relations are established. The importance degrees of each criteria can be shown with ''W'' and $\lambda = (\lambda_1, \lambda_2, ..., \lambda_l)$ is the weight of each DM and $\sum_{k=1}^{l} \lambda_k = 1, \quad \lambda_k \in [0,1]$. In this study, IFWA operator (Xu, 2007) is utilized to aggregate DMs' evaluations for rating the levels of importance for criteria and alternatives. Here, let $W_j^{(k)} = \left[\mu_j^{(k)}, v_j^{(k)}, \pi_j^{(k)}\right]$ be an IFS that is given by the kth DM to the criteria $X_j$. The aggregation process is done by using Eq. (2) and the criteria weights are calculated with the IFWA operator as follows:

$$W_j = IFWA_\lambda\left(W_j^{(1)}, W_j^{(2)}, ..., W_j^{(l)}\right) = \lambda_1 W_j^{(1)} \oplus \lambda_2 W_j^{(2)} \oplus \lambda_3 W_j^{(3)} \oplus \cdots \oplus \lambda_l W_j^{(l)}$$
$$= \left[1 - \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(v_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k}, -\prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k}\right]$$
$$W = \left[W_1, W_2, W_3, ..., W_j\right], \quad W_j = \left[\mu_j, v_j, \pi_j\right]\left(j = 1, 2, ..., n\right) \tag{2}$$

**Step 5:** Establish the aggregated weighted Intuitionistic Fuzzy (IF) decision matrix. Once the criteria weights (W) and the aggregated IF decision matrix are found, then the aggregated weighted IF decision matrix is formed as given below (Atanassov, 1986);

$$R \otimes W = \left\{x, \mu_{A_i}(x) \cdot \mu_w(x), v_{A_i}(x) + v_w(x) - v_{A_i}(x) \cdot v_w(x) \text{I} x \in X\right\} \tag{3}$$
$$\pi_{A_i} w(x) = 1 - v_{A_i}(x) - v_w(x) - \cdot \mu_{A_i}(x) \cdot \mu_w(x) + v_{A_i}(x) \cdot v_w(x) \tag{4}$$

And finally, the aggregated weighted IF decision matrix is obtained as follows:

$$R^* = \begin{bmatrix} \mu_{A_1} w(x_1), v_{A_1} w(x_1), \pi_{A_1} w(x_1) & \cdots & \mu_{A_1} w(x_n), v_{A_1} w(x_n), \pi_{A_1} w(x_n) \\ \mu_{A_2} w(x_1), v_{A_2} w(x_1), \pi_{A_2} w(x_1) & \cdots & \mu_{A_2} w(x_n), v_{A_2} w(x_n), \pi_{A_2} w(x_n) \\ \vdots & \ddots & \vdots \\ \mu_{A_m} w(x_1), v_{A_m} w(x_1), \pi_{A_m} w(x_1) & \cdots & \mu_{A_m} w(x_n), v_{A_m} w(x_n), \pi_{A_m} w(x_n) \end{bmatrix}$$

$$R' = \begin{bmatrix} r'_{11} & r'_{12} & r'_{13} & \cdots & r'_{1m} \\ r'_{21} & r'_{22} & r'_{23} & \cdots & r'_{2m} \\ r'_{31} & r'_{32} & r'_{33} & \cdots & r'_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r'_{n1} & r'_{n2} & r'_{n3} & \cdots & r'_{nm} \end{bmatrix} \tag{5}$$

$r'_{ij} = \left(\mu^*_{ij}, \nu^*_{ij}, \pi^*_{ij}\right) = \left(\mu_{A_iw}\left(x_j\right), \nu_{A_iw}\left(x_j\right), \pi_{A_iw}\left(x_j\right)\right)$ is an element of the AW-IF decision matrix.

**Step 6:** Check the consistency of each intuitionistic preference relation. To do so, the consistency ratio (CR) is calculated. Here, the aim is to estimate whether the pair-wise comparisons are consistent. If CR is less than 0.10, then it suggests that the comparisons are acceptable, otherwise (i.e. if CR is greater than 0.10) they are not acceptable and the values should be revised. Consistency value formula is shown in Eq. (6).

$$CR = \frac{\left(\lambda_{\max} - n\right)/\left(n-1\right)}{RI} \tag{6}$$

Assume that $\left(\lambda_{\max} - n\right)$ is the average of $\pi_x$ values, which is the aggregated IF matrix of each criterion. Here, n denotes the size of the matrix.

**Step 7:** Calculate IF entropy weights of the aggregated weighted IF decision matrix, as follows:

$$\overset{=}{w}_i = -\frac{1}{n \ln 2}\left[\mu_i \ln \mu_i + \nu_i \ln \nu_i - \left(1-\pi_i\right)\ln\left(1-\pi_i\right) - \pi_i \ln 2\right] \tag{7}$$

And the final entropy weights of each IF matrix are defined using the following Eq. (8):

$$w_i = \frac{1-\overset{=}{w}_i}{n - \sum_{j=1}^{n} \overset{=}{w}_i} \quad \text{where} \quad \sum_{j=1}^{n} \overset{=}{w}_i = 1 \tag{8}$$

**Step 8:** Integrate IF-TOPSIS into the aggregated weighted intuitionistic fuzzy decision matrix. Ranking in IF-TOPSIS requires preliminary values of the relative criteria importance, which can be extracted from the intuitionistic fuzzy decision matrix. After establishing necessary values, compute the distance values of the alternatives from the positive and negative ideal solution points. Assume that $J_1$ is the benefit criteria and $J_2$ denotes the cost criteria. With these, $A^+$ represents the IF positive ideal solution, whereas $A^-$ represents the IF negative ideal solution which can be found with the following equations:

$$A^+ = \left(\mu_{A^+W}\left(x_j\right), \nu_{A^+W}\left(x_j\right)\right) \; and \quad A^- = \left(\mu_{A^-W}\left(x_j\right), \nu_{A^-W}\left(x_j\right)\right) \tag{9}$$

Where;

$$\mu_{A^+W}\left(x_j\right)=\left(\left(\max_i \mu_{A_i \cdot W}\left(x_j\right) \mid j\in J_1\right),\left(\min_i \mu_{A_i \cdot W}\left(x_j\right) \mid j\in J_2\right)\right)$$
$$\nu_{A^+W}\left(x_j\right)=\left(\left(\min_i \nu_{A_i \cdot W}\left(x_j\right) \mid j\in J_1\right),\left(\max_i \nu_{A_i \cdot W}\left(x_j\right) \mid j\in J_2\right)\right)$$
$$\mu_{A^-W}\left(x_j\right)=\left(\left(\min_i \mu_{A_i \cdot W}\left(x_j\right) \mid j\in J_1\right),\left(\max_i \mu_{A_i \cdot W}\left(x_j\right) \mid j\in J_2\right)\right)$$
$$\nu_{A^-W}\left(x_j\right)=\left(\left(\max_i \nu_{A_i \cdot W}\left(x_j\right) \mid j\in J_1\right),\left(\min_i \nu_{A_i \cdot W}\left(x_j\right) \mid j\in J_2\right)\right) \quad (10)\text{-}(13)$$

**Step 9:** Calculate the separation measures of IF sets of the alternatives.
In this part, Hamming distance is utilized to measure separation measures of the alternatives. For each of the alternatives, the distances from the positive and negative ideal solution points are computed as follows (Xu, Z.S, 2007; Xu and Chen, 2008; Boran, F.E. 2011):

$$S^+=\frac{1}{2}\sum_{j=1}^{n}\left[\left|\mu_{A_i \cdot W}\left(x_j\right)-\mu_{A^+W}\left(x_j\right)\right|+\left|\nu_{A_i \cdot W}\left(x_j\right)-\nu_{A^+W}\left(x_j\right)\right|+\left|\pi_{A_iW}\left(x_j\right)-\pi_{A^+W}\left(x_j\right)\right|\right] \quad (14)$$

$$S^-=\frac{1}{2}\sum_{j=1}^{n}\left[\left|\mu_{A_i \cdot W}\left(x_j\right)-\mu_{A^-W}\left(x_j\right)\right|+\left|\nu_{A_i \cdot W}\left(x_j\right)-\nu_{A^-W}\left(x_j\right)\right|+\left|\pi_{A_iW}\left(x_j\right)-\pi_{A^-W}\left(x_j\right)\right|\right] \quad (15)$$

Here, $S_i^+$ represents the IF positive ideal solution and $S_i^-$ represents the IF negative ideal solution.

**Step 10:** Find the relative closeness coefficient $\left(CC_i\right)$ for the intuitionistic ideal solution. Here, this coefficient for an alternative $A_i$ with respect to $A^+$ can be calculated as below:

$$CC_i=\frac{S_{i-}}{S_{i-}+S_{i+}} \quad (16)$$

**Step 11:** Rank the alternatives in descending order according to their, $CC_i$ values.

In this method, the selected alternative is the one that has the highest $CC_i$ value. The aim here is to be as close as possible to the positive ideal solution point and at the same time to be as far as possible from the negative ideal solution point.

## 3. Application

The aim of the study was to evaluate neuroimaging techniques by considering multiple contradictory criteria. For this purpose, AHP and TOPSIS methods based on Intuitionistic fuzzy set were used in this study. 11 steps of the application are as follows:

**Step 1:** In this study, a decision group consisting of three experts from (academicians working in the field of neuroergonomics) are selected. A total of eight neuroimaging techniques were determined to be evaluated (MRI ($A_1$), DTI ($A_2$), fMRI ($A_3$), fNIRs ($A_4$), EEG ($A_5$), ERP ($A_6$), TMS ($A_7$) ve tDCS ($A_8$)). The criteria were determined by literature research and brainstorming with decision makers under two main headings, including economic and technical criteria. The hierarchy of criteria is shown in Figure 2.

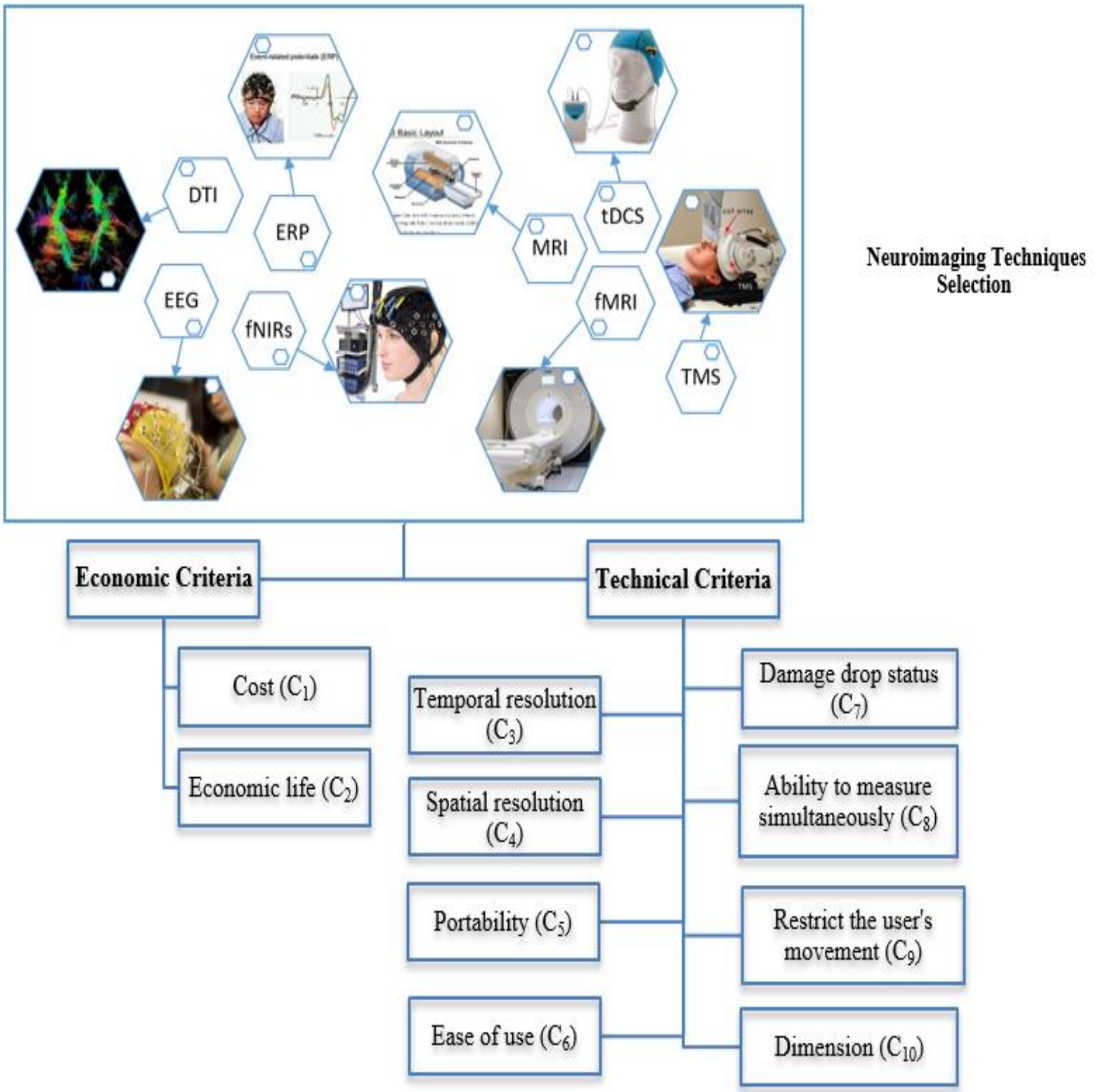


**Figure 2.** The hierarchy of criteria to be used in the evaluation of neuroimaging techniques

**Step 2:** In the decision process, DMs make pair-wise comparisons with the help of Table 2. Here, there are 9 evaluation scale with its intermediate values.

**Step 3:** Establish DMs weight. In this study there are three decision makers and the weights of DMs are determined by using Table 3 and Eq (1). The weights of DMs found to be 0.3125, 0.375 and 0.3125 (Table 4).

**Table 4**. Linguistic preference and weights of DMs

| DM1 | DM2 | DM3 |
|---|---|---|
| I | VI | I |
| 0.3125 | 0.375 | 0.3125 |

**Step 4:** Determine DM's intuitionistic preference relations using Table 2. In this step DMs respond to a series of comparison questions and they analyze alternatives and criteria according to their expertise. Due to space limitation all matrices cannot be given. As an example, 3 DM's evaluations of technical criteria and their IFS form are shown in Table 5.

**Table 5**. Linguistic evaluation matrix of DMs with respect to technical criteria

| **$DM_1$** | **$C_3$** | **$C_4$** | **$C_5$** | **$C_6$** | **$C_7$** | **$C_8$** | **$C_9$** | **$C_{10}$** |
|---|---|---|---|---|---|---|---|---|
| **$C_3$** | - | | | | | | | |
| **$C_4$** | IV2 | - | | | | | | |
| **$C_5$** | MI | TSI | - | | | | | |
| **$C_6$** | IV2 | TIV4 | TIV2 | - | | | | |
| **$C_7$** | VSI | IV4 | EMI | IV4 | - | | | |
| **$C_8$** | IV3 | TSI | TIV2 | VSI | TIV4 | - | | |
| **$C_9$** | TVSI | TEMI | TIV3 | TMI | TEMI | TIV3 | - | |
| **$C_{10}$** | TIV2 | TVSI | TIV3 | TIV1 | TEMI | TIV3 | SI | - |
| **$DM_2$** | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ | $C_8$ | $C_9$ | $C_{10}$ |
| **$C_3$** | - | | | | | | | |
| **$C_4$** | MI | - | | | | | | |
| **$C_5$** | IV1 | TSI | - | | | | | |
| **$C_6$** | IV2 | TVSI | TSI | - | | | | |
| **$C_7$** | SI | IV3 | IV4 | IV4 | - | | | |
| **$C_8$** | SI | TSI | IV1 | VSI | TIV3 | - | | |
| **$C_9$** | TSI | TIV4 | TIV2 | TMI | TVSI | TIV3 | - | |
| **$C_{10}$** | TSI | TIV3 | TIV1 | TMI | TVSI | TSI | SI | - |
| **$DM_3$** | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ | $C_8$ | $C_9$ | $C_{10}$ |
| **$C_3$** | - | | | | | | | |
| **$C_4$** | SI | - | | | | | | |
| **$C_5$** | MI | TIV3 | - | | | | | |
| **$C_6$** | IV2 | TVSI | TIV2 | - | | | | |
| **$C_7$** | IV3 | VSI | IV4 | IV4 | - | | | |
| **$C_8$** | IV3 | TSI | TIV2 | VSI | TIV4 | - | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **$C_9$** | TIV3 | TEMI | TSI | TMI | TIV4 | TSI | - | |
| **$C_{10}$** | TIV2 | TIV3 | TIV2 | TIV1 | TIV4 | TSI | IV2 | - |

**Step 5:** By using IFWA operator and Eq (2)-(4) kth DMs opinions are aggregated into a collective form (Table 6). As an example, aggregated μ, ν and π values for $DM_1$ are calculated as:

- $\mu$ ; $1-((1-0.02)^{0.3125} x\ (1-0.28)^{0.3125} x\ (1-0.27)^{0.3125} x\ (1-0.28)^{0.3125} x\ (1-0.18)^{0.3125} x\ (1-0.23)^{0.3125} x\ (1-0.62)^{0.3125} x\ (1-0.22)^{0.3125}) = 0.566$

- $\nu$ ; $0.18^{0.3125} x\ 0.22^{0.3125} x\ 0.13^{0.3125} x\ 0.22^{0.3125} x\ 0.62^{0.3125} x\ 0.47^{0.3125} x\ 0.18^{0.3125} x\ 0.628^{0.3125} = 0.032$

- $\pi$; $1-(0.566+0.032)=0.402$

**Table 6**. Aggregated IF judgment matrices of 3 DMs assessments with respect to technical criteria

| | **$DM_1$** | | | **$DM_2$** | | | **$DM_3$** | | |
|---|---|---|---|---|---|---|---|---|---|
| | μ | ν | π | μ | ν | π | μ | ν | π |
| **$C_3$** | 0.566 | 0.032 | 0.402 | 0.587 | 0.008 | 0.406 | 0.525 | 0.036 | 0.439 |
| **$C_4$** | 1.000 | 0.000 | 0.000 | 0.810 | 0.012 | 0.178 | 1.000 | 0.000 | 0.000 |
| **$C_5$** | 0.504 | 0.050 | 0.447 | 0.425 | 0.014 | 0.561 | 0.407 | 0.058 | 0.535 |
| **$C_6$** | 0.314 | 0.077 | 0.609 | 0.400 | 0.052 | 0.548 | 0.334 | 0.071 | 0.595 |
| **$C_7$** | 1.000 | 0.000 | 0.000 | 0.923 | 0.005 | 0.072 | 0.951 | 0.006 | 0.043 |
| **$C_8$** | 0.679 | 0.035 | 0.285 | 0.683 | 0.017 | 0.300 | 0.629 | 0.039 | 0.332 |
| **$C_9$** | 0.348 | 0.118 | 0.534 | 0.502 | 0.035 | 0.463 | 0.405 | 0.071 | 0.524 |
| **$C_{10}$** | 0.418 | 0.054 | 0.528 | 0.543 | 0.014 | 0.443 | 0.442 | 0.033 | 0.525 |

**Step 6:** The consistency is checked with in all matrices by using Eq (6). In order to ensure consistency, all the matrices should be checked one by one. As an example, the first matrix’ consistency is calculated as:

$$CR_1 = \frac{((0.402+0.000+0.447+0.609+0.000+0.285+0.534+0.528)/8)/7)}{1.41} = 0.036 < 0.10$$

indicating that the matrix is consistent. Here, RI=1.41. $CR_2$ and $CR_3$ are calculated the same way and found as 0.038.

**Step 7:** The intuitionistic fuzzy entropy weights of $DM_1$ are calculated with the help of Eq (7).

$$\bar{\bar{w}}_1 = -\frac{1}{8In2}\left[0.566*(In0.566)+0.032*(In0.032)-(1-0.402)*In(1-0.402)-0.402*In2\right]$$

$$= 0.073$$

$\bar{\bar{w}}_2, \bar{\bar{w}}_3, \bar{\bar{w}}_4, \bar{\bar{w}}_5, \bar{\bar{w}}_6, \bar{\bar{w}}_7, \bar{\bar{w}}_8$ are calculated the same way and found as 0.000, 0.086, 0.111, 0.000, 0.061, 0.114, 0.096 for DM1, respectively. The final entropy weights of $DM_1$ are calculated by using Eq (8):

$$w_1 = \frac{1-0.073}{8-(0.0173+0.000+0.086+0.111+0.000+0.061+0.114+0.096)} = 0.124\,.$$

$w_2, w_3, w_4, w_5, w_6, w_7, w_8$ are calculated the same way and found as 0.134, 0.123, 0.119, 0.134, 0.126, 0.119, 0.121. These are shown in Table 7.

**Table 7**. The technical criteria weights for the aggregated final entropy

| | **$DM_1$** | **$DM_2$** | **$DM_3$** | **Final Entropy** |
|---|---|---|---|---|
| **$C_3$'** | 0.124 | 0.125 | 0.124 | 0.125 |
| **$C_4$'** | 0.134 | 0.129 | 0.134 | 0.132 |
| **$C_5$'** | 0.123 | 0.122 | 0.121 | 0.122 |
| **$C_6$'** | 0.119 | 0.120 | 0.120 | 0.120 |
| **$C_7$'** | 0.134 | 0.131 | 0.133 | 0.133 |
| **$C_8$'** | 0.126 | 0.126 | 0.125 | 0.126 |
| **$C_9$'** | 0.119 | 0.122 | 0.121 | 0.121 |
| **$C_{10}$'** | 0.121 | 0.124 | 0.123 | 0.123 |

By using Eq (2), collective IF judgement matrices with regards to technical criteria are shown in Table 8. Here, GDM plays an important role in the decision process.

**Table 8**. The aggregated matrix for technical criteria

| | **μ** | **ν** | **π** |
|---|---|---|---|
| **$C_3$** | 0.562 | 0.020 | 0.419 |
| **$C_4$** | 1.000 | 0.000 | 0.000 |
| **$C_5$** | 0.445 | 0.032 | 0.522 |
| **$C_6$** | 0.354 | 0.065 | 0.581 |
| **$C_7$** | 1.000 | 0.000 | 0.000 |

| | | | |
|---|---|---|---|
| **$C_8$** | 0.666 | 0.028 | 0.306 |
| **$C_9$** | 0.427 | 0.064 | 0.509 |
| **$C_{10}$** | 0.475 | 0.028 | 0.497 |

The consistency should be checked and CR=0.036<0.1 indicating that the matrix is consistent.

In order to find final entropy weights of all criteria, all the matrices should be calculated one by one. The final entropy weights and the final evaluation criteria weights are summarized in Table 9.

**Table 9**. The final entropy and evaluation weights of criteria with IF-AHP

| Main Criteria | Weights | Sub-Criteria | Final entropy | Final C.W. |
|---|---|---|---|---|
| Economic | 0.482 | | | |
| | | $C_1$ | 0.506 | 0.244 |
| | | $C_2$ | 0.491 | 0.237 |
| Technical | 0.518 | | | |
| | | $C_3$ | 0.125 | 0.064 |
| | | $C_4$ | 0.132 | 0.068 |
| | | $C_5$ | 0.122 | 0.063 |
| | | $C_6$ | 0.120 | 0.062 |
| | | $C_7$ | 0.133 | 0.069 |
| | | $C_8$ | 0.126 | 0.065 |
| | | $C_9$ | 0.121 | 0.062 |
| | | $C_{10}$ | 0.123 | 0.064 |

As a result of the calculations, the criteria among the main criteria which had the highest weights was technical criteria. Among the technical criteria, the most significant ones were Damage drop status ($C_7$), Spatial resolution ($C_4$), Ability to measure simultaneously ($C_8$) and Temporal resolution ($C_3$), When the global weights of the sub-criteria were considered, the rank of priority were as follows: Cost ($C_1$), Economic life ($C_2$), Damage drop status ($C_7$), Spatial resolution ($C_4$).

**Step 8:** Integrate IF-TOPSIS into decision making problem by using Eq. (9)-(13). The DMs linguistic preferences are presented in Table 10. Aggregations of these values are provided in Table 11.

**Table 10**. DMs linguistic evaluation data used in IF-TOPSIS

| | **$A_1$** | | | **$A_2$** | | | **$A_3$** | | | **$A_4$** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **$DM_1$** | **$DM_2$** | **$DM_3$** | **$DM_1$** | **$DM_2$** | **$DM_3$** | **$DM_1$** | **$DM_2$** | **$DM_3$** | **$DM_1$** | **$DM_2$** | **$DM_3$** |
| $C_1$ | IV4 | EMI | IV4 | IV4 | VSI | VSI | IV4 | IV4 | IV4 | EI | EI | IV1 |
| $C_2$ | IV4 | EMI | IV4 | IV4 | VSI | VSI | IV4 | IV4 | IV4 | EI | EI | IV1 |
| $C_3$ | EI | EI | EI | EI | EI | EI | MI | IV1 | MI | IV2 | MI | IV2 |
| $C_4$ | VSI | IV4 | VSI | VSI | IV4 | VSI | VSI | IV4 | VSI | MI | IV2 | MI |
| $C_5$ | EI | EI | EI | EI | EI | EI | EI | EI | EI | VSI | IV4 | VSI |
| $C_6$ | IV4 | IV4 | EMI | MI | IV2 | MI | IV1 | MI | IV1 | IV1 | MI | IV1 |
| $C_7$ | IV1 | MI | IV1 | IV1 | MI | IV1 | EI | IV1 | EI | EI | IV1 | EI |
| $C_8$ | IV4 | EMI | IV4 | VSI | VSI | IV4 | VSI | VSI | IV4 | VSI | VSI | IV4 |
| $C_9$ | IV4 | EMI | IV4 | IV3 | VSI | VSI | IV4 | EMI | IV4 | EI | IV1 | EI |
| $C_{10}$ | IV4 | EMI | IV4 | IV2 | IV2 | SI | VSI | IV4 | VSI | EI | IV1 | EI |
| | **$A_5$** | | | **$A_6$** | | | **$A_7$** | | | **$A_8$** | | |
| | **DM1** | **DM2** | **DM3** | **DM1** | **DM2** | **DM3** | **DM1** | **DM2** | **DM3** | **DM1** | **DM2** | **DM3** |
| $C_1$ | IV1 | IV1 | EI | EI | IV1 | IV1 | MI | IV2 | MI | EI | EI | EI |
| $C_2$ | IV1 | IV1 | EI | EI | IV1 | IV1 | MI | IV2 | MI | EI | EI | EI |
| $C_3$ | VSI | IV3 | VSI | VSI | IV3 | VSI | IV4 | VSI | IV4 | IV1 | EI | IV1 |
| $C_4$ | IV1 | IV1 | EI | IV1 | IV1 | EI | VSI | IV4 | VSI | IV1 | IV1 | EI |
| $C_5$ | MI | IV2 | MI | MI | IV2 | MI | MI | IV1 | IV1 | VSI | IV4 | IV4 |
| $C_6$ | MI | IV1 | MI | IV4 | IV4 | EMI | VSI | IV4 | VSI | IV3 | VSI | IV3 |
| $C_7$ | EI | IV1 | EI | EI | IV1 | EI | EI | IV1 | EI | EI | IV1 | IV1 |
| $C_8$ | EI | IV1 | IV1 | EI | IV1 | IV1 | MI | IV2 | MI | EI | IV1 | IV1 |
| $C_9$ | VSI | IV4 | VSI | VSI | IV4 | VSI | IV2 | SI | IV2 | MI | IV1 | MI |
| $C_{10}$ | EI | IV1 | IV1 | VSI | IV3 | VSI | SI | IV3 | SI | EI | IV1 | IV1 |

**Table 11**. Aggregated μ, ν and π values of DMs assessments with respect to alternatives

| | **$A_1$** | | | **$A_2$** | | | **$A_3$** | | | **$A_4$** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **μ** | **ν** | **π** | **μ** | **ν** | **π** | **μ** | **ν** | **π** | **μ** | **ν** | **π** |
| $C_1$ | 1.000 | 0.000 | 0.000 | 0.689 | 0.220 | 0.090 | 0.800 | 0.100 | 0.100 | 0.085 | 0.194 | 0.721 |
| $C_2$ | 1.000 | 0.000 | 0.000 | 0.689 | 0.220 | 0.090 | 0.800 | 0.100 | 0.100 | 0.085 | 0.194 | 0.721 |
| $C_3$ | 0.020 | 0.180 | 0.800 | 0.020 | 0.091 | 0.889 | 0.104 | 0.254 | 0.641 | 0.207 | 0.276 | 0.516 |
| $C_4$ | 0.701 | 0.144 | 0.154 | 0.701 | 0.213 | 0.086 | 0.701 | 0.144 | 0.154 | 0.209 | 0.274 | 0.517 |
| $C_5$ | 0.020 | 0.180 | 0.800 | 0.020 | 0.091 | 0.889 | 0.020 | 0.180 | 0.800 | 0.620 | 0.144 | 0.235 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_6$ | 1.000 | 0.000 | 0.000 | 0.165 | 0.218 | 0.617 | 0.087 | 0.244 | 0.669 | 0.142 | 0.244 | 0.614 |
| $C_7$ | 0.087 | 0.244 | 0.669 | 0.087 | 0.160 | 0.753 | 0.035 | 0.197 | 0.767 | 0.087 | 0.197 | 0.715 |
| $C_8$ | 1.000 | 0.000 | 0.000 | 0.689 | 0.287 | 0.024 | 0.689 | 0.150 | 0.161 | 0.605 | 0.150 | 0.246 |
| $C_9$ | 1.000 | 0.000 | 0.000 | 0.578 | 0.286 | 0.136 | 1.000 | 0.000 | 0.000 | 0.087 | 0.197 | 0.715 |
| $C_{10}$ | 1.000 | 0.000 | 0.000 | 0.256 | 0.295 | 0.449 | 0.701 | 0.144 | 0.154 | 0.087 | 0.197 | 0.715 |

| | $A_5$ | | | $A_6$ | | | $A_7$ | | | $A_8$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | μ | ν | π | μ | ν | π | μ | ν | π | μ | ν | π |
| $C_1$ | 0.048 | 0.213 | 0.739 | 0.048 | 0.213 | 0.739 | 0.165 | 0.274 | 0.561 | 0.020 | 0.180 | 0.800 |
| $C_2$ | 0.048 | 0.213 | 0.739 | 0.048 | 0.213 | 0.739 | 0.165 | 0.274 | 0.561 | 0.020 | 0.180 | 0.800 |
| $C_3$ | 0.570 | 0.197 | 0.233 | 0.570 | 0.197 | 0.233 | 0.746 | 0.125 | 0.130 | 0.045 | 0.210 | 0.745 |
| $C_4$ | 0.048 | 0.213 | 0.739 | 0.048 | 0.213 | 0.739 | 0.701 | 0.144 | 0.154 | 0.048 | 0.213 | 0.739 |
| $C_5$ | 0.165 | 0.274 | 0.561 | 0.165 | 0.274 | 0.561 | 0.082 | 0.242 | 0.676 | 0.756 | 0.120 | 0.124 |
| $C_6$ | 0.104 | 0.254 | 0.641 | 1.000 | 0.000 | 0.000 | 0.701 | 0.144 | 0.154 | 0.532 | 0.210 | 0.258 |
| $C_7$ | 0.035 | 0.197 | 0.767 | 0.048 | 0.213 | 0.739 | 0.060 | 0.230 | 0.710 | 0.060 | 0.230 | 0.710 |
| $C_8$ | 0.048 | 0.213 | 0.739 | 0.048 | 0.213 | 0.739 | 0.165 | 0.274 | 0.561 | 0.048 | 0.213 | 0.739 |
| $C_9$ | 0.701 | 0.144 | 0.154 | 0.701 | 0.144 | 0.154 | 0.263 | 0.276 | 0.461 | 0.104 | 0.254 | 0.641 |
| $C_{10}$ | 0.048 | 0.213 | 0.739 | 0.570 | 0.197 | 0.233 | 0.386 | 0.254 | 0.359 | 0.048 | 0.213 | 0.739 |

By using aggregated matrix in Table 6, the weighted decision matrix is calculated in Table 12.

**Table 12**. Weighted decision matrix

| | $A_1$ | | | $A_2$ | | | $A_3$ | | | $A_4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | μ | ν | π | μ | ν | π | μ | ν | π | μ | ν | π |
| $C_1$ | 0.059 | 0.149 | 0.792 | 0.040 | 0.337 | 0.623 | 0.047 | 0.234 | 0.719 | 0.005 | 0.315 | 0.680 |
| $C_2$ | 0.101 | 0.109 | 0.791 | 0.069 | 0.305 | 0.625 | 0.080 | 0.198 | 0.722 | 0.009 | 0.282 | 0.709 |
| $C_3$ | 0.011 | 0.196 | 0.793 | 0.011 | 0.108 | 0.880 | 0.059 | 0.269 | 0.672 | 0.117 | 0.290 | 0.593 |
| $C_4$ | 0.701 | 0.144 | 0.154 | 0.701 | 0.213 | 0.086 | 0.701 | 0.144 | 0.154 | 0.209 | 0.274 | 0.517 |
| $C_5$ | 0.009 | 0.207 | 0.785 | 0.009 | 0.120 | 0.871 | 0.009 | 0.207 | 0.785 | 0.276 | 0.172 | 0.552 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_6$ | 0.354 | 0.065 | 0.581 | 0.058 | 0.268 | 0.673 | 0.031 | 0.293 | 0.676 | 0.050 | 0.293 | 0.657 |
| $C_7$ | 0.087 | 0.244 | 0.669 | 0.087 | 0.160 | 0.753 | 0.035 | 0.197 | 0.767 | 0.087 | 0.197 | 0.715 |
| $C_8$ | 0.666 | 0.028 | 0.306 | 0.459 | 0.307 | 0.235 | 0.459 | 0.173 | 0.368 | 0.403 | 0.173 | 0.424 |
| $C_9$ | 0.427 | 0.064 | 0.509 | 0.247 | 0.332 | 0.421 | 0.427 | 0.064 | 0.509 | 0.037 | 0.248 | 0.714 |
| $C_{10}$ | 0.475 | 0.028 | 0.497 | 0.122 | 0.315 | 0.564 | 0.333 | 0.168 | 0.498 | 0.042 | 0.220 | 0.739 |

| | **$A_5$** | | | **$A_6$** | | | **$A_7$** | | | **$A_8$** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **μ** | **ν** | **π** | **μ** | **ν** | **π** | **μ** | **ν** | **π** | **μ** | **ν** | **π** |
| $C_1$ | 0.003 | 0.512 | 0.486 | 0.003 | 0.331 | 0.667 | 0.010 | 0.382 | 0.608 | 0.001 | 0.302 | 0.696 |
| $C_2$ | 0.005 | 0.543 | 0.452 | 0.005 | 0.299 | 0.697 | 0.017 | 0.353 | 0.631 | 0.002 | 0.269 | 0.729 |
| $C_3$ | 0.320 | 0.213 | 0.467 | 0.320 | 0.213 | 0.467 | 0.419 | 0.142 | 0.439 | 0.025 | 0.225 | 0.749 |
| $C_4$ | 0.048 | 0.213 | 0.739 | 0.048 | 0.213 | 0.739 | 0.701 | 0.144 | 0.154 | 0.048 | 0.213 | 0.739 |
| $C_5$ | 0.073 | 0.297 | 0.629 | 0.073 | 0.297 | 0.629 | 0.037 | 0.266 | 0.697 | 0.337 | 0.149 | 0.515 |
| $C_6$ | 0.037 | 0.303 | 0.661 | 0.354 | 0.065 | 0.581 | 0.248 | 0.200 | 0.552 | 0.188 | 0.261 | 0.551 |
| $C_7$ | 0.035 | 0.197 | 0.767 | 0.048 | 0.213 | 0.739 | 0.060 | 0.230 | 0.710 | 0.060 | 0.230 | 0.710 |
| $C_8$ | 0.032 | 0.235 | 0.733 | 0.032 | 0.235 | 0.733 | 0.110 | 0.294 | 0.596 | 0.032 | 0.235 | 0.733 |
| $C_9$ | 0.300 | 0.199 | 0.502 | 0.300 | 0.199 | 0.502 | 0.112 | 0.322 | 0.565 | 0.045 | 0.302 | 0.654 |
| $C_{10}$ | 0.023 | 0.235 | 0.742 | 0.271 | 0.220 | 0.509 | 0.184 | 0.275 | 0.541 | 0.023 | 0.235 | 0.742 |

**Step 9:** The separation measures are calculated by using Eq (14)-(15). For each of the alternatives their distance from the positive and negative ideal solution points are calculated, the results of which are given in Table 13.

**Step 10:** By using Eq (16), the $CC_i$ is calculated.

**Step 11:** The alternatives are ranked in descending order, based on their $CC_i$ values. The selected alternative should have the highest $CC_i$ value. These are shown in Table 13.

**Table 13**. Separation measures and relative closeness values for the alternatives

| Alternatives | $S_{i-}$ | $S_{i+}$ | $CC_i$ | Ranking |
|---|---|---|---|---|
| $A_1$ | 2.314 | 1.992 | 0.537 | 3 |
| $A_2$ | 2.521 | 2.142 | 0.541 | 2 |
| $A_3$ | 1.852 | 2.332 | 0.443 | 6 |
| $A_4$ | 2.211 | 2.021 | 0.522 | 4 |
| $A_5$ | 1.517 | 2.797 | 0.352 | 8 |
| $A_6$ | 1.647 | 2.568 | 0.391 | 7 |
| $A_7$ | 2.486 | 1.659 | 0.600 | 1 |

| | | | | |
|---|---|---|---|---|
| $A_8$ | 2.014 | 2.409 | 0.455 | 5 |

As a result of IF-AHP&IF-TOPSIS integrated approach, alternative $A_7$ (TMS) was chosen as the best alternative. TMS, which allows for the temporary inhibition or activation of certain brain regions. Thus, it is an undamaged technique that examines the causal role of different brain regions in various cognitive functions. The TMS is more advantage technique in comparison to the other methods in terms of spatial resolution and temporal resolution.

## 4. Comparison Analysis

The comparison analysis was performed using IF-VIKOR method. The reason for selecting VIKOR method was that it involves sensitivity analysis in its steps and TOPSIS and VIKOR methods have similar working principles. The steps of IF-VIKOR method is taken from Büyüközkan et al. (2019). The positive and negative ideal solution were found. Then, the group utility value and individual regret value were found and were shown in Table 14.

**Table 14**. Separation measures and relative closeness values for the alternatives (IF-VIKOR)

| | $S(A_i)$ | $R(A_i)$ |
|---|---|---|
| $A_1$ | 0.606 | 0.301 |
| $A_2$ | 0.644 | 0.222 |
| $A_3$ | 0.644 | 0.244 |
| $A_4$ | 0.501 | 0.226 |
| $A_5$ | 0.605 | 0.234 |
| $A_6$ | 0.556 | 0.234 |
| $A_7$ | 0.443 | 0.199 |
| $A_8$ | 0.537 | 0.244 |

The degree of closeness coefficient for each of the alternatives was calculated. Each alternative’s distance to the ideal solution was shown in Table 15.

**Table 15**. The degree of closeness coefficients of alternatives with different α values

| $Q(A_i)$ | α | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $A_1$ | 1.000 | 0.981 | 0.962 | 0.943 | 0.924 | 0.906 | 0.887 | 0.868 | 0.849 | 0.830 | 0.811 |
| | $A_2$ | 0.218 | 0.296 | 0.374 | 0.453 | 0.531 | 0.609 | 0.687 | 0.765 | 0.844 | 0.922 | 1.000 |
| | $A_3$ | 0.438 | 0.494 | 0.550 | 0.606 | 0.662 | 0.718 | 0.774 | 0.830 | 0.887 | 0.943 | 0.999 |
| | $A_4$ | 0.259 | 0.262 | 0.265 | 0.268 | 0.271 | 0.275 | 0.278 | 0.281 | 0.284 | 0.287 | 0.290 |
| | $A_5$ | 0.340 | 0.387 | 0.433 | 0.479 | 0.526 | 0.572 | 0.618 | 0.665 | 0.711 | 0.757 | 0.804 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_6$ | 0.340 | 0.363 | 0.385 | 0.407 | 0.429 | 0.451 | 0.474 | 0.496 | 0.518 | 0.540 | 0.563 |
| $A_7$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $A_8$ | 0.441 | 0.444 | 0.447 | 0.450 | 0.453 | 0.455 | 0.458 | 0.461 | 0.464 | 0.467 | 0.470 |

A set of compromise solutions for the best suitable choice for neuroimaging techniques was found as sorting of alternatives in decreasing order for $Q(A_i)$ (Table 16). For different α values, the ranking was changed.

**Table 16**. Ranking order of alternatives as α values

| α | Ranking order |
|---|---|
| 0.00 | $A_7>A_2>A_4>A_6>A_5>A_3>A_8>A_1$ |
| 0.10 | $A_7>A_4>A_2>A_6>A_5>A_8>A_3>A_1$ |
| 0.20 | $A_7>A_4>A_2>A_6>A_5>A_8>A_3>A_1$ |
| 0.30 | $A_7>A_4>A_6>A_8>A_2>A_5>A_3>A_1$ |
| 0.40 | $A_7>A_4>A_6>A_8>A_5>A_2>A_3>A_1$ |
| 0.50 | $A_7>A_4>A_6>A_8>A_5>A_2>A_3>A_1$ |
| 0.60 | $A_7>A_4>A_8>A_6>A_5>A_2>A_3>A_1$ |
| 0.70 | $A_7>A_4>A_8>A_6>A_5>A_2>A_3>A_1$ |
| 0.80 | $A_7>A_4>A_8>A_6>A_5>A_2>A_1>A_3$ |
| 0.90 | $A_7>A_4>A_8>A_6>A_5>A_1>A_2>A_3$ |
| 1.00 | $A_7>A_4>A_8>A_6>A_5>A_1>A_3>A_2$ |

VIKOR method has two conditions. If these conditions are satisfied, obtained solution by using index is compromise solution.

- **Condition 1:** Acceptable advantage, because of $Q(A_1)-Q(A_7)\geq DQ=1/(8-1)=0.143$, the condition 1 is satisfied.
- **Condition 2:** Acceptable stability, the best alternative $A_7$ which was used rank according to index, in addition, if best alternative $A_7$ which were used rank according to $S(A_i)$ and $R(A_i)$ index, this conciliatory solution stable decide process.

As a result of IF-AHP & TOPSIS method, the best alternative $A_7$ selected was the first in every ranking and as a result of managing of admission requirements although the alternative rankings change for different α values in VIKOR method.

## 5. Conclusion and Discussion

Neuroergonomics enables to monitor which parts of the brain perform which functions during the activity of the individual by using neuroimaging techniques, how the brain reacts to the changes in the working process. With the examination of the human brain, questions such as "what are the cognitive boundaries of human, what are the factors that form the mental workload of the employee, how this mental load can be reduced, how to ensure the efficiency in human-machine interaction, how to reduce employee errors" may be answered. For these reasons, it is quite important to know neuroimaging techniques well and use them correctly. This issue becomes popular with the development of undamaged neuroimaging techniques. Neuroimaging techniques differ by criteria such as the measured material, device portability, the amount of restriction of the person, damage status, spatial and temporal resolution.

The aim of this study is to evaluate the neuroimaging techniques used in neuroergonomics by using AHP and TOPSIS methods based on Intuitionistic Fuzzy Set (IFS). There is no study on the evaluation of neuroimaging techniques used in neuroergonomics by using any of the multi criteria decision making methods. Therefore, this study is an original study contributing to literature.

The best alternative $A_7$ was chosen as a result of IF-AHP & TOPSIS and IF-AHP & VIKOR calculations. Non-invasive TMS ($A_7$) technique with high spatial and temporal resolution is highly effective in determining as the best alternative.

The study has some limitations: There is no research paper in the field of neurergonomics in Turkey as a result of the literature search. There are only a few review studies. It is difficult to find data because it is also difficult to find an expert working in this field and has a knowledge of techniques. Exact real values could not be used in the decision matrices since there is no experimental data. Intuitionistic Fuzzy Set based decision making methods were used for this reason. IFS provides information on the membership, non-membership and hesitancy functions. It is useful tool to deal with uncertainty and fuzziness of complex problems. Because of these features and obtain to a more complete evaluation and more precise results. IF-AHP&TOPSIS method is appropriate for the problem of the neuroimaging techniques selection.

In future studies, it is thought that neuroimaging techniques will be evaluated by using other MCDM methods by increasing the number of alternative/criteria. Moreover, in future studies, techniques shall be evaluated by using MCDM techniques based on Hesitant Fuzzy Set that allows the decision maker to express his/her opinion for uncertain problems by using more than one term, not a single term.

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